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A FRAMEWORK FOR DEFAULT LOGIC WITH CASE REASONING*,**

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Abstract Brewka's framework for derault logic is extended such that this new framework is possessed of more powerful expressivity when reasoning by cases. Under this extended framework a characterization of extensions of a default theory is given. From this characterization the algorithms for main reasoning tasks in default logic with case reasoning are derived.

Key words default logic, compatible class of sets of defaults, reasoning by cases, characterization, algorithm.

Brewka[1] pointed out that Reiter's default logic (DL)[2] has its drawbacks: existence of extensions is not guaranteed; justifications applied within one extension are not joint consistent with generated extension; the use of inference rules as defaults makes it impossible to reason by cases; DL's skeptical inference relation is not cumulative. He also showed that his cumulative default logic (CDL)[3] solved problems of joint consistency and cumulativity, but introduced a new problem——the floating conclusion problem. To overcome these drawbacks he further extended the idea underlying CDL and presented a framework for default reasoning[1]. In this approach defaults are elements of the logic language, but not inference rules; and reasoning about defaults can be performed. Specifically, he used Reiter's defaults of form $\alpha:\beta_1,\cdots,\beta_n/\gamma$ as the elements of the framework language, where α, β_i and γ are closed first order formulas. DEF is the set of all defaults. A default theory(DT) T is a subset of DEF, i.e. $T\in P(\mathrm{DEF})$. Defaults of forms true: true/γ and true: $\beta_1,\cdots,\beta_n/\gamma$ are called facts and assumables respectively. We will often use γ as shorthand for the fact true: true/γ as well as $\beta_1,\cdots,\beta_n/\gamma$ for the assumable true: $\beta_1,\cdots,\beta n/\gamma$.

A default logic is a pair(R, Ext), where R is a set of(monotonic) inference rules on defaults; and Ext: $P(\mathrm{DEF})\mapsto P(P(\mathrm{DEF}))$ is a function mapping a set of defaults D to a set of subsets of D, the extension of D.

Extensions of the closure $C_R(T)$ will also simple be called extensions of T.

To illustrate the expressiveness of this framework Brewka introduced two sets R_{DL} and R_{L_1} of rules to define logics L_{DL} and L_1 respectively. Brewka also showed that drawbacks mentioned by him had been avoided. Unfortunately, Brewka's framework still has some drawbacks when reasoning by cases. Consider the following

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example:

Example 1.

(i) emu: runs/runs;

(ii) emu ∨ runs.

It seems natural to obtain the derivation of the assumable runs/runs in the single L_1 extension. But this is impossible in Brewka's framework.

Before now a lot of authors (e.g. refs. [4,5]) tried to include reasoning by cases in default logic. They modified Reiter's definition of extensions by allowing sets of defaults instead of isolated defaults only. Nic[4] gave the modified definition of extensions for a default theory. It is easy to see that the idea underlying Nic's modified definition is similar to that in Brewka's framework. So the same drawback as that shown in Example 1 has not been overcome yet. In this paper we modify Brewka's set R_{L_1} of inference rules by adding Prerequisites Rule, whose special case is Tautologies Rule. Then, in such a version of default logic (denoted as DDL), a characterization of DDL extensions under modified set R_{DDL} of inference rules is given. Moreover, algorithms for some main reasoning tasks in DDL are got. Finally, we compare our work with related ones to show that our work is not only independent but also satisfactory.

1 Characterization of extensions and algorithms

In this section we give a modified set of inference rules on defaults and a characterization of extensions of default theories.

Definition 1.1. For any set of defaults D we define

$$\mathrm{ASS}(D) = \{\alpha:\beta_1,\cdots,\beta_n/\gamma \in D \mid \alpha = \mathrm{true}\},$$

$$\mathrm{KERN}(D) = \{d \in D \mid d = \mathrm{true}:\mathrm{true}/\gamma\}.$$

Let $\dot{D}$ be a class of sets of defaults. We define

$$\mathrm{PRE}(\dot{D}) = \{\alpha^1 \vee \cdots \vee \alpha^i \vee \cdots \mid \{\alpha^i:\beta_1^i,\cdots\beta_{n(i)}^i/\gamma^i \mid i \geq 1\} \in \dot{D}\},$$

$$\mathrm{CON}(\dot{D}) = \{\gamma^1 \vee \cdots \vee \gamma^i \vee \cdots \mid \{\alpha^i:\beta_1^i,\cdots\beta_{n(i)}^i/\gamma^i \mid i \geq 1\} \in \dot{D}\},$$

$$\mathrm{CCS}(\dot{D}) = \{\beta_1^1,\cdots,\beta_{n(1)}^1,\cdots,\beta_1^i,\cdots\beta_{n(i)}^i,\cdots, \mid \{\alpha^i:\beta_1^i,\cdots,\beta_{n(i)}^i/\gamma^i \mid i \geq 1\} \in (\dot{D})\}.$$

In particular, we view a set of defaults D as the class $\{\{d\} \mid d \in D\}$ of sets with only one default d when using notations , PRE(D), CCS(D) and CON(D).

Definition 1.2. Let F_1 and F_2 be any set of closed formulas. $F_1 \equiv F_2$ if and only if:

(i) for any $\alpha \in F_1$, α = true or there exists $\beta \in F_2$ such that $\alpha \equiv \beta$;

(*ii*) *for any* $\beta \in F_2$, β = true or there exists $\alpha \in F_1$ such that $\beta \equiv \alpha$ where $\alpha \equiv \beta$ if and only if $\alpha \vdash \beta$ and $\beta \vdash \alpha$.

Definition 1.3. R_{DDL} consists of the following inference rules:

DR_0 prerequisites:

$\Rightarrow \alpha:\mathrm{true}/\alpha$ where α = true or α is any closed formula;

DR_1 weakening:

$\mathrm{true}:\beta_1,\cdots,\beta_n/\gamma, \gamma \vdash \sigma \Rightarrow \mathrm{true}:\beta_1,\cdots,\beta_n/\sigma$;

DR_2 combination:

$\mathrm{ture}:\beta_1,\cdots,\beta_m/\gamma_1, \mathrm{ture}:\beta_{m+1},\cdots,\beta_n/\gamma_2 \Rightarrow \mathrm{ture}:\beta_1,\cdots,\beta_n/\gamma_1 \wedge \gamma_2$;

DR_3 chaining:

true: $\beta_1, \cdots, \beta_m/\gamma, \gamma: \beta_{m+1}, \cdots, \beta_n/\delta \Rightarrow$ true: $\beta_1, \cdots, \beta_n/\delta$;

DR_4 equivalence:

$\alpha: \beta_1, \cdots, \beta_n/\gamma, \alpha \equiv \sigma, \{\beta_1, \cdots, \beta_n\} \equiv \{\tau_1, \cdots, \tau_m\} \Rightarrow \sigma: \tau_1, \cdots, \tau_m/\gamma$;

DR_5 disjoining:

true: $\beta_1, \cdots, \beta_m/\gamma$, true: $\tau_1, \cdots, \tau_n/\gamma \Rightarrow$ true: $\beta_1 \vee \tau_1, \cdots, \beta_1 \vee \tau_n, \cdots, \beta_m \vee \tau_1, \cdots, \beta_m \vee \tau_n/\gamma$;

DR_6 cases:

$\alpha: \beta_1, \cdots, \beta_m/\gamma, \sigma: \beta_{m+1}, \cdots, \beta_n/\delta \Rightarrow \alpha \vee \sigma: \beta_1, \cdots, \beta_n/\gamma \vee \delta$.

In order to eliminate the drawback shown in Example 1 we try to define an extension by means of the set R_{DDL}. Let T be a default theory, C the closure of T under R_{DDL} and E a set of assumables. We say E is a DL^* extension of T if and only if (i) $E \in$ ASS(C) and (ii) for each $d = \beta_1, \cdots, \beta_n/\gamma \in$ ASS(C): $d \in E$ if and only if CON(E) $\cup \{\beta_i\}$ is consistent for all $i: 1 \leqslant i \leqslant n$.

Returning to Example 1, it is clear that the assumable runs/runs appears in the only DL^* extension. In fact, we can apply DR_0 and get (iii) runs: true/runs. Application of DR_6 to (i) and (iii) leads to (after equivalence transformation of using DR_4) (iv) emu $\vee$ runs: runs/runs. Similarly we apply DR_3 to (ii) and (iv) and obtain (v) runs/runs.

However we find that the rule DR_0 seems to be too strong, since it makes some unwanted results occur.

Example 2. Consider the default theory $D = \{\alpha: \beta/\gamma\}$. We get a DL^* extension containing $\beta/\neg\alpha \vee \gamma$ even if D does not contain any fact. The assumable $\beta/\neg\alpha \vee \gamma$ seems to be an unwanted answer since it would lead to the unexpected contraposition $\beta/\neg\alpha$ when adding the default (fact) $\neg\gamma$. This drawback is due to the application of rule DR_0: two defaults $\alpha: \beta/\gamma$ and $\neg\alpha: /\neg\alpha$ should be used to generate the assumable $\beta/\neg\alpha \vee \gamma$.

Generally, we think that α should be in CON(D) of a considered default theory D for additional defaults with forms $\alpha: /\alpha$. Thus we modify rule DR_0 in the following way when considering an extension of a default theory T.

DR_0^*:

$\Rightarrow \alpha$: true/α where α = true or $\alpha \in$ CON(T).

So we get the set $R_{DDL}(T)$ of inference rules for any given default theory T when replacing rule DR_0 by DR_0^* in R_{DDL}.

Definition 1.4. Let T be a default theory, C the closure of T under $R_{DDL}(T)$ and E a set of assumables.

(i) (DDL logic) E is a DDL extension of T if and only if:

(a) $E \subseteq$ ASS(C).

(b) For each $d = \beta_1, \cdots, \beta_n/\gamma \in$ ASS(C), $d \in E$ if and only if CON(E) $\cup \{\beta_i\}$ is consistent for all $i: 1 \leqslant i \leqslant n$.

(ii) (DL_1 logic) E is a DL_1 extension of T if and only if E is a maximal subset of C such that:

(a) $E \subseteq$ ASS(C).

(b) KERN(C) $\subseteq E$.

(c) CCS(E)$\cup$CON(E) is consistent.

Since we are only interested in existence of extensions of default theories, we always suppose that CON ($KERN(T)$) is consistent for any default theory T through this paper. To characterize extensions of default theories we need the following definitions:

Definition 1.5. A class $\dot{D}\in P(P(\mathrm{DEF}))$ of sets of defaults is compatible if and only if CON($\dot{D}$) $\nvdash\neg\beta$ for any $\beta\in$ CCS($\dot{D}$). $\dot{D}$ is joint compatible if and only if CCS($\dot{D}$)$\cup$CON($\dot{D}$) is consistent.

Let D be any set of defaults. We denote $D\cup\{\alpha:\mathrm{true}/\alpha\mid\alpha\in\mathrm{CON}(D)\}$ as $TP(D)$.

Definition 1.6. $\Xi$and $\Lambda: P(DEF)\mapsto P(P(\mathrm{DEF}))$ are two operators mapping a set of defaults D to a class of subsets of $TP(D)$ such that:

$\Xi(D)=\bigcup_{n\geqslant 0}(D)_n$,

$\Lambda(D)=\{D'\in\Xi(D)\mid D'$ is minimal$\}$,

where $(D)_0=\{\{d\}\mid d\in\mathrm{KERN}(D)\}$, for $n\geqslant 0$;

$(D)_{n+1}=\{D'\subseteq TP(D)\mid D'\}$ is a minimal nonempty set such that $\mathrm{CON}((D)_n)\vdash\mathrm{PRE}(\{D'\})\}$.

Note that the above D' is always finite by compactness of first order logic.

In what follows we give a sufficient condition of existence of DDL extensions. To do this several lemmas are needed.

Lemma 1.1 *Let D be a set of defaults, C and C′colsures of D under* $R_{\mathrm{DDL}}(D)$ *and* $R_{DDL}(TP(D))$ *respectively. Then* ASS(C) = ASS(C') *and E is a DDL extension of D if and only if E is one of* $TP(D)$.

Proof. It is only needed to prove that ASS(C) = ASS(C'). Obviously, ASS(C)$\subseteq$ASS((C'). It is easy to see that the set $R_{\mathrm{DDL}}(D)$ of rules is the same as the set $R_{\mathrm{DDL}}(TP(D))$ of rules since CON(T) = CON(TP (D)). So we get ASS(C')$\subseteq$ASS(C)form $TP(D)\subseteq C$.

Lemma 1.2. *Let D be a set of defaults, C its closure under* $R_{\mathrm{DDL}}(D)$. *Then* $Th(\mathrm{CON}(ASS(C)))$ = CON($ASS(C)$).

Proof. It is clear by R_{DDL}.

Using induction on $R_{\mathrm{DDL}}(D)$ the following lemma is obvious.

Lemma 1.3. *Suppose that* $D\in P(\mathrm{DEF})$ *and C is its closure under* $R_{\mathrm{DDL}}(D)$. *For any* $\alpha:\beta_1,\cdots,\beta_n/\gamma\in C$: *if*: $\alpha\not\equiv$ *true then there exist defaults* $\alpha^i:\beta_1^i,\cdots,\beta_{n(i)}^i/\gamma^i\in TP(D)(1\leqslant i\leqslant k)$ *such that* $\alpha^i\not\equiv$ *true*, $\alpha\equiv\alpha^1\vee\cdots\vee\alpha^k$, $\gamma=\gamma^1\vee\cdots\vee\gamma^k$ *and* $\{\beta_1^i,\cdots,\beta_{n(i)}^i\mid 1\leqslant i\leqslant k\}\equiv\{\beta_1,\cdots,\beta_n\}$.

Lemma 1.4. *For any* $D\in P(\mathrm{DEF})$,

(i) $Th(\mathrm{CON}(\Xi(D)))=Th(\mathrm{CON}(\Lambda(D)))$;

(ii) $\Xi(D)=\Xi(TP(D))$, $\Lambda(D)=\Lambda(TP(D))$;

(iii) *if* D' *is a minimal nonempty subset of* $TP(D)$ *such that* $\mathrm{CON}(\Xi(D))\vdash\mathrm{PRE}(\{D'\})$, *then* $D'\in\Xi(D)$;

(iv) $D'\in\Lambda(D)$ *if and only if* D'*is a minimal nonempty subset of* $TP(D)$ *such that* $\mathrm{CON}(\Lambda(D))\vdash\mathrm{PRE}(\{D'\})$; *and*

(v) $\Xi(\bigcup\Xi(D))=\Xi(D)$, $\Lambda(\bigcup\Lambda(D))=\Lambda(D)$.

Proof.

(i) Clearly, $\Lambda(D)\subseteq\Xi(D)$, which means $\mathrm{CON}(\Xi(D))\vdash\mathrm{CON}(\Lambda(D))$. For any $D'\in\Xi(D)$ there is $D''\in\Lambda(D)$ such that $D''\subseteq D'$, which implies that $\mathrm{CON}(\Lambda(D))\vdash\mathrm{CON}(\Xi(D))$.

(ii) It is only needed to note that $TP(D) = TP(TP(D))$.

(iii) It is immediate by compactness of first order logic.

(iv) and (v) are obvious by (i) and (iii).

Lemma 1.5. *Let D be a set of defaults, $D^* = \bigcup \Xi(D)$, $D^{**} = \bigcup \Lambda(D)$, C^* and C^{**} the closures of D^* and D^{**} under $R_{DDL}(D^*)$ and $R_{DDL}(D^{**})$ respectively.*

(i) For any $m \geq 0$: if $\{\alpha^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leq i \leq k\} \in (D)_m$ then there is $\beta_1, \cdots, \beta_n / \gamma^1 \vee \cdots \vee \gamma^k \in C^*$ such that $\{\beta_1^i, \cdots, \beta_{n(i)}^i \mid 1 \leq i \leq k\} \subseteq \{\beta_1, \cdots, \beta_n\}$.

(ii) For any $\{\alpha^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leq i \leq k\} \in \Lambda(D)$ there is $\beta_1, \cdots, \beta_n / \gamma^1 \vee \cdots \vee \gamma^k \in C^{**}$ such that $\{\beta_1^i, \cdots, \beta_{n(i)}^i \mid 1 \leq i \leq k\} \subseteq \{\beta_1, \cdots, \beta_n\}$.

Proof. It is easy to show the lemma by induction on m.

Lemma 1.6. *Suppose that $D \in P(\mathrm{DEF})$, $D^* = \bigcup \Xi(D)$ and C^* is the closure of D^* under $R_{DDL}(D^*)$. Then $\mathrm{CON}(\mathrm{ASS}(C^*)) = Th(\mathrm{CON}(\Xi(D))) = Th(\mathrm{CON}(\Lambda(D)))$.*

Proof. It is clear that $Th(\mathrm{CON}(\Xi(D))) \subseteq \mathrm{CON}(\mathrm{ASS}(C^*))$ by Lemma 1.5. To prove the converse inclusion it is only needed to show by induction that $\mathrm{CON}(\Xi(D)) \vdash \gamma$ for any $d = \beta_1, \cdots, \beta_n / \gamma \in \mathrm{ASS}(C^*)$.

Base. It is clear. Since $d \in D^*$ and $TP(D^*) \subseteq TP(D)$, then $\{d\} \in (D)_1 \subseteq \Xi(D)$.

Step. Induction conclusions for DR_0^*, DR_1, DR_2, DR_4 and DR_5 are obviously true. As for DR_3 and DR_6, we can inductively show them. As an example, consider DR_3. Assume $\beta_1, \cdots, \beta_m / \sigma, \sigma : \beta_{m+1}, \cdots, \beta_n / \gamma \in C^*$. Without losing generality, suppose $\sigma \not\equiv$ true (otherwise the lemma is trivially satisfied). By inductive hypothesis we have $\mathrm{CON}(\Xi(D)) \vdash \sigma$. By using Lemma 1.3 we get $\{\sigma^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leq i \leq j\} \in TP(D^*)$ such that $\sigma \equiv \sigma^1 \vee \cdots \vee \sigma^j$, $\gamma = \gamma^1 \vee \cdots \vee \gamma^j$, $\{\beta_1^1, \cdots, \beta_{n(j)}^j\} \equiv \{\beta_{m+1}, \cdots, \beta_n\}$. Without losing generality we assume that $\{\sigma^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leq i \leq l\}$ $(l \leq j)$ is a minimal nonempty set such that $\mathrm{CON}(\Xi(D)) \vdash \sigma^1 \vee \cdots \vee \sigma^l$. Then $\{\sigma^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leq i \leq l\} \in \Xi(D)$, by Lemma 1.4. So, $\mathrm{CON}(\Xi(D)) \vdash \gamma$.

Lemma 1.7. *Let $D \in P(\mathrm{DEF})$, $D^* = \bigcup \Xi(D)$, C and C^* be closures of D and D^* under $R_{DDL}(D)$ and $R_{DDL}(D^*)$ respectively. For any $\beta_1, \cdots, \beta_n / \gamma \in \mathrm{ASS}(C)$, there is $\beta'_1, \cdots, \beta'_k / \gamma \in \mathrm{ASS}(C^*)$. Moreover $\mathrm{CON}(\mathrm{ASS}(C)) = Th(\mathrm{CON}(\Lambda(D)))$.*

Proof. It proceeds in a similar way to inductive method used in Lemma 1.6.

Lemma 1.8. *Let $T \in P(\mathrm{DEF})$, $T^* = \bigcup \Lambda(D)$, C and C^* be closures of T and T^* under $R_{DDL}(T)$ and $R_{DDL}(T^*)$ respectively. If E^* is a DDL extension of T^* then $E = E^* \bigcup \{\beta_1, \cdots, \beta_n / \gamma \in \mathrm{ASS}(C) \mid E^* \bigcup \{\beta_i\}$ is consistent for all $1 \leq i \leq n\}$ is one of T. If E is a DDL extension of T then $E^* = E \bigcap C^*$ is one of T^*.*

Proof. Immediate from Lemmas 1.1, 1.7 and Definition 1.4.

By Lemmas 1.6 and 1.8 we obtain the following sufficient condition of existence of DDL extensions.

Theorem 1.1 *Let T be a default theory. If $\Lambda(T)$ is compatible then T has exactly one DDL extenion.*

Now wre give a necessary condition of existence of DDL extensions.

Definition 1.7. Let $T \in P(\mathrm{DEF})$ be a default theory, E a DDL extension of T. Define $GD(E, T) = \{T' \subseteq TP(T) \mid T'$ is a minimal nonempty set such that $\mathrm{CON}(E) \vdash \mathrm{PRE}(\{T'\})$ and $\mathrm{CON}(E) \nvdash \neg \beta$ for each $\beta \in \mathrm{CCS}(\{T'\})\}$.

Lemma 1.9. *Let $T \in P(\mathrm{DEF})$ be a default theory, E a* DDL *extension of T and C the closure of T under $R_{\mathrm{DDL}}(T)$. Then*

(i) $\mathrm{CON}(E) = Th(\mathrm{CON}(GD(E,T)))$.

(ii) *For any $\alpha : \beta_1, \cdots, \beta_n / \gamma \in C$: if $\alpha \not\equiv true$, $\mathrm{CON}(E) \vdash \alpha$ and $\mathrm{CON}(E) \cup \{\beta_i\} (\leqslant i \leqslant n)$ is consistent then there exist defaults $\alpha^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \in TP(\bigcup GD(E,T))(1 \leqslant i \leqslant k)$ such that $\alpha^i \not\equiv true$, $\alpha^1 \vee \cdots \vee \alpha^k \vdash \alpha$ and $\gamma^1 \vee \cdots \vee \gamma^k \vdash \gamma$.*

Proof.

(i) It is easy to see that $\mathrm{CON}(GD(E,T)) \subseteq \mathrm{CON}(E) = Th(\mathrm{CON}(E))$ by compactness of first order logic and the definition of $GD(E,T)$. We show by induction on $R_{\mathrm{DDL}}(T)$ that if $\beta_1, \cdots, \beta_n / \gamma \in E$ then CON $(GD(E,T)) \vdash \gamma$. Induction base is clear. For induction step we consider DR_3 as an example. Assume $\beta_1, \cdots, \beta_m / \sigma, \sigma : \beta_{m+1}, \cdots, \beta_n / \gamma \in C$. Without losing generality suppose $\sigma \not\equiv$ true. By inductive hypothesis we have $\mathrm{CON}(GD(E,T)) \vdash \sigma$ since $\beta_1, \cdots, \beta_m / \sigma \in E$. By Lemma 1.3 it is easy to prove that $\mathrm{CON}(GD(E,T)) \vdash \gamma$ in a similar way to that in Lemma 1.6.

(ii) Immediate from Definition 1.7 and Lemma 1.3.

Lemma 1.10. *Let $T \in P(\mathrm{DEF})$ be a default theory and E a DDL extension of T. Then*

(i) *$GD(E,T) = \Lambda(\bigcup GD(E,T))$;*

(ii) *$GD(E,T)$ is compatible;*

(iii) *for any nonempty set $D \in P(TP(T)) - GD(E,T))$, if there is no $D' \in GD(E,T)$ such that $D' \subseteq D$ then $\mathrm{CON}(GD(E,T)) \nvdash \mathrm{PRE}(\{D\})$ or threr is some $\beta \in \mathrm{CCS}(\{D\})$ such that $\mathrm{CON}(GD(E,T)) \vdash \neg \beta$.*

Proof.

(i) First we have CON $(E) \supseteq Th(\mathrm{CON}(\Lambda(\bigcup GD(E,T))))$, which is easy to get by inductively showing that, for all $m \geqslant 0$, $\mathrm{CON}(GD(E,T)) \vdash \mathrm{CON}((\bigcup(GD(E,T))_m)$.

Next, we prove by induction on $R_{\mathrm{DDL}}(T)$, that for any $d = \beta_1, \cdots, \beta_n / \gamma \in \mathrm{ASS}(C)$ where C is the closure of T under $R_{DDL}(T)$, if $\mathrm{CON}(E) \cup \{\beta_i\} (1 \leqslant i \leqslant n)$ is consistent then $\mathrm{CON}(\Lambda(\bigcup \mathrm{GD(E,T)})) \vdash \gamma$, which means that $\mathrm{CON}(\Lambda(\bigcup \mathrm{GD(E,T)})) \vdash \mathrm{CON(E)}$。

Base. If $\mathrm{d} \in \mathrm{T}$ and $\mathrm{CON(E)} \cup \{\beta_i\} (1 \leqslant \mathrm{i} \leqslant \mathrm{n})$ is consistent then $\{d\} \in GD(E,T)$. So $\{d\} \in \Lambda(\bigcup GD(E,T))$.

Step. Consider the case for DR_3. Assume that $\beta_1, \cdots, \beta_m / \sigma, \sigma : \beta_{m+1}, \cdots, \beta_n / \gamma \in C$ and that $\mathrm{CON}(E) \cup \{\beta_i\} (1 \leqslant i \leqslant n)$ is consistent. Without losing generality, suppose $\sigma \not\equiv$ true (otherwise the lemma is trivially satisfied). By inductive hypothesis we have $\mathrm{CON}(\Lambda(\bigcup GD(E,T))) \vdash \sigma$. So $\mathrm{CON}(E) \vdash \sigma$. By using Lemma 1.9 we get $\{\sigma^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leqslant i \leqslant j\} \in TP(\bigcup GD(E,T))$ such that $\sigma^1 \vee \cdots \vee \sigma^j \vdash \sigma$ *and* $\gamma^1 \vee \cdots \vee \gamma^j \vdash \gamma$. Without losing generality we assume that $\{\sigma^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leqslant i \leqslant p\} (p \leqslant j)$ is a minimal nonempty set such that $(\mathrm{CON}(\Lambda(\bigcup GD(E,T)))) \vdash \sigma^1 \vee \cdots \vee \sigma^p$. Then $\{\sigma^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \mid 1 \leqslant i \leqslant p\} \in \Lambda(\bigcup GD(E,T))$. So $\mathrm{CON}(\Lambda(\bigcup GD(E,T))) \vdash \gamma^1 \vee \cdots \vee \gamma^p \vdash \gamma$.

Similarly the induction conclusion can be proved for DR_6 using Lemma 1.3 and Definition 1.7. The other cases are trivial.

Finally it is easy to estabish that $GD(E,T) = \Lambda(\bigcup GD(E,T))$ by Lemma 1.4.

The items (ii) and (iii) of the lemma are trivial.

Theorem 1.2 (Characterization of DDL Extensions). *A default theory T has a DDL extension if and only if there exists a set of defaults $T^* \subseteq T$ such that:*

(i) $\Lambda(T^*)$ *is compatible;*

(ii) $\bigcup \Lambda(T^*) \cap T = T^*$;

(iii) *for any $D \in P(TP(T)) - \Lambda(T^*)$, if there is no $D' \in \Lambda(T^*)$ such that $D' \subseteq D$ then* $\mathrm{CON}(\Lambda(T^*)) \not\vdash \mathrm{PRE}(\{D\})$ *or there is some $\beta \in \mathrm{CCS}(\{D\})$ such that* $\mathrm{CON}(\Lambda(T^*)) \vdash \neg \beta$.

Proof. The part "only if" is immediate by Lemma 1.10. Now consider the part "If". From Theorem 1.1 T^* has exactly one extension $E^* = \mathrm{ASS}(C^*)$ and $\mathrm{CON}(E^*) = Th(\mathrm{CON}(\Lambda(T^*)))$ where C^* is the closure of T^* under $R_{\mathrm{DDL}}(T^*)$. Let $T^{**} = \{\alpha : \beta_1, \cdots, \beta_n / \gamma \in T - T^* \mid \mathrm{CON}(E^*) \cup \{\beta_i\}$ is consistent for every $1 \leqslant i \leqslant n\}$ and $T' = T^* \cup T^{**}$. Suppose that C and C' are closures of T and T' under $R_{\mathrm{DDL}}(T)$ and $R_{\mathrm{DDL}}(T')$ respectively. Let $E = \mathrm{ASS}(C')$. It is easy to see that $E^* \subseteq E$ and that $\mathrm{CON}(E^*) \cup \{\beta\}$ is consistent for each $\beta \in \mathrm{CCS}(E)$. We show by induction on $R_{\mathrm{DDL}}(T')$ that if $d = \beta_1, \cdots, \beta_n / \gamma \in E$ then $\mathrm{CON}(E^*) \vdash \gamma$, which implies $\mathrm{CON}(E^*) = \mathrm{CON}(E)$.

Base. This is obvious. In fact, if $d \in T'$ then $d \in T^*$. Since $d \in T^{**}$ then $\{d\} \in P(TP(T)) - \Lambda(T^*)$ and there is no $D \in \Lambda(T^*)$ such that $D \subseteq \{d\}$. By condition (iii) of the theorem there is some β_i $(1 \leqslant i \leqslant n)$ such that $\mathrm{CON}(E^*) \vdash \neg \beta_i$ which leads to a contradiction.

Step. As an example we consider DR_3. Assume $\beta_1, \cdots, \beta_m / \sigma, \sigma : \beta_{m+1}, \cdots, \beta_n / \gamma \in C'$. By inductive hypothesis $\mathrm{CON}(E^*) \vdash \sigma$. Without losing generality suppose $\sigma \not\equiv$ true. By Lemma 1.3 there are defaults $d^i = \sigma^i : \beta_1^i, \cdots, \beta_{n(i)}^i / \gamma^i \in TP(T^*)$ $(1 \leqslant i \leqslant j)$ such that $\sigma \equiv \sigma^1 \vee \cdots \vee \sigma^j$, $\gamma = \gamma^1 \vee \cdots \vee \gamma^j$, $\{\beta_1^1, \cdots, \beta_{n(j)}^j\} \equiv \{\beta_{m+1}, \cdots, \beta_n\}$. We conclude that there is $D \in \Lambda(T^*)$ such that $D \subseteq \{d^i \mid 1 \leqslant i \leqslant j\}$, which implies $\mathrm{CON}(\Lambda(T^*)) \vdash \gamma$, i.e. $\mathrm{CON}(E^*) \vdash \gamma$. In fact, if there is no $D \in \Lambda(T^*)$ such that $D \subseteq \{d^i \mid 1 \leqslant i \leqslant j\}$ then we will again get a contradiction by condition (iii).

Finally, it is easy to show by induction on $R_{\mathrm{DDL}}(T)$ that if $\beta_i, \cdots, \beta_n / \gamma \in \mathrm{ASS}(C)$ and $\mathrm{CON}(E) \cup \{\beta_i\}$ is consistent for each $1 \leqslant i \leqslant n$ then $d \in E$. So, E is a DDL extension of T.

From the above proof it is easy to get the following result:

Corollary 1.1. *Let E be a DDL extension of a default theory T and γ any closed formula. $\gamma \in \mathrm{CON}(E)$ if and only if there is a subset T^* of T such that T^* satisfies three conditions of Theorem 1.2 and $\mathrm{CON}(\Lambda(T^*)) \vdash \gamma$ holds.*

Theorem 1.3. *A default theory T has only one extension if and only if there is a unique subset T^* of T such that conditions (i)—(iii) in Theorem 1.2 are satisfied. Here "unique" means that default sets T_1 and T_2 are the same if for any $\alpha : \beta_1, \cdots, \beta_n / \gamma \in T_1$ there is $\sigma : \tau_1, \cdots, \tau_m / \theta \in T_2$ such that $\alpha \equiv \sigma$, $\gamma \equiv \theta$, $\{\beta_1, \cdots, \beta_n\} \equiv \{\tau_1, \cdots, \tau_m\}$, and vice versa.*

Proof. It is clear from Theorem 1.2.

Now we deal with the so-called membership problem and reasoning tasks in DDL. Considering that the explicit management of justification of a default is not always a desirable feature, often one just may wish to know whether a closed formula γ appears in some DDL extension or all DDL extensions, regardless of its (their) justifications. Based on these ideas, we define the following membership problem and reasoning tasks.

Definition 1.8. Given a default theory T.

Skeptical Reasoning (SR): decide whether a given formula γ occurs in $\mathrm{CON}(E)$ for all DDL extensions E

of T.

Credulous Reasoning (CR): decide whether a given formula γ occurs in CON(E) for some DDL extension E of T.

It follows from Corollaruy 1.1 the process of finding a DDL extension of a default theory T consists of three phases. First we guess asubset T^* of T(a candidate of "generating defaults"). Then we check if T^* satisfies three conditions of Theorem 1.2. Finally we generate a DDL extension. For the above reasoning tasks we only check if a given formula γ is a member of CON(E) for each (some) DDL extension E of T, that is, check if CON (Λ (T^*)) $\vdash \gamma$ holds for each (some) candidate T^* satisfying conditions. The membership problem is solved by the following Boolean Function MEMBER (γ, T, T'), whose correctness follows immediately from Corollary 1.1. To do this, we give two procedures LAMBDA (T') and GD (T, T'), which returns $\Lambda(T')$ and the other one of which yields true when T' satisfies the three conditions of Theorem 1.2.

For any class C of sets we denote the function computing all minimal sets among C by MIN(C). Clearly, MIN(C) requires polynomial time in $|C|$. Note that the given procedures here apply as primitive operations inference tests in classical logic ($\vdash$). We assume that these tests are provided as oracles.

```
FUNCTION LAMBDA(T')
  result: = ∅
  χ: = ∅
  REPEAT ψ: = ∅;
    FOR EACH T* ⊆ TP(T') - χ DO
        IF CON(result) ⊢ PRE({T*})THEN ψ: = ψ∪{T*};
    χ: = χ∪ψ;
      result: = MIN(result)
  UNTIL ψ: = ∅;
  result: = MIN(result)
  RETURN(result)
```

Note that the above procedure runs in polynomial time except for an exponential number of inference tests.

```
BOOLEAN FUNCTION GD (T, T')
ψ: = LAMBDA(T')
FOR EACH β∈ CCS(T') DO
  IF CON(ψ) ⊢ ¬ β THEN RETURN(false)
If∪ψ≠ T' THEN RETURN(false)
FOR EACH T* ⊆TP(T) - ψ DO
  FOR EACH D∈ ψ AND ALL β∈ CCS({T*})DO
      IF D∩T* ≠ D AND CON(ψ) ⊬¬ β AND CON(ψ) ⊢PRE({T*}) THEN RETURN(false)
RETURN(true)
```

The function GD (T, T') runs in polynomial time except for an exponential number of inference tests. So we get procedures for solving membership problem and reasoning tasks respectively.

```
FUNCTION MEMBER (γ, T, T')
  IF GD(T, T') AND CON(LAMBDA(T')) ⊢ γ THEN RETURN(true)
```

```
    ELSE RETURN(false).

  BOOLEAN FUNCTION SR(γ, T, T')
    FOR EACH T' ⊆ T DO
        IF GD (T, T') AND NOT MEMBER (γ, T, T') THEN RETURN(false)
      RETURN(true)
BOOLEAN FUNCTION CR(γ, T, T')
    FOR EACH T' ⊆ T DO
        IF GD (T, T') AND MEMBER (γ, T, T') THEN RETURN(true)
      RETURN(false)
```

2 Discussion

The research reported in this paper was originally motivated by some defects in Reiter's default logic and Brewka's framework for cumulative default logics. In particular, Reiter's DL cannot adequately handle reasoning by cases; Brewka's L_1 can perform reasoning by cases in simple examples but it could not deal with some more general cases. To avoid these defects we presented a variant of Brewka's framework in this paper. In this new framework we defined and characterized DDL extensions and gave algorithms for some reasoning tasks in DDL logic. At first appearance, our work reported in the paper seems to be similar to those in refs. [6—9], but there is subtle difference. Contrary to Reiter's definition of extensions and Brewka's one of CDL extensions, neither Brewka's definition of L_1 extensions nor one of DDL extensions is based on a fixed point construction. This makes it possible to deal with reasoning by cases and hence makes it more difficult and complex to characterize DDL extensions.

We note that, more recently, a similar modification of extensions for Reiter's DL has been provided independently by Yves[10] (C-extension). Clearly, the idea underlying Moinard's modification is the same as that underlying our framework in the paper. Although his modification simplifies the complexity of the search for extensions, yet it uses too many defaults of forms $\alpha:/\alpha$ and stronger justifications to generate C-extensions. As shown in Example 2, adding default $\alpha:/\alpha$ would lead to unwanted answer $\beta/\alpha\rightarrow\gamma$. Correspondingly, consider a default theory $(\varnothing, \{\alpha:\beta/\gamma\})$, its C-extension contains $\alpha\rightarrow\gamma$. This seems to be unsatisfactory intuitively. In particular it leads to unexpected results shown in Examples 3.11 and 3.12 of ref. [10]. Moreover, it is easy to see that our framework behaves with those basic examples in sections 2 and 3 of ref. [10], which were used as a benchmark for any proposition of default logic dealing with reasoning by cases. So, the notion of DDL extensions is not only independent of one of C-extensions but also more satisfactory than the latter.

Noting one-to-one correspondence between R-extensions and consequents of L_{DL} extensions[1], we compare our framework with Yves's version from another point of view. There are examples which show that a default theory T has a L_{DL} extension but it has no DDL extension and vice versa. On the other hand, we can prove that any default theory has a C-extension. This shows that the notion of C-extension is weaker than that of R-extension in a sense, since there is a default theory which has a C-extension but has no R-extension. The default theory $T = (\varnothing, \{:\neg b/b\})$ is such an example.

As shown in Broken Arms example for DL[11], of course, DDL logic still left the problem with mutually inconsistent justifications of defaults unsolved since it produces a DDL extension in which both arms are usable. So we will further develop DDL and DL_1 logics and find some computationally tractable subclasses of de-