ENGINEERING DESIGN

A Synthesis of Stress Analysis and Materials Engineering

SECOND EDITION

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PREFACE

In this second edition the philosophy and intent of the first edition have been maintained. The addition of new chapters and new material to existing chapters reflects the current literature and the growing influence of computer technology on problem solving. A new feature is the addition of problems at the end of each chapter to facilitate using the book as a text for a two-semester course.

Many friends and associates offered aid and encouragement for which we are grateful. In particular, we are indebted to students who worked on and contributed suggestions for many of the homework problems; to Dr. James Foxworthy, Dean of the College of Science and Engineering, and Dr. Joseph Callinan, Chairman of the Mechanical Engineering Department, both of Loyola Marymount University, and to the management of the Engineering Department of the Du Pont Company.

The influence of the late Dr. Joseph Marin set the tone for the first edition. The second edition has benefited from the added influence of Professor Robert M. Rivello, author of *Theory and Analysis of Flight Structures*, and Professor John W. Jackson, who taught a year-long course from the first edition.

Like the first edition, the second edition could not have come to fruition without the assistance of many persons who contributed by their discussions. We are particularly indebted to the authors whose works are quoted in the text and to our wives and families for their support. It is also a pleasure to acknowledge the editorial and secretarial assistance contributed by Roxana Letamendi and Carol McMillan.

JOSEPH H. FAUPEL

Wilmington, Delaware

FRANKLIN E. FISHER

Los Angeles, California

April 1980

PREFACE TO FIRST EDITION

Many books, particularly those written by university professors, evolve over a span of years and represent a distillation of knowledge into a compact exposition of a discipline for others to follow. On the other hand, books are frequently written in response to a need in a particular field, and they represent the author's views of the subject at the time of writing; this book had its beginning somewhere in between.

The material given here is an integrated treatment of stress analysis and materials engineering based on lectures given for several years in the graduate division of mechanical engineering at the University of Delaware. The contents represent a combination of these fields at a level somewhat beyond that given in elementary texts, yet not approaching the degree of sophistication found in advanced treatises. The reader will find here the conventional subjects treated in elementary texts, but, in many cases, with extensions. For example, some beam problems are solved by numerical integration, Maclaurin's series, Laplace transform, and others. In addition, such subjects as minimum weight analysis, ductility and brittleness of materials, analysis of composite, honeycomb and reinforced materials, designing with plastics, metal-working and limit analysis in the plastic range, prestressing for strength, strength under combined stress, dynamic behavior of materials, stability and buckling, thermal stress analysis, creep, stress rupture, fatigue and stress concentration are presented from the point of view of the practicing engineer.

By the very nature of the contents, the format of this book departs considerably from what might be considered standard, but no apology is made for this. My purpose has been to present practical information in a form useful to a diverse audience and not to sacrifice clarity or usefulness for consistency or form; discussions with teachers, engineers, and scientists, many of whom were my students, have confirmed this view. Although directed principally to practicing engineers, this book can also serve as a reference for students and as a text for an intermediate or graduate course in engineering design.

The preparation required the cooperation and assistance of many people and I am indebted to my many friends and associates for their kind encouragement and assistance. I am particularly indebted to all those whose works are quoted in the text and to my graduate-school-day professors, Dr. Joseph Marin and Dr. Maxwell Gensamer. It is also a pleasure to acknowledge the generous support of H. C. Vernon, Director of the Du Pont Company's Engineering Research Laboratory, and Miss Jennie Di Bartolomeo, who supplied valuable editorial and secretarial assistance.

J. H. FAUPEL

Wilmington, Delaware
August 1964

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chapter 1

MATERIALS AND PROPERTIES

1-1 THE NATURE AND PROPERTIES OF MATERIALS

Theoretical strength of materials

The deformation of materials can be considered on at least three levels of the division of matter. At the atomic and molecular levels, strength is associated with elemental forms of matter of the order of 10^{-8} in. held together by electronic forces. At the microscopic level, groups of elemental particles of the order of 10^{-8} to 10^{-4} in. are also held together by electronic forces, but these forces are reduced by defects and imperfections in the structure. At the macroscopic or phenomenological level, which is of more interest to the engineer, the structure is held together by some average force determined by the defects and the electronic forces. Theoretically, the strength of a material should be reflected by the forces at the atomic level. However, because of defects in the structure, the practical strength of materials is several orders of magnitude less than theory would predict.

An estimation of the theoretical strength of a material may be made as follows. Consider Fig. 1.1, which shows an ideal elemental form of matter consisting of two rows of atoms. The shear stress necessary to produce slip by a distance x will be a function of x. Assuming that the attractive and repulsive forces are balanced in the position shown, there will be zero stress required to maintain the arrangement. If particle C is moved to a position m–n between A and B (Fig. 1.2), a force results which is zero at m–n, positive to the left of m–n and negative to the right; by assuming a sinusoidal stress variation, Fig. 1.3 is obtained. This figure shows how stress varies in the lattice structure. The shear stress is zero at 0, $d/2$, and d. For intermediate locations

$$\tau = f(\tau) \sin 2\pi x/d \qquad 1.1$$

or from Fig. 1.3, for small values of x,

$$\tau = (\tau_c) \sin 2\pi x/d = \tau_c(2\pi x/d) \qquad 1.2$$

where τ_c is the peak stress developed (maximum theoretical shear strength of the material). By virtue of Hooke's law and since shear strain equals x/a, the

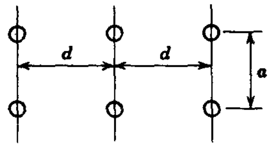


Fig. 1.1 Ideal elemental arrangement of atoms.

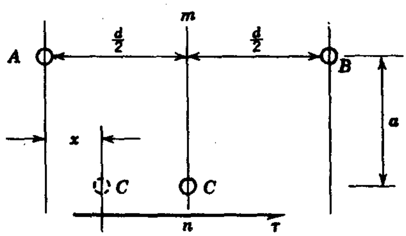


Fig. 1.2 Atoms displaced by applied load.

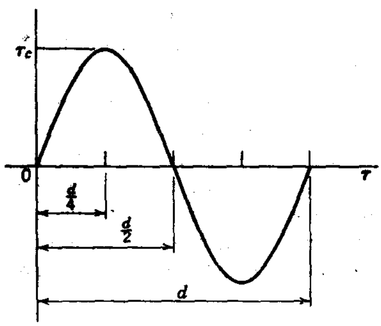


Fig. 1.3 Stress variation in lattice.

stress is given by the relationship

$$\tau = G(x/a) = \tau_c \sin 2\pi x/d \qquad 1.3$$

where G is the shear modulus of elasticity. From Eqs. 1.2 and 1.3 the theoretical maximum shear strength of a material is

$$\tau_c = G(d/2\pi a) \qquad 1.4$$

This means that for mild steel, for example, where $d = a$, the shear stress would be about $(11.5 \times 10^6)/2\pi$ or 1.8×10^6 psi. The observed value is about 3.0×10^4 psi, or about $\frac{1}{60}$ of the theoretical value. This large discrepancy is believed due to the presence of *dislocations* in the material structure (4).

Strain behavior

Most structural materials* exhibit one or more of the following types of strain: linear elastic, nonlinear elastic, viscoelastic, plastic, and anelastic. In linear

*Typical tensile stress-strain curves for some structural metals are shown in Figs. 1.4*B* and 1.4*C*.

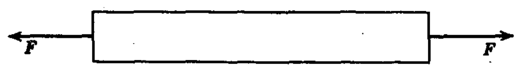
elastic deformation, stress and strain are related by Hooke's law

$$\text{strain} = \text{stress/constant}$$

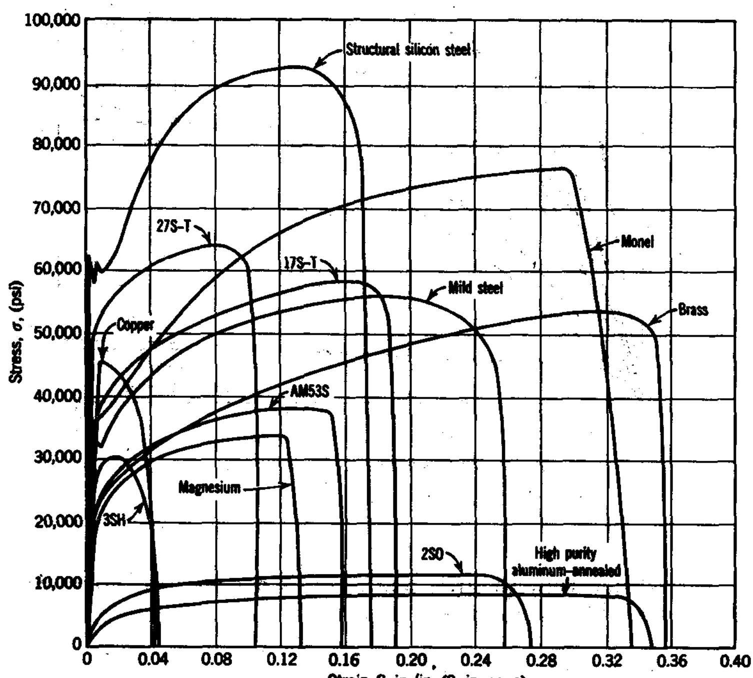
For uniaxial loading, Fig. 1.4*A*, this relationship becomes

$$\varepsilon = \sigma/E \qquad 1.5$$

where ε is strain, σ is the applied stress, and E is Young's modulus of elasticity. In nonlinear elastic behavior, the material behaves elastically in that no permanent residual deformation results when the applied load is removed from the

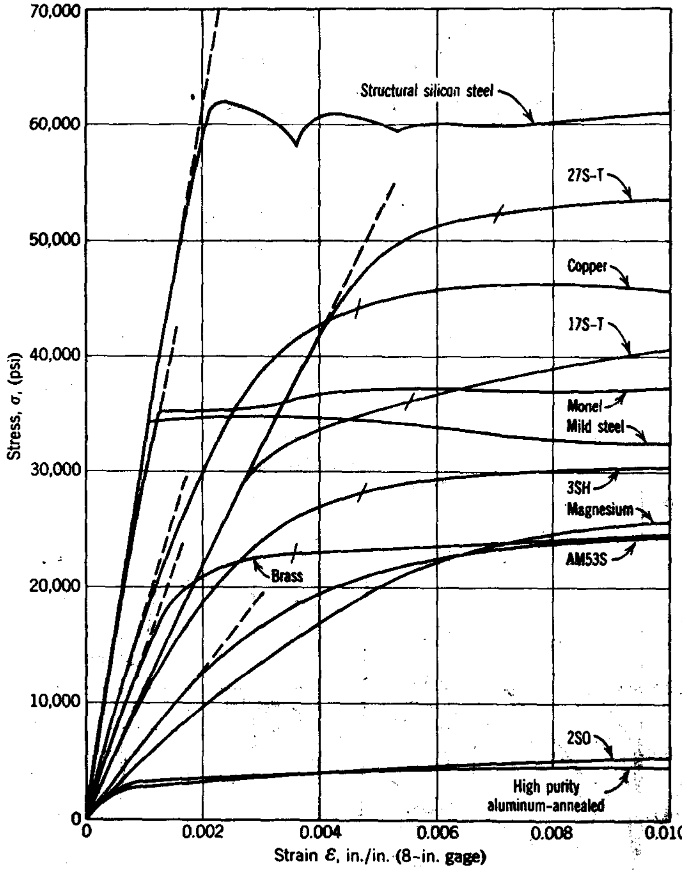


(*A*) Prismatic bar



(*B*) Typical stress-strain curves.
27S-T = Old alloy no longer available
17S-T = 2017-T4
3SH = 3003-H14
2SO = 1100-O

Fig. 1.4 Tensile or uniaxial loading of a bar of material. [After Templin and Sturm, (27), Courtesy of The Institute of Aeronautical Sciences.]



(C) Expanded scale for curves in B

Fig. 1.4 (Continued).

body. The relation between stress and strain is not linear as expressed by Eq. 1.5 but takes the general form

$$\varepsilon = (\sigma/\lambda)^n \qquad 1.6$$

where λ is a pseudoelastic modulus and n is a constant, both determined experimentally. This relationship is used in Chapter 15. In viscoelastic behavior there are strong resemblances to linear elasticity. In linear elastic strain the strain stops if the loading stops, whereas in viscoelastic deformation straining continues even though the loading stops and a residual strain remains when all the load is removed. Viscoelasticity is discussed in Chapter 5. In plastic deformation there is always a residual deformation on removal of the applied load. Anelastic behavior is characterized by noninstantaneous strain at small stress levels that is recoverable after a period of time (34). For practical structural applications the engineer will not ordinarily be concerned with anelasticity.

Simple static properties of materials

Load-deformation characteristics of materials are of great practical interest, and several common types of curves obtained in tension tests of materials have been observed. "Brittle" materials are characterized by a curve such as shown in Fig. 1.5*A*; the material deforms in accordance with Hooke's law (Eq. 1.5) to

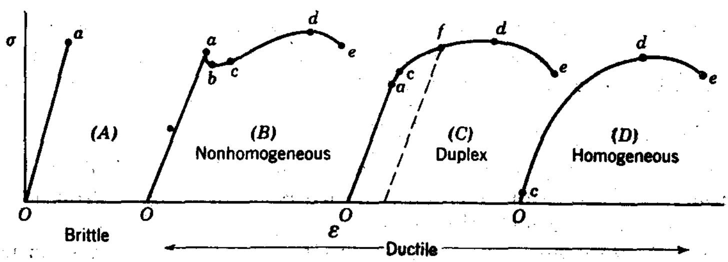


Fig. 1.5 Types of tension stress-strain curves.

fracture, point *a*. Materials such as some grades of tool steel, glass, and many ceramics, when tested in tension at room temperature, exhibit the characteristics shown in Fig. 1.5*A*. Most "ductile" materials exhibit behavior characterized by Figs. 1.5*B*, 1.5*C*, or 1.5*D*. *Nonhomogeneous* deformation (Fig. 1.5*B*) is most closely associated with low-carbon steel. Initially, the deformation is linearly elastic, line *Oa*; this is followed by a sudden drop in stress to *b* and then gradual stress increase through *c* to a maximum at *d* and finally fracture at *e*. In *duplex deformation* (Fig. 1.5*C*) there is an initial straight-line portion *Oa*, followed by gradual stress increase *c* and *f*, reaching a maximum at *d* and fracturing at *e*. In *homogeneous* deformation (Fig. 1.5*D*) there is no linear portion but continuously "smooth" stress buildup to *d* followed by fracture at *e*.

Modulus of elasticity. The modulus of elasticity E in Eq. 1.5 is a measure of the *inherent rigidity* of a material and is defined by the straight-line portion *Oa* of Fig. 1.5, and by *OA* in Fig. 1.6; thus, from Fig. 1.6,

$$E = \sigma/\varepsilon = HA/OH \qquad 1.7$$

Secant and tangent moduli. The *secant modulus*, frequently used in the theory of plasticity (Chapter 6), is a variable defined by Eq. 1.7 but represented by line *OF* in Fig. 1.6; that is,

$$\text{secant modulus} = E_s = \sigma/\varepsilon = CJ/OJ \qquad 1.8$$

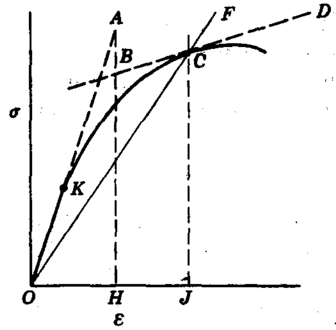


Fig. 1.6 Various moduli of elasticity.

The *tangent modulus* measures the rate of change of stress to strain and is represented by a tangent to the stress-strain curve at the point in question. For example, at point C in Fig. 1.6, the tangent modulus is

$$E_T = (d\sigma/d\varepsilon)_c \tag{1.9}$$

In the elastic range (OK, Fig. 1.6), all moduli values are identical; for the special case of stress-strain relationships such as shown in Fig. 1.5D, the modulus is defined as the tangent to the curve at zero stress.

Modulus of rupture. The modulus of rupture is the fracture stress in bending calculated by means of the formula $\sigma = M/Z$ discussed in Chapter 2.

Proportional limit. The proportional limit is defined as the terminus of the straight-line portion of the stress-strain curve; that is, it is the upper limit for which Hooke's law is valid (a in Fig. 1.5).

Elastic limit. The elastic limit defines a range of strain characterized by the absence of residual deformation on release of load (c in Fig. 1.5).

Upper yield point. An upper yield stress (a, Fig. 1.5B) is characteristic of mild steel, but it is believed to be a fictitious value obtained as a result of the deformation characteristics of steel and inertia effects in the testing machine.

Lower yield point. The lower yield point is also characteristic of low-carbon steel and is defined by b in Fig. 1.5B. Sometimes there is a "flat" at b (deformation at constant stress) as shown in Fig. 1.7. For behavior characterized by Fig. 1.7

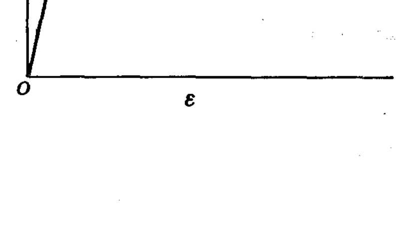


Fig. 1.7 Tensile deformation at constant stress.

the portion of the curve from a to c defines yielding and is frequently referred to as the *yield stress, lower yield stress,* or *fundamental yield stress.*

Offset yield strength. For materials characterized by the plots in Figs. 1.5C and 1.5D, there is no well-defined stress at which yielding occurs. Actually, the elastic limit defines this point, but this is frequently nearly impossible to discern. To overcome this difficulty the *offset yield strength* has been introduced and is defined as the stress corresponding to the intersection of the stress-strain curve with a parallel to the straight-line portion, with origin at some finite strain value (f in Fig. 1.5, for example). The strain offset is usually 0.01% or 0.20%. This stress has also been referred to as the *proof stress.*

Tensile strength. Tensile strength, also called the *ultimate strength,* is the greatest stress sustained per unit load by the material. It is characterized by the beginning of "necking down," or local instability, and is identified by d in Fig. 1.5.

Fracture strength. Fracture strength is, as its name implies, the stress at fracture (point e, Fig. 1.5). This value has little design significance, for it is, like the upper yield point, a fictitious value dependent on or influenced by structure, testing conditions, and the testing machine.

Working stress. The working or design stress is the yield or ultimate stress divided by a factor of safety.

Ductility. Ductility is a measure of the deformability of a material. In tension, the ductility is measured by *elongation* and *reduction of area* as

$$\varepsilon = \frac{L - L_0}{L_0} \qquad 1.10$$

$$q = \frac{A_0 - A}{A_0} \qquad 1.11$$

where ε is unit strain (in./in.), q is reduction of area, L is the final gauge length, L_0 is the initial gauge length, A is the final cross-sectional area, and A_0 is the initial cross-sectional area.

Shear modulus of elasticity. The shear modulus of elasticity, also called the *modulus of rigidity,* is the constant of proportionality between shear stress τ and shear strain γ in the elastic range of strain and has the same significance to shear as E, Young's modulus, has to tension stresses. Thus by definition

$$\gamma = \tau/G \qquad 1.12$$

It is shown in Chapter 3 that there is a relation between E and G that takes the form

$$G = \frac{E}{2(1 + \nu)} \qquad 1.13$$

where ν is Poission's ratio.

Poisson's ratio. Poisson's ratio ν is a measure of the unit strain of a material in the directions normal to the applied load. For example, in tension, if σ_x is the tensile stress and ε_x the strain, then

$$\varepsilon_x = \sigma_x/E \tag{1.14}$$

and the unit lateral contraction in the other two cartesian directions is

$$\varepsilon_y = \varepsilon_z = -\nu(\sigma_x/E) \tag{1.15}$$

Poisson's ratio, like modulus, is a constant depending on the material in the elastic range, and, as shown later, equal to 0.5 for all incompressible materials and materials operating in the plastic range.

Bulk modulus. Bulk modulus B, also called *modulus of volume expansion* and *modulus of volume elasticity*, is a measure of the elastic volume change in a material and is defined as

$$B = \frac{E}{3(1-2\nu)} \tag{1.16}$$

Compressibility. Compressibility C of a material (elastically) is associated with the reciprocal of bulk modulus; thus,

$$C = 1/B \tag{1.17}$$

Resilience. Resilience is defined as that property of a material by virtue of which it can release elastic strain energy as stress is removed. The measure of resilience most commonly used is the area enclosed by a stress-strain curve within the elastic limit. This is called the *modulus of elastic resilience* U.

RESILIENCE IN TENSION. A hypothetical tensile stress-strain curve is shown in Fig. 1.8. In this figure resilience would be considered as the area OAB. In terms of mechanical properties, if σ_y is the elastic limit stress and ε_y is the elastic limit strain, then

$$U = (OB)(AB)/2 = \varepsilon_y\sigma_y/2 \tag{1.18}$$

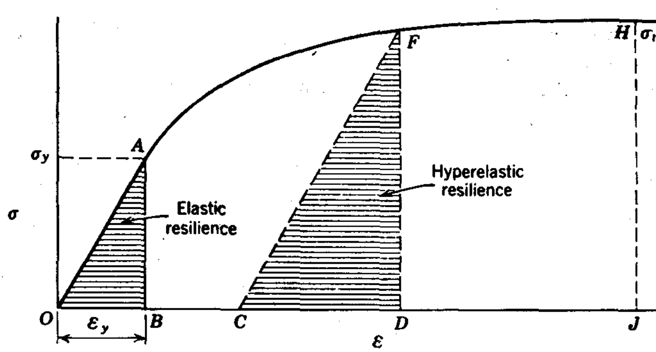


Fig. 1.8 Definition of areas of resilience.