《数学教育·数学史与数学文化史·信息科学》国际会议论文汇编

Proceedings of The International Conference On Mathematics Education History of Mathematics and Cultural History of Mathematics · Informatics

(1998.4.26--29)

北京师范大学
Beijing Normal University
1998年4月

Theme of Conference

(1) Researches on Mathematics education of university, secondary school, primary school and kindergarten.

(2) Researches on history of mathematics, and cultural history of mathematics. Especially, in theme (2), there will be included the researches on the historical exchange of technology between China and West Europe, and on Euclid Elements.

(3)Researches on educational roles in Informatics.

Committees

Chairmen	Prof. Zhong Shanji	(Beijing Normal University)
	Prof. KiyoshiYokochi	(Chief Organizer / Visiting Prof. of Beijing Normal University)
	Prof. Klaus-D. Graf	(Freie Universitat Berlin)
	Prof. Li Di	(Institute for the History of Science, Inner Mongolia Normal University)
Vice Chairmen	Prof. Luo Jianjin	(Inner Mongolia Normal University)
	Prof. Mo De	(Inner Mongolia Normal University)
	Prof. Qian Peiling	(Beijing Normal University)
	Prof. Liu Jiemin	(Beijing Normal University)
Secretary General:	Prof. Wang Yanhua	(Beijing Normal University)

会议主题

（一）数学教育与实践的研究

（二）数学史与数学文化史的研究。其中包括中国与西欧科技交流史的研究和关于《欧几里得原本》的研究。

（三）数学教育手段,特别是计算机辅助教学的研究

会议组织委员会主要成员：

主　席：钟善基（北京师范大学）
　　　　李　迪（内蒙古师范大学）
　　　　横地清（日本《数学教育学会》）
　　　　克．杰．格拉夫（德国柏林自由大学）
副主席：莫　德（内蒙古师范大学）
　　　　罗见今（内蒙古师范大学）
　　　　钱佩玲（北京师范大学）
　　　　刘洁民（北京师范大学）
秘书长：王焱华（北京师范大学）

CALCULATING AND CODING -

BASIC PROBLEMS AT THE TIME OF KANG XI AND LEIBNIZ

Klaus-Dieter Graf, Institut für Informatik, Freie Universität Berlin, Germany

(1) BAI Shangsu, LI Di, Michael WILLIAMS and Klaus-Dieter GRAF have reported about the calculating machines which are stored in the Beijing Palace Museum. They use two different techniques: disks and rods (Neper). There are many interesting links from these machines to several western scholars working on the problem of mechanical calculation with different techniques in the 17th century (Christian counting).

(2) Wilhelm Schickard is one of the most fascinating figures in this field, since his machine from 1623 is the first one known in history and, at the same time, it combines two different techniques: disks for addition and subtraction, rods for multiplication and division.

(3) The technique of addition machines mostly copied the paper-and-pencil algorithm, proceeding digitwise and pushing "carries" forward. This caused enormous mechanical problems, never really solved by Schickard, Pascal or Leibniz. One can imagine that a different algorithm could have led to a more efficient machine, avoiding pushing forward the carries. But this algorithm was only discovered in the 20th century by John van Neumann.

(4) Coding information by using a minimal set of digits has been pursued by many scholars in history. In the 17th century at least three methods existed or were developed, which handled three different kinds of information:
a) natural and mental phenomena, coded with the hexagrams of I Ging,
b) alphabetic texts, coded with Bacon's Code,
c) numbers, coded with Leibniz' dyadic number system.
Today we see that the general principle behind these methods is binary coding.

(5) Knowing the mechanical problems when calculating with decimal numbers, Leibniz thought about a calculating machine handling dyadic (dual) numbers. He described such a machine; but only in our century Mackensen constructed a model, which works fine.

(6) Epilogue: There has always been rumour about Leibniz having sent one of his decimal calculating machines to emperor Kang Xi as a gift. Which are the real informations?

目　　录

THE ROAD TO OBLIQUE PROFILE AND OBLIQUE PROJECTION IN ANCIENT PICTURES IN CHINA

kiyoshi Yokochi
Guest Professor at Beijing Normal University
Home address:2-8-4 Matsugaoka, Nakano-ku, Tokyo 165-0024, JAPAN
Fax:+81-3-3388-2350

Summary: First, I will explain the reason why I was driven to study the above theme, from the standpoints of children's pictures and ancient Japanese pictures. Next, I will explain the main part of the theme.

1. Prologue

1.1. Children's Pictures

Photo 1 through Photo 3 are pictures of gathering berries drawn by nursery school children at the age 3, 4 and 5, respectively. Attending to the drawing method of children's faces and the ground, we see the following.

(1) At the age 3: Children draw person's face as a full face. And they are indifferent to drawing the ground.

(2) At the age 4, children begin to draw a face as profile and to draw the ground as a horizontal line running along under the margin of the drawing paper(Hereafter this drawing method is referred to as "horizontal-line ground").

(3) At the age 5, children begin to draw the ground's depth (Hereafter this drawing method is referred to as "depth ground").

Photo.4 is a picture of soccer drawn by a nursery school child at the age 5. We see the following.

(4) The face of a child in the rear of a group of children is drawn as oblique profile.

Generally speaking, oblique profile is very rarely found in pictures at the age 5.

Photo 5 and Photo 6 are pictures of a car drawn by kindergarten children at the age 5. We see the following.

(5) In Photo 5, the side base line of the engine room and the front base line of its body run nearly on a common horizontal line. Generally speaking, when the children at the age 5 draw a rectangular parallelepiped standing erect, they tend to make the base lines of its side and front run on a common horizontal line(Hereafter the drawing method is referred to as "elevation-depth projection").

(6) In Photo 6, the figure of a car is nearly oblique projection. Generally speaking, oblique projection is rarely found in pictures at the age 5.

The above children's development of drawing methods led me to study the case with human race in general.

1.2 Ancient Pictures in Japan

In ancient Japan, pictures drawn by painting specialists appear in about the 7th century, affected by pictures in ancient China and Korea. Needless to say, we can find many naive pictures before the 7th century. I will show some ancient pictures.

Photo 7 is pictures of hunting and a warehouse drawn on "Dotaku"(accurate reproduction) in about B.C.1st-A.D.1st century.

We see the following.

(1) In hunting, profile appears.

(2) In the warehouse, horizontal-line ground appears. And the base lines of the right side and the left side of the warehouse run on a common horizontal line with the base of the front of the warehouse. This drawing method is a sort of elevation-depth projection.

Photo 8 and Photo 9 are pictures drawn on silk embroidery of "Tenjukoku" at Chuguji Temple, woven in 622(repaired correctly in 1275). We see the following.

(3) In Photo 8, both horizontal-line ground and depth ground appear. And the drawing method of the bell tower is elevation-depth projection.

(4) In Photo 9, priests' faces are drawn as oblique profile.

Photo 10 is the halo of "Yumedono-Kannon" engraved in the early 7th century at Horyuji Temple. We see the following.

(5) The pagoda is drawn as excellent oblique projection.

Generally speaking, in ancient Japanese pictures, oblique profile and oblique projection appear in about the 7th century.

The above development of drawing methods in Japanese ancient pictures led me to study the case with ancient China.

2. Development of Ancient Pictures in China

2.1. Pictures on Painted Pottery Bowls in Neolithic Age

In the following, I will mainly explain drawing methods in Han period(B.C.202-A.D.220), because we find broad and rapid development of drawing methods in the period.

First, let us see pictures on painted pottery bowls in Neolithic age(about B.C.4000-B.C.1000), attending to the drawing method of a person's face.

Photo 11 through Photo 13 are bowls excavated in Qinghai Sheng. We see the following.

(1) In Photo 11, dancers' faces are drawn in full face. In Photo 12, carriers' faces are drawn in profile. In Photo 13, dancers' faces are drawn in profile.

Generally speaking, in painted pottery bowls, profiles are rarely found. However, the profiles are full of spirit.

2.2 Pictures Found in the Western Han Tombs Mawangdui, Hunan Sheng

The tombs at Mawangdui were built in about B.C.170. Pictures found in the tombs show very important aspects of drawing methods at that time. In the following I will explain some of them.

Photo 14 and Photo 15 are pictures from Tomb No.1. We see the following.

(1) In Photo 14, the drawing method of person's face is profile. On the other hand, in Photo 15, the method is oblique profile.

(2) In Photo 14, the drawing method of ground is horizontal-line ground. In Photo 15, the method is depth ground.

(3) In Photo 14, three lady attendants are drawn in the following way. The feet of two attendants at the back are translated horizontally so as to be on a common horizontal line with the feet of the attendant at the front. This drawing method is a sort of elevation-depth projection.

(4) In Photo 15, we find oblique projection in structures in front of the two persons.

Photo 16 and Photo 17 are pictures from Tomb No.3. We see the following.

(5) In Photo 16, we can find many oblique profiles and depth ground.

(6) In Photo 17, the feet of the horse-vehicles at the back are translated perpendicularly so as to be on common perpendicular lines at the left side and the right side with the feet of the horse-vehicle at the front. This drawing method, too, is a sort of elevation-depth projection. Hereafter this method is referred to as "perpendicular elevation-depth projection". On the other hand the former method is referred to as "horizontal elevation-depth projection".

Judging from several pictures in those days, we may say the following.

Horizontal elevation-depth projection comes into existence on the basis of horizontal ground. Perpendicular elevation-depth projection comes into existence on the basis of depth ground. Oblique projection comes into existence on the basis of synthesizing both the horizontal elevation-depth projection and the perpendicular one. And oblique profile comes into existence in relation to the concept of oblique projection. In Western Han period(B.C.202-A.D.8), both of oblique profile and oblique projection began to gain ground. At the same time, profile, horizontal elevation-depth projection and perpendicular one are used as valuable methods. like in the case of Photo 14.

2.3 Stone Reliefs Found in Eastern Han Tombs

I will show several stone reliefs in Eastern Han(A.D.25-220). Photo 18 and Photo 19 are stone reliefs from Jiaxiang, Shandong. We see the following.

(1) In Photo 18, the drawing methods of the face and the ground are profile and horizontal-line ground, respectively.

(2) In Photo 19, the drawing method of the vehicle is horizontal elevation-depth projection. On the other hand, the drawing method of three horses synthesizes the horizontal elevation-depth projection and the perpendicular one. And it is near oblique projection.

Photo 20 and Photo 21 are stone reliefs from Yinan, Shandong. We see the following.

(3) In Photo 20, the drawing methods of the persons' faces and the horse vehicle are oblique profile and oblique projection, respectively.

(4) In Photo 21, the drawing method of the house is oblique projection.

Generally speaking, in Eastern Han period, oblique profile and oblique projection were established. At the same time, old drawing methods were used sticking to formality.

3. Conclusion

It may be given as conclusion that in both cases of children and ancient Japan, development of drawing methods of a face, the ground and the rectangular parallelepiped bears a close parallel to the ones in ancient China.

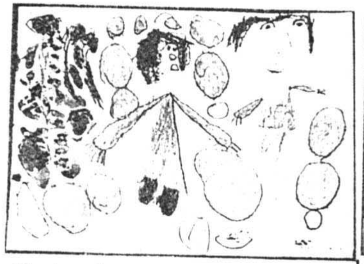
Photo 1


Photo 2

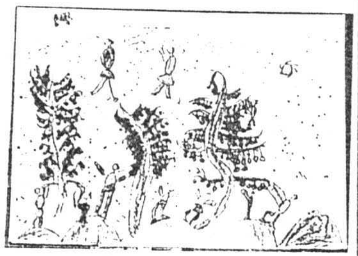
Photo 3

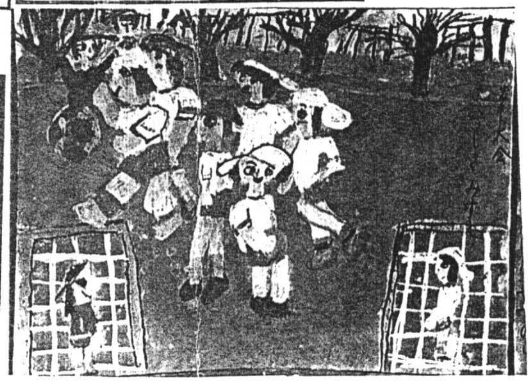
Photo 4

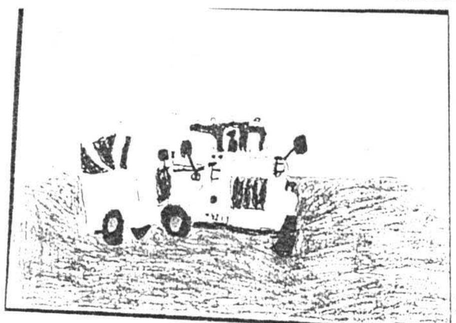
Photo 5

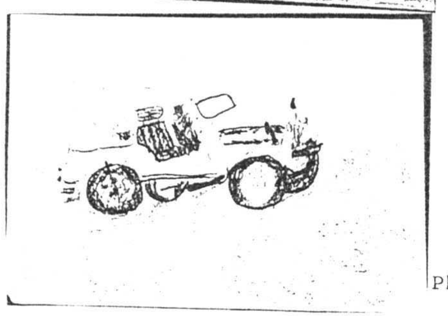
Photo 6


Photo 7

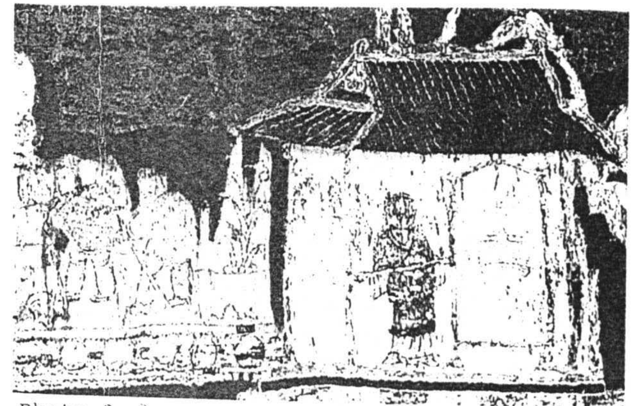
Photo 8

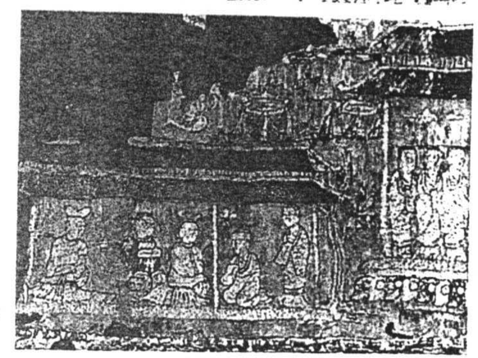
Photo 9

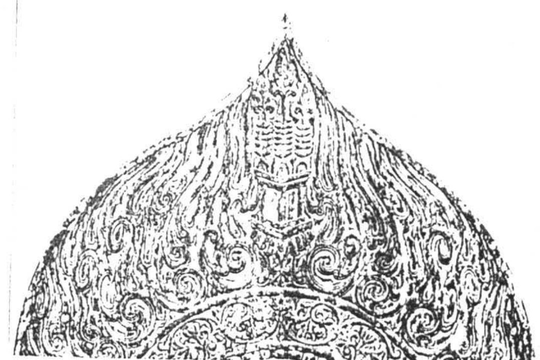
Photo 10

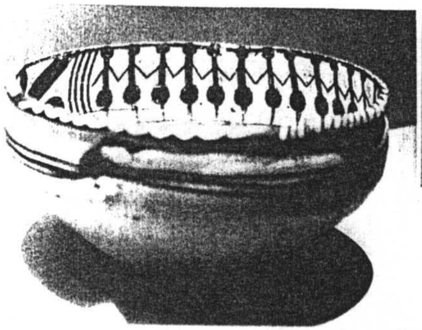
Photo 11

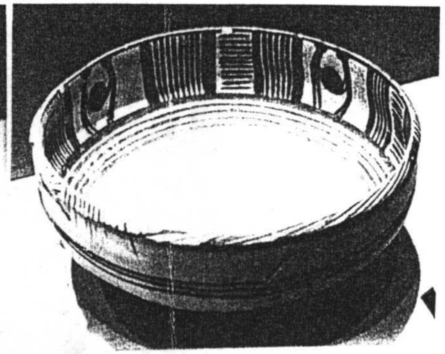
Photo 12


Photo 13


Photo 14


Photo 15

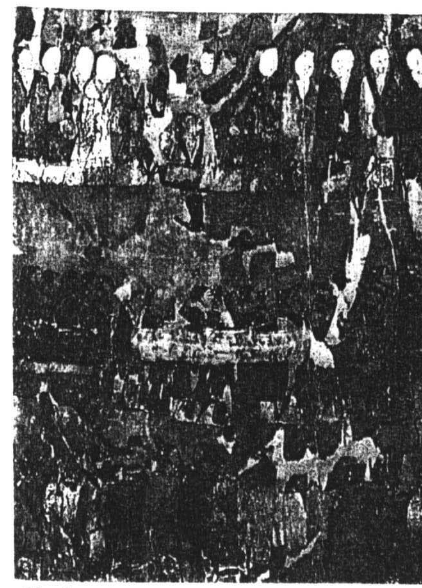
Photo 16


Photo 17

Photo 18

Photo 19

Photo 20

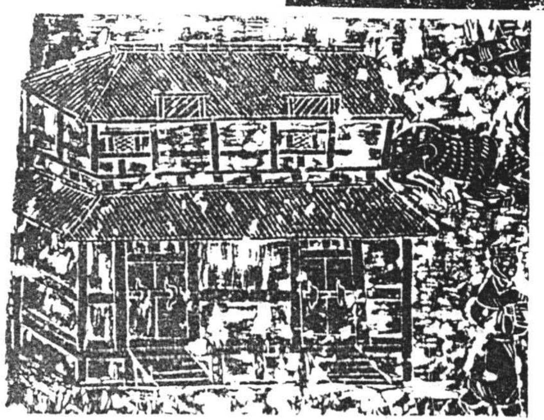

Photo 21

THE ELEMENTS OF CHINESE MATHEMATICS IN THE BEGINNING WORKS OF WASAN

TAKENOUCHI, Osamu

Osaka International University

Abstract Japanese traditional mathematics called Wasan (和算) started in the seventeenth century. In the beginning period, people must have studied seriously the Chinese mathematics which were then highly developed. In this talk, the author would like to elucidate what kind of literature they could refer to through the works of Seki-Takakazu.

1 Introduction

Seki-Takakazu (関孝和, 1642～1708) is the most notable figure in the history of mathematics in Japan. At the time when Wasan was created, people must have studied seriously the Chinese mathematics which were then highly developed. Among the individuals who built the first epoc of Wasan, the important names are Yoshida-Mitsuyoshi (吉田光由), Imamura-Chisho (今村知商), and Seki. Yoshida's Jingoki (塵劫記) and Imamura's Jugairoku (竪亥録) were widely used during the Edo era. But, from the point of view of mathematics, the most important ones are the various ingenious works of Seki.

Hitherto, it is commonly recognized that the literature of Chinese mathematics existing and was available at that time were:

Líu Huī (劉徽) : Jiǔzhāng suànshù (九章算術) : ca 3rd century

Nine Chapters on the Mathematical Arts

Yáng Huī (楊輝) : Yáng Huī suàngfā (楊輝算法) : 1274

Yáng Huī's Methods of Computation

Zhū Shìjié (朱世傑) : Suàngxué qīméng (算学啓蒙) : 1298

Introduction of Mathematical Studies

Chéng Dàwèi (程大位) : Suàngfā tōngzóng (算法統宗) : 1592

Systematic Treatise on Arithmetic

But, examining the works of Seki, the present author now has come to think that his works are based on several other important masterpieces of the ancient Chinese mathematics.

Philologically, it is not certain that the following important literature existed in our country:

Lǐ Yě (李冶) : Céyuán hǎijíng (測圓海鏡) : 1248

Sea Mirror of Circle Measurements

Qín Jiǔsháo (秦九韶) Shùshūjiǔzhāng (数書九章) : 1247

Mathematical Treatise in Nine Sections

Zhū Shìjié (朱世傑) : Sìyuán yùjiàn (四元玉鑑) : 1303

Precious Mirror of the Four Elements

There are no records which prove the existence of these books in seventeenth century Japan. But, the present author feels the strong influence of these books on Seki's works. The intention of this talk is to describe the basis for this feeling.

2 Katsuyo Sampo (括要算法) Principal Mathematical Reasonings : 1712

Seki wrote many works which are collected in his "Collected Works".

Katsuyo Sampo was posthumously published by his students. It consists of four volumes for a total of 200 pages. The content of each volume is as follows.

Vol.1 a. Interpolation method

b. Sum of powers of natural numbers

Vol.2 Elements of number theory, including indeterminate equations

Generalization of Chinese remainder theorem

Vol.3 Regular polyhedra

Given the length of an edge of a regular polyhedron, find the length of radii of inscribed and circumscribed circles. The number of edges is from 3 to 20.

Along with geometric considerations, the celestial element method was an important tool to give results.

Vol.4 a. Circle number π

(1) Find the length of the circumference of a regular polyhedron inscribed in a fixed circle, number of edges is 4, 8, 16, $\cdots$, 32768, 65536, 131072.

(2) His famous acceleration method

Let $a =$ the length of the circumference of a regular polyhedron with 32768 edges

$b =$ that with 65536 edges

$c =$ that with 131072 edges

Then,

$$\begin{aligned}\text{approximative value of } \pi &= b + \frac{(b-a)(c-b)}{(b-a)-(c-b)} \\ &= 3.14159265359 \text{ slightly less}\end{aligned}$$

He made repeated use of this acceleration method in this paper.

(3) Approximation by fractions.

He made successively fractions $\frac{3}{1}, \frac{7}{2}, \frac{10}{3}, \cdots$, and examined the scale of approximation to the number π. And finally he got

$$\text{approximative value of } \pi = \frac{355}{113}$$

This fraction was found by Zǔ Chōngzhī (祖冲之) already in the fifth century, but in Chinese literature thereafter, his discovery was ignored and the fraction $\frac{22}{7}$ was only used as the rate of Zǔ Chōngzhī. Seki seems to have been unaware of this result. Only some twenty or thirty years later, his student Takebe reported that he found the literature telling the discovery of Zǔ Chōngzhī.

b. Arc length of a circle

Given an arc of a circle, find its length from the sagitta to this arc.

He did not succeed to establish a complete result. This was accomplished by one of his students Takebe-Katahiro.

He employed here another interpolation method.

c. Volume of a sphere

3 Sum of powers of natural numbers

The following is the well-known formula due to Jakob Bernoulli (1714).

$$\begin{aligned}1^p + 2^p + \cdots + n^p &= \frac{1}{p+1} n^{p+1} + \frac{1}{2} n^p + B_1 \frac{p}{2} n^{p-1} - B_2 \frac{p(p-1)(p-2)}{2 \cdot 3 \cdot 4} n^{p-3} + \cdots \\ &= \frac{1}{p+1} \left\{ n^{p+1} + \frac{1}{2} {}_{p+1}C_1 n^p + B_1 \, {}_{p+1}C_2 n^{p-1} - B_2 \, {}_{p+1}C_3 n^{p-3} + \cdots \right\}\end{aligned}$$

Here $B_1, B_2, \cdots$ are the numbers known as Bernoulli numbers:

$$B_1 = \frac{1}{6}, \quad B_2 = \frac{1}{30}, \quad B_3 = \frac{1}{42}, \quad B_4 = \frac{1}{30}, \quad \cdots$$

Seki himself obtained this formula in the form as in Table 1.

One should notice here, that he apparently knew the table of binomial coefficients which is commonly called the Pascal Triangle.

He showed also Table 2.

This table may be observed showing the construction of the binomial coefficients.

Now, here are questions.

Question 1. How did he know the table of binomial coefficients?

Question 2. How did he know that the binomial coefficients have this structure?

In the Chinese literature, the table of binomial coefficients first appeared in Sìyuán yùjiàn (四元玉鑑). See Table 3.

No explanation is made in this book, but as the title of the table is "Ancient Method (古法)", this table may have been widely known in his day.

Further, it should be noticed that in the tenth chapter of vol. 2 of this book, which consists of 3 volumes, is devoted to the summation problem of the sequence of natural numbers. And we can recognize there that Zhū Shìjié (朱世傑) made explicit use of the property of the table of binomial coefffients. Indeed, he showed

$$3^3 + 4^3 + \cdots + n^p = 27_{n-2}C_1 + 37_{n-2}C_2 + 24_{n-2}C_3 + 6_{n-2}C_4$$

Here, 27, 37, 24 and 6 are the successive differences of 3^3, 4^3, 5^3 and 6^3. This formula can be derived by using the property of the table of binomial coefficients. He also gave the structure of the binomial coefficients by examples.

Therefore, it is my strong feeling that this part of the book gave a big hint to Seki.

4 Tiān Yuán Shù (天元術) Celestial Element Method

In the thirteenth century in China, amazing progress was made in the domain of algebra. That is the celestial element method. It is the theory of algebraic equations, according to which mathematicians like Lǐ Yě (李冶), Qín Jiǔsháo (秦九韶), Zhū Shìjié (朱世傑) were able to write outstanding works.

Now, our Seki made the extensive use of the celestial element method in his works.

Here again is a question.

Question 3. By what means, could he study the celestial element method?

According to commonly accepted opinion, he mastered the method by making studies of Yáng Huī suàngfā (楊輝算法) and Suàngxué qǐméng (算学啓蒙). But, in these books, applications are made whereas no precise accounts of the method are given.

The present author considers that Qín Jiǔsháo (秦九韶)'s Shùshūjiǔzhāng (数書九章) was for him a great aid to understanding the celestial element method.

Another reason of the author's belief about the influence of this book to Seki is the study of the theory of numbers. As is shown in section 2, Seki made some studies about the residual relations of natural numbers. We see that Shùshūjiǔzhāng (数書九章) and Suàngfā tōngzóng (算法統宗) are important literature concerning the residual relations. But it seems to the author that the writing style of Seki more closely resembles that of the former.

About the celestial element method, Lǐ Yě (李冶)'s Céyuán hǎijíng (測圓海鏡) also gives precise accounts. But it is more likely that the emphasis should be placed on the geometric side, and, though Seki himself made an extensive study of geometric objects, the author feels there is a good deal of difference between their studies, and at present, he does not think this book influenced Seki.

Seki invented a method of describing various quantities aside the rod of celestial element method. (Table 4) This may be called the aside writing method (傍書法), which made the understanding of the meaning of the expression easier. In Sìyuán yùjiàn (四元玉鑑), Zhū Shìjié (朱世傑) had developed methods managing 2, 3 and 4 unknowns. Perhaps, after having investigated the managing way of multiple elements method, in which the meaning of various expressions should be kept in mind during the calculation, the idea of writing the quantities aside came to Seki's mind. This is, so to say, the symbolic manipulation of celestial element method. This aside writing method was thereafter accepted by the workers of Wasan, and helped much their research.

5 Conclusion

There are discussions about what kind of literature of Chinese mathematics existed at the beginning period of Wasan.

Looking after the Seki's works, the author intends to get some idea about this problem. He thinks that there were several important books of mathematics more than those hitherto considered.