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BULLETIN OF THE EARTHQUAKE
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1. *The Variable Period Hypothesis and Q of the Chandler Wobble Reexamined.*

By Shuhei OKUBO*,

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(Received February 16, 1982)

Abstract

The Chandler wobble is one of the elastic-gravitational normal modes of the Earth. The eigenperiod is about 435 sidereal days, larger than the other modes by a factor 10^4, which gives the Chandler wobble an exotic status in the group of normal modes. The quality factor of the Chandler wobble, Q_w, plays a critically important role in discussing the mantle rheology since the frequency dependence of the mantle Q is affected more seriously by the low frequency Chandler Q_w than by the other seismic Q's.

A problem should be resolved in advance so as to estimate Q_w safely from the spectral analysis of the polar motion. Namely, significance of the variable Chandler period hypothesis. We test the hypothesis in two ways. First, we reexamine throughly the observational grounds of the hypothesis by applying the same scheme as employed by the proponents of the hypothesis to synthetic polar motion of a constant Chandler period. It is revealed that most of the evidence for the hypothesis is not definitive and it is also explained by the invariant Chandler period model as well. Next, we trace the time variation of the spectral structure of the Chandler wobble. For this purpose, we extend the high-resolution Instantaneous Frequency Analysis to be applicable to a complex-valued time series. Applying the technique to IPMS and BIH polar motion data, we find that the result favors the time-invariant Chandler period model.

After confirming that there is no observational difficulty in the time-invariant Chandler period model, we estimate Q_w by critically applying Maximum Entropy Spectral Analysis (MESA) to ILS and IPMS data. It is found that Q_w lies in the range of $50 \leqq Q_w \lesssim 100$ and Graber's result of $Q_w = 600$ is due to the erroneous choice of the length of the prediction error filter.

Finally, we discuss the effect of the mantle anelasticity upon the period and Q of the Chandler wobble. We calculate the complex

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Love number k for the realistic Q models by Rayleigh's principle and relate it to Q_w. The ratio of Q_w to lower mantle Q_m is found to be about 1.5 and it is shown to be consistent with the energy budget arguments. If the Chandler wobble energy is totally dissipated in the mantle, Q_m should be frequency-dependent to account for the observed Q_w of 50~100. If the frequency dependence of Q_m is the power law, the power exponent is found to be 0.1~0.2. Anelasticity of the mantle is shown to lengthen the theoretical Chandler period by 7~11 days due to physical dispersion. Adding 29.8 days of ocean effect to the theoretical period of oceanless Earth yields 438~443 sidereal days as the Chandlerian period, in excellent agreement with the observed one.

1. Introduction

The Chandler wobble is a free vibration of the Earth's rotational axis around the figure axis (MUNK & MACDONALD, 1960). Its geometrical expression is given by the Poinsot representation (Fig. 1a, b). The angular momentum axis H is fixed in space in the absence of external torques. The Earth is attached to the outer cone (body cone), and its figure axis x_3 rotates about the instantaneous rotation axis ω with a nearly diurnal period. As the body cone swings around the H axis keeping contact with the inner cone (space cone), the instantaneous rotation axis ω oscillates around the Earth's figure axis (Fig. 1c, d). This is the wobble and it is observed astronomically as latitude variation.

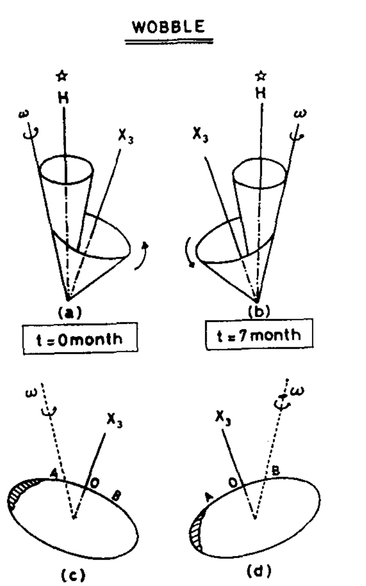

Fig. 1. (a), (b) Poinsot representation of the Chandler wobble. (c), (d) Spatial configurations of the Earth corresponding to (a) and (b).

The period and Q of the Chandler wobble are two of the most fundamental parameters that constrain the elastic and possibly anelastic properties of the solid Earth. Realizing the geophysical importance of these two parameters, numerous investigators have tried to determine those values as accurately as possible (RUDNICK, 1956; JEFFREYS, 1968; CURRIE, 1974; OOE, 1978). According to these researchers, the Chandler period is about 435 sidereal days (ROCHESTER, 1973; LAMBECK, 1980), and it is quantitatively explained by applying the elastic-gravitational normal mode theory to realistic Earth models and assuming the equilibrium pole tide (DAHLEN, 1976 ; SMITH, 1977 ; SASAO, OKUBO & SAITO, 1980 ; SMITH & DAHLEN, 1981). However, there is a fierce controversy among researchers whether the Chandler period is steady or variable in time (Table 1). Not a few authors, including Chandler himself, postulated multiple or variable period hypothesis (CHANDLER, 1892 ; MELCHIOR, 1957 ; SEKIGUCHI, 1972, 1976 ; GAPOSCHKIN, 1972 ; CARTER, 1981), but they are not yet confirmed because of the dubious nature of the analytical technique. It is rather astonishing that the hypothesis has survived nearly 90 years without either being completly rejected or confirmed. One of the objects of this study is to draw conclusion on this problem.

Table 1. History of the controversy on the variable/multiple Chandler period.

variable/multiple		invariant/single	
Chandler	(1892)	Newcomb	(1892)
Kimura	(1918)		
Hattori	(1949)		
Melchior	(1957)	Munk & MacDonald	(1960)
Colombo & Shapiro	(1968)		
Gaposchkin	(1972)	Pedersen & Rochester	(1972)
Sekiguchi	(1976)	Ooe	(1978)
Carter	(1981)		

Contrary to the agreement on the estimates of the Chandler period (except for the time-variable period hypothesis), there is no consensus about the Chandler wobble Q (hereafter denoted by Q_w) (Table 2). Estimated values range from 25 to a high of 600. RUDNICK (1956) estimated $Q_w=25$ from the periodgram analysis of International Latitude Service (ILS) data of 54.4 years long. CURRIE (1974) gave $Q_w=72$ from Maximum Entropy Spectral Analysis (MESA) of ILS data of 73 year duration. GRABER (1976) also applied MESA to International Polar Motion Service (IPMS) data of 15 year long and obtained $Q_w=600$. Differences of the adopted technique and the length of analyzed data affect the estimate for Q_w considerably as shown above. The matter becomes more complicated if the time-variable Chandler period hypothesis is the case. It is expected that the ordinary harmonic analysis of the polar motion with a variable period and higher Q_w ($Q_w \gg 100$) would yield apparently lower Q_w value ($Q_w < 100$). Since the frequency dependence of the mantle Q (hereafter

Table 2. Previous estimates of the Chandler wobble Q.

$Q_w<30$	$50<Q_w<100$	$Q_w>500$
Rudnick (1956)	Jeffreys (1968)	Graber (1976)
Walker & Young (1957)	Claerbout (1969)	
	Currie (1974)	
Munk & MacDonald (1960)	Wilson & Haubrich (1976)	
	Ooe (1978)	

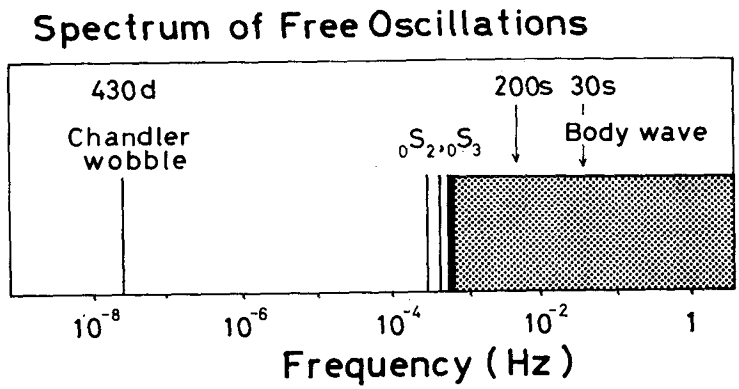

Fig. 2. Schematic line spectrum of the Earth's elastic-gravitational normal modes.

denoted by Q_m) is often discussed based upon the estimate of Q_w, an accurate determination of Q_w is critically important (ANDERSON & MINSTER, 1979) (Fig. 2).

In summary, models of the Chandler wobble can be classified into three groups.

(1) Multiple or variable Chandler period with a high Q_w ($Q_w \gg 100$).
(2) Single Chandler period with a high Q_w ($Q_w \gg 100$).
(3) Single Chandler period with a low Q_w ($Q_w < 100$).

In this paper, we will test all the evidence which seems to support the variable period hypothesis and show that it is also explained by the model (3). In order to test the hypothesis more directly, we trace the time-variation of the spectral structure of the polar motion. For this purpose, we extend the Instantaneous Frequency Analysis to be applicable to a complex-valued time series, The result does not reveal significant fluctuation of the Chandlerian period and favors the invariant period model.

After confirming that there is no observational obstacle to the invariant Chandler period model, we can safely estimate Q_w from the spectral analysis of the polar motion. The most favorable value is found to be $50 \leqq Q_w \leqq 100$ from the simulation approach of MESA. We will also elucidate the cause of the wide discrepancy between the estimated Q_w's by the previous authors.

If the wobble energy is totally dissipated within the mantle, some relation should hold between Q_w and Q_m at the Chandler frequency. Q_w and Q_m are defined as

$$Q_w = 2\pi E_w / \Delta E$$

$$Q_m = 2\pi E_s / \Delta E$$

where E_w and E_s are the total energy of the wobble and the strain energy, respectively. ΔE is the amount of energy dissipated in one cycle of the oscillation. E_w is larger than E_s since E_w includes the kinetic, the gravitational and the strain energies. Hence Q_m is smaller than Q_w. Most of the earlier investigators supposed $E_w \cong 10 \cdot E_s$ from the kinematical arguments and obtained $Q_w \cong 10 \cdot Q_m$ (STACEY, 1969, 1977; MERRIAM & LAMBECK, 1979). However, we find that E_w/E_s is at most 2 and Q_w/Q_m is also $1 \sim 2$ from the more rigorous treatment. We confirm the result by computing the complex Chandler frequency for the realistic Earth models with complex elastic moduli by Rayleigh's principle. Using the relation between Q_m and Q_w and the observed Q_w value, the frequency dependence of the mantle Q will be discussed.

Anelasticity of the mantle induces what is called physical dispersion. Hence the elastic moduli (real part) at the Chandler frequency is different from those appropriate for the seismic frequency range. We will assess the effect of physical dispersion on the Chandler period and show that the Chandler period is lengthend by about $7 \sim 11$ days.

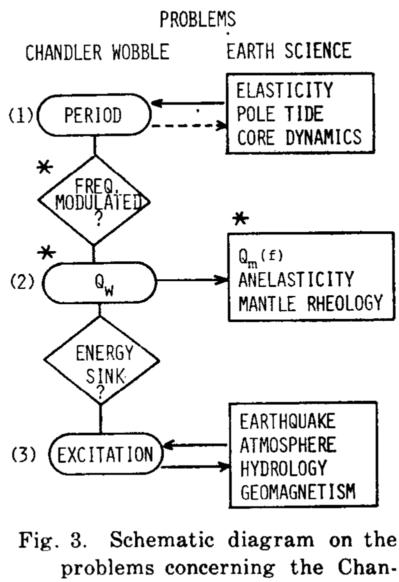

Fig. 3. Schematic diagram on the problems concerning the Chandler wobble. * denotes the aim in this study.

2. Formulation of the Earth's rotation

§ 1. Derivation of the basic wobble equation

The rotation of the Earth is a complicated problem owing to its radially inhomogeneous structure and the presence of the fluid outer core. We shall briefly review the formulation of the rotation dynamics of the oceanless, slightly elliptical, realistic Earth after SASAO, OKUBO & SAITO (1980).

(1) Basic state

We use a reference frame fixed to the mean principal axes of the mantle rotating with an angular velocity $\vec{\omega}$.

$$\vec{\omega}=\Omega(m_1, m_2, 1+m_3)$$

We take $\vec{i}_1$, $\vec{i}_2$ and $\vec{i}_3$ as the basis vectors in this frame. The hydrostatic equilibrium state in the fluid core is expressed as

$$\nabla P_0=\rho_0\nabla\phi_0$$

where P, ρ and ϕ denote the pressure, the mass density and the gravitational potential (including centrifugal potential), respectively. Subscript 0 designates the basic state. We assume the coincidence of the equipotential and equidensity surfaces in the basic state. The distance from the origin to the surface, r_0, is given by

$$r_0=r(1-(2/3)\cdot\varepsilon(r)\cdot P_2(\cos\theta))$$

where r and θ are the distance parameter and the colatitude, respectively. $\varepsilon(r)$ is the geometrical ellipticity of the equipotential surface and P_2 is the Legendre function of degree 2.

$\varepsilon(r)$ is given by integrating the Clairaut equation.

$$\frac{d^2\varepsilon}{dr^2}-\frac{6}{r^2}\varepsilon+\frac{8\pi G\rho_0}{g_0}\left(\frac{d\varepsilon}{dr}+\frac{\varepsilon}{r}\right)=0$$

where g_0 is given by

$$g_0(r)=4\pi G\int_0^r\rho_0(b)b^2\,db/r^2$$

The boundary conditions for $\varepsilon(r)$ are

$$r\frac{d\varepsilon}{dr}+2\varepsilon=(5/2)\,\Omega^2r/g_0 \qquad \text{at } r=a$$

$$\frac{d\varepsilon}{dr}=0 \qquad \text{at } r=0$$

(2) Velocity field in the fluid core

The velocity field in the fluid core is assumed to be composed of a uniform rotation and a small correcting term $\vec{v}$.

$$\vec{v}_f=\vec{\omega}_f\times\vec{r}+\vec{v}$$

$$\vec{\omega}_f = \Omega(m_1^f, m_2^f, m_3^f)$$

where $\vec{\omega}_f$ denotes the angular velocity of the fluid core. The hydrodynamic equations of motion and continuity for the perturbed state are given by

$$\rho_0\left(\frac{\partial \vec{v}_f}{\partial t} + \frac{d\vec{\omega}}{dt} \times \vec{r} + 2\Omega \vec{i}_3 \times \vec{v}_f\right) = -\nabla P_1 + \rho_1 \nabla \phi_0 + \rho_0 \nabla \phi_1$$

$$\frac{\partial \rho_1}{\partial t_1} + \rho_0 \nabla \cdot \vec{v}_f + \vec{v}_f \cdot \nabla \rho_0 = 0 \qquad (2\text{-}1)$$

where the subscript 1 designates the perturbed state. The Poisson equation is

$$\nabla^2 \phi_1 = -4\pi G \rho_1$$

The perturbed potential is composed of three terms.

$$\phi_1 = \phi_e + \phi_m + \phi_d$$

$$\phi_e = (\Omega^2/3) r^2 \,\mathrm{Re}\,[\tilde{\phi} Y_2^1]$$

$$\phi_m = -(\Omega^2/3) r^2 \,\mathrm{Re}\,[\tilde{m} Y_2^1]$$

$$\nabla^2 \phi_d = -4\pi G \rho_1$$

where Y_2^1 is the spherical surface harmonic of degree 2, order 1. ϕ_e is the external tidal potential and ϕ_m is the pole tide potential. ϕ_d is the gravitational potential arising from the elastic deformation of the Earth. $\tilde{m}$ designates the complex representation of the wobble and is defined as

$$\tilde{m} = m_1 + i m_2$$

Introducing new quantities

$$\phi_f = -(\Omega^2/3) r^2 \,\mathrm{Re}\,[\tilde{m}_f Y_2^1] = -\Omega^2 (m_1^f xz + m_2^f yz)$$

$$\tilde{m}_f = m_1^f + i m_2^f$$

(2-1) is rewritten as

$$\frac{\partial \vec{v}}{\partial t} + \frac{d(\vec{\omega} + \vec{\omega}_f)}{dt} \times \vec{r} + 2\Omega \vec{i}_3 \times \vec{v} + \Omega(\vec{i}_3 \times \vec{\omega}_f) \times \vec{r} = -\nabla \boldsymbol{P} - \boldsymbol{R} \nabla r_0 \qquad (2\text{-}2)$$

$$\boldsymbol{P} = P_1/\rho_0 - \phi_1 - \phi_f$$

$$\boldsymbol{R} = P_1 (d\rho_0/dr_0)/\rho_0^2 - \rho_1 (d\phi_0/dr_0)/\phi_0 .$$

Multiplying (2-2) by $\rho_0 \vec{r}$ vectorically and integrating the product over the whole core, we obtain

$$\frac{d\vec{H}_f}{dt}-\vec{\omega}_f\times\vec{H}_f=-\vec{\omega}\times\int\rho_0\vec{r}\times\vec{v}dv-\int\rho_0\boldsymbol{P}\vec{r}\times d\vec{S}$$

$$+\int(\boldsymbol{P}d\rho_0/dr_0-\rho_0\boldsymbol{R})\vec{r}\times\nabla r_0 dV$$

$$\vec{H}_f=A_f(\vec{\omega}+\vec{\omega}_f)+(C_f-A_f)\Omega\vec{i}_3+c^f_{31}\Omega\vec{i}_1$$

$$+c^f_{32}\Omega i_2+\int\rho_0\vec{r}\times\vec{v}dV \tag{2-3}$$

where A_f and C_f are the least and the greatest principal moments of inertia and $-c^f_{31}$ and $-c^f_{32}$ are the products of inertia of the fluid core. We may choose $\vec{\omega}_f$ so that $\int\rho_0\vec{r}\times\vec{v}dV$ vanishes without loss of generality. SASAO, OKUBO & SAITO showed that $\boldsymbol{P}$ and $\boldsymbol{R}$ are of the order of $\Omega|\vec{v}|$ and the integrals on the right hand side of (2-3) are negligible since $\boldsymbol{P}$ and $\boldsymbol{R}$ are further multiplied by factors of order $\varepsilon(r)$. Thus we obtain

$$\frac{d\vec{H}_f}{dt}-\vec{\omega}_f\times\vec{H}_f=0 \tag{2-4}$$

(3) Basic wobble equations

The Liouville equation of the whole Earth is

$$\frac{d\vec{H}}{dt}+\vec{\omega}\times\vec{H}=\vec{L} \tag{2-5}$$

$$\vec{H}=A\vec{\omega}+(C-A)\Omega\vec{i}_3+A_f\vec{\omega}_f+c_{31}\Omega\vec{i}_1+c_{32}\Omega\vec{i}_2$$

$$\vec{L}=\int\rho_0\vec{r}\times\nabla\phi_e dV$$

Equations (2-4) and (2-5) give

$$A_f[D\tilde{m}+(D+i(1+e_f)\Omega)\tilde{m}_f]+D\tilde{c}_3{}^f=0$$

$$A(D-ie\Omega)\tilde{m}+(D+i\Omega)(A_f\tilde{m}_f+\tilde{c}_3)=-iAe\Omega\tilde{\phi} \tag{2-6}$$

where D stands for d/dt. e and e_f are the dynamical flattenings of the Whole Earth and the fluid core, respectively.

$$e=(C-A)/A$$

$$e_f=(C_f-A_f)/A_f$$

$\tilde{c}_3$ and $\tilde{c}^f_3$ are defined as

$$\tilde{c}_3=c_{31}+ic_{32}$$

$$\tilde{c}^f_3=c^f_{31}+ic^f_{32}$$

SASAO, OKUBO & SAITO showed that $\tilde{c}_3$ and $\tilde{c}_3{}^f$ arising from the elastic deformation are given by

$$\tilde{c}_{3,\mathrm{e}}=-A[\kappa(\tilde{\phi}-\tilde{m})-\xi\tilde{m}_{\mathrm{f}}]$$

$$\tilde{c}_{3}{}^{\mathrm{f}}{}_{,\mathrm{e}}=-A_{\mathrm{f}}[\gamma(\tilde{\phi}-\tilde{m})-\beta\tilde{m}_{\mathrm{f}}] \tag{2-7}$$

where κ, ξ, β and γ are the physical constants characteristic of each Earth model.

Substituting (2-7) into (2-6) yields

$$\begin{pmatrix} (1+\gamma)D\quad, & (1+\beta)D+i(1+e_{\mathrm{f}})\Omega \\ (1+\kappa)D+i(\kappa-e)\Omega\,, & (A_{\mathrm{f}}/A+\xi)(D+i\Omega) \end{pmatrix}\begin{pmatrix} \tilde{m} \\ \tilde{m}_{\mathrm{f}} \end{pmatrix}=\begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \end{pmatrix}=\begin{pmatrix} \gamma D\tilde{\phi}-D\tilde{c}_3{}'^{\mathrm{f}}/A_{\mathrm{f}} \\ \kappa D\tilde{\phi}-ie\Omega\tilde{\phi}-(D+i\Omega)\tilde{c}_3{}'/A \end{pmatrix} \tag{2-8}$$

where $\tilde{c}_3{}'$ and $\tilde{c}_3{}'^{\mathrm{f}}$ are given by

$$\tilde{c}_3{}'=\tilde{c}_3-\tilde{c}_{3,\mathrm{e}}$$

$$\tilde{c}_3{}'^{\mathrm{f}}=\tilde{c}_3{}^{\mathrm{f}}-\tilde{c}_3{}^{\mathrm{f}}{}_{,\mathrm{e}}$$

Now we can derive eigenfrequencies of the free motion by solving the secular equation. Since κ, ξ, β and γ are of order 10^{-3} or less, a first approximation for the secular equation is

$$\begin{vmatrix} \sigma\quad, & \sigma+\Omega \\ \sigma+(\kappa-e)\Omega\,, & (A_{\mathrm{f}}/A)(\sigma+\Omega) \end{vmatrix}=0$$

$$(\sigma+\Omega)(\sigma-A(e-\kappa)\Omega/A_{\mathrm{m}})=0 \tag{2-9}$$

where σ denotes angular frequency. A_{m} is the least principal moment of inertia of the mantle and it is given by

$$A_{\mathrm{m}}=A-A_{\mathrm{f}}\,.$$

The roots of the equation (2-9) correspond to the angular frequencies of the Chandler wobble and the free core nutation (nearly diurnal wobble). The Chandlerian angular frequency for the oceanless, perfectly elastic Earth is given by

$$\sigma_{\mathrm{e}}=(A/A_{\mathrm{m}})\cdot(e-\kappa)\Omega \tag{2-10}$$

Now we shall assess the effect of oceans upon the rotation dynamics. Mobility of oceans induces variation of c'_{13} and c'_{23} synchronous to wobble due to the pole tide.

$$\tilde{c}'_{3,\mathrm{o}}=c'_{13,\mathrm{o}}+ic'_{23,\mathrm{o}}=(\sigma'/\Omega)A_{\mathrm{m}}\cdot\tilde{m} \tag{2-11}$$

where subscript o refers to the effect of oceans. σ' is a physical constant. Substituting (2-11) into (2-8) and solving the secular equation, we obtain

$$\sigma_c = \sigma_e - \sigma' \tag{2-12}$$

Equation (2-12) implies the lengthening of the Chandler period. Numerical analysis showed that the Chandler period is lengthened by about 29.8 days (DAHLEN, 1976 with correction by SMITH & DAHLEN, 1981).

Finally, we shall take the mantle anelasticity into account by assuming complex elastic moduli. Since κ is related to the Earth's elasticity through the static Love number k by

$$\kappa = ka^5\Omega^2/(3GA)$$

complex elastic moduli make κ complex as well as k. Hence, the Chandler angular frequency given by (2-12) also becomes complex.

$$\begin{aligned}\tilde{\sigma}_c &= \tilde{\sigma}_e - \sigma' \\ &= (A/A_m)(e - \tilde{k}a^5\Omega^2/(3GA))\Omega - \sigma' \end{aligned} \tag{2-13}$$

where $\sim$ stands for a complex value.
The Chandler wobble Q_w is derived from the imaginary part of $\tilde{\sigma}_c$.

$$Q_w = -(A_m/A)(3GA/a^5\Omega^3)\sigma_c/(2\,\mathrm{Im}\,[\tilde{k}]) \tag{2-14}$$

$$\sigma_c = \mathrm{Re}\,[\tilde{\sigma}_c]\,.$$

§ 2. Solution to the basic wobble equations

The solution to the wobble equation (2-8) is most easily obtained in the frequency domain. A Fourier transform of (2-8) allowing for (2-11) and (2-13) yields.

$$\begin{aligned} M(\sigma) &= \Phi(\sigma)/(\sigma - \tilde{\sigma}_c) \\ \Phi(\sigma) &= (A_f/A)\psi_1(\sigma) - \psi_2(\sigma) \end{aligned} \tag{2-15}$$

If $\Phi(\sigma)$ has a white spectrum, Q_w can be estimated from the spectrum of $m(t)$ by

$$Q_w = \sigma_c/\Delta\sigma \tag{2-16}$$

where $\Delta\sigma$ is the full half width of $|M(\sigma)|^2$.

$$|M(\sigma_c \pm \Delta\sigma/2)|^2 = |M(\sigma_c)|^2/2\,.$$

Random excitation of the Chandler wobble is not a bad approximation as shown by SEKIGUCHI (1976). Furthermore, the flatness of $\Phi(\sigma)$ just around σ_c is sufficient in order to estimate Q_w from (2-16). Most investigators so far have assumed the whiteness of Φ and presented estimates for Q_w. We will also take this view and estimate Q_w in Chapter 5.

3. Test of the variable Chandler period hypothesis

§ 1. The variable Chandler period hypothesis

Numerous authors suggested that the Chandler period is variable in time since the discovery of the Chandler wobble in the late 19-th century (CHANDLER, 1892; KIMURA, 1918; MELCHIOR, 1954, 1957; SEKIGUCHI, 1972, 1976; GAPOSCHKIN, 1972; CARTER, 1981). The hypothesis is characterized by the following empirical laws (MELCHIOR, 1957).

(1) The period of the Chandler wobble fluctuates. The maximum departure from the mean value is approximately 4%.

(2) Period and amplitude of the Chandler motion are proportional. The correlation coefficient is more than 0.8.

(3) A long Chandler period is correlated with a small amplitude of the annual motion.

If the Chandler motion is indeed variable, its intrinsic Q can never be estimated from the ratio of the spectral half width to the Chandler frequency. This is because the variable Chandler frequency inevitably broadens the width of the originally sharp line spectrum. Let the frequency be modulated by a fraction α. Ordinary harmonic analysis of this frequency-modulated time series is expected to yield a relatively broad peak in the frequency band of $f_c(1-\alpha)<f<f_c(1+\alpha)$ where f_c denotes the Chandler frequency. In this case, Q may be judged to be $Q_{app}=1/\alpha$. If α is 0.04 as suggested from the Melchior's first law, Q_{app} becomes 25, which has nothing to do with the intrinsic Q_w value.

The above argument makes the hypothesis very attractive, since it offers the explanation of an anomalously low Q of the wobble derived from the spectral analysis. However, the hypothesis has been suffering from serious defects, theoretically as well as observationally. The theoretical difficulty is that no physically plausible mechanism is presented which can cause the fluctuation of the Chandler period (NEWCOMB, 1892, MUNK & MACDONALD, 1960). Although a nonequilibrium pole tide is postulated as a possible cause, the theory still remains kinematical (DICKMAN, 1979; CARTER, 1981). From the observational point of view, it seems to us that the hypothesis is constructed on rather shaky grounds. In particular, the feasibility of the analyses indicating the variable period have not yet been fully tested. Hence, it is very probable that they may yield spurious "laws" described above even when they are applied to a synthetic polar motion with a invariant Chandler period and a finite Q_w.

We believe it is decisively important to test the observational "evidence" of the hypothesis at this stage because the mantle Q_m at the Chandler frequency is seriously affected by the reality of the variable period. The above consideration leads us to testing the hypothesis by applying the same technique employed by the variable period hypothesis to synthetic polar motion. Tested are the following methods.

(1) Running Harmonic Analysis
(2) Revolution Angle Analysis
(3) Autocorrelation Approach
(4) Beat Period Analysis
(5) Running Maximum Entropy Spectral Analysis

The above methods represent most of the earlier analytical techniques although they may not be complete. If they do not reveal spurious time-variability for the invariant Chandler frequency model, they are considered to pass the test and vice versa. Before applying these methods, we shall describe in the next section how to generate synthetic polar motions.

§ 2. Generation of synthetic polar motion

As is well-known, the polar motion is composed of three parts. They are

(1) The Chandler wobble, the excitation mechanism of which is not yet resolved. The mean amplitude is about 0.″15.

(2) The annual wobble, which is most probably excited by seasonal change of atmospheric and hydrological effects (WILSON & HAUBRICH, 1976a). Its mean amplitude is 0.″10.

(3) Secular drift of the order of 0.″003/yr in the direction of 70°W.

In order to generate synthetic polar motion assuming an invariant frequency and a finite Q_w, excitation function Φ should be specified (see eq. (2-15)). Since the frequency component just around the Chandler frequency dominates the behavior of the excited wobble as seen from (2-15), purely random excitation is sufficient for the present purpose. The amplitude and phase of the excitation spectrum, $A(f)$ and $\theta(f)$, are defined as

$$A(f)=A_0=\text{const.}$$

$$\theta(f)=\text{uniformly random in the range of } 0\leqq\theta<2\pi\,.$$

The excitation spectrum $\Phi(f)$ is given by

$$\begin{aligned}\Phi(f)&=A(f)\exp(i\cdot\theta(f))\\&=A_0\exp(i\cdot\theta(f))\end{aligned}$$

A_0 should be taken as a scaling factor for the moment. The Fourier spectrum of the synthesized Chandler wobble is then computed from (2-15)