KEY TO

ADVANCED ENGINEERING MATHEMATICS

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PART 1

高 等 工 程 數 學 詳 解

上　册　目　次

第一章　一階常微分方程式

節1-1

於1-5題，試證所予函數為所予微分方程式之一解

1. $xy'-3y=0$，　$y=cx^3$

解：$y'=3cx^2$

$xy'-3y=3cx^2\cdot x-3cx^3=0$

故 $y=cx^3$ 為已知微分方程式之一解

2. $y''+4y=0$，$y=A\cos 2x+B\sin 2x$

解：$y'=-2A\sin 2x+2B\cos 2x$

$y''=-4A\cos 2x-4B\sin 2x$

$y''+4y=-4A\cos 2x-4B\sin 2x+4A\cos 2x+4B\sin 2x$

$=0$

即 $y=A\cos 2x+B\sin 2x$ 為已知微分方程式之一解

3. $y''-y=0$，$y=c_1e^x+c_2e^{-x}$

解：$y'=c_1e^x-c_2e^{-x}$，$y''=c_1e^x+c_2e^{-x}$

$y''-y=c_1e^x+c_2e^{-x}-c_1e^x-c_2e^{-x}=0$

$y=c_1e^x+c_2e^{-x}$ 滿足 $y''-y=0$，故為其解

4. $y'''=6$，$y=x^3+ax^2+bx+c$

解：$y'=3x^2+2ax+b$，　$y''=6x+2a$，　$y'''=6$

故 $y=x^3+ax^2+bx+c$ 滿足 $y''=6$ 而為其解

5. $y''+2y'+2y=0$，　$y=e^{-x}(A\cos x+B\sin x)$

解：$y'=-e^{-x}(A\cos x+B\sin x)+e^{-x}(-A\sin x+B\cos x)$

$=e^{-x}[-(A+B)\sin x+(-A+B)\cos x]$

$y''=-e^{-x}[-(A+B)\sin x+(-A+B)\cos x]+e^{-x}[-(A+B)\cos x-(-A+B)\sin x]$

$=e^{-x}[2A\sin x-2B\cos x]$

$y''+2y'+2y=e^{-x}(2A\sin x-2B\cos x)+2e^{-x}[-(A+B)\sin x+(-A+B)\cos x]+2e^{-x}(A\cos x+B\sin x)$

$=-2e^{-x}(A\cos x+B\sin x)+2e^{-x}(A\cos x+B\sin x)$

$=0$

故 $y=e^{-x}[A\cos x+B\sin x]$ 滿足 $y''+2y'+2y=0$，而為其解

解下列各微分方程式

6. $y' = \sin x$

解：因 $\frac{dy}{dx} = \sin x$， $dy = \sin x\, dx$

$\int dy = \int \sin x\, dx$，故 $y = -\cos x + c$ （c = 常數）

7. $y' = e^x$

解：因 $\frac{dy}{dx} = e^x$，$dy = e^x dx$

$\int dy = \int e^x dx$，故 $y = e^x + c$（c = 常數）

8. $y'' = 3$

解：$y'' = \frac{dy'}{dx} = 3$ $\quad dy' = 3\, dx$

$y' = 3x + c_1$，故 $y = \frac{3}{2}x^2 + c_1 x + c_2$

（c_1 及 c_2 均爲常數）

9. $y'' = \cos 2x$

解：$y' = \frac{1}{2}\sin 2x + c_1$，故 $y = -\frac{1}{4}\cos 2x + c_1 x + c_2$

（c_1 及 c_2 均爲常數）

下列各題，試證所予函數爲所予微分方程式之解

10. $y' + y = 0$，$y = ce^{-x}$

解：$y' = -ce^{-x}$

故 $y' + y = -ce^{-x} + ce^{-x} = 0$

11. $y' + y = 2$，$y = ce^{-x} + 2$

解：$y' = -ce^{-x}$

故 $y' + y = -ce^{-x} + ce^{-x} + 2 = 2$

12. $xy' - 4y = 0$，$y = cx^4$

解：$y' = 4cx^3$

故 $xy - 4y = 4cx^4 - 4cx^4 = 0$

13. $yy' = -x$，$x^2 + y^2 = c$

解：$y^2 = c - x^2$ $\quad \therefore y = \sqrt{c - x^2}$

$$y' = \frac{1}{2}(c - x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{c - x^2}}$$

故 $yy'=\sqrt{c-x^2}\left(-\dfrac{x}{\sqrt{c-x^2}}\right)=-x$

14. $y'+2xy=0$，　$y=ce^{-x^2}$

解：$y'=-2cx\,e^{-x^2}$

故 $y'+2xy=-2cx\,e^{-x^2}+2cxe^{-x^2}=0$

下列各例，試證所予函數爲所予微分方程式之解，若欲特殊解滿足所予條件，求各 c 值。

15. $xy'=y$，$y=cx$，　$y=\pi$ 當 $x=2$

解：$y'=c$

$\therefore xy'=cx=y$

又 $\pi=c\times 2$，　$c=\dfrac{\pi}{2}$

故 $y=\dfrac{\pi}{2}x$

16. $y'=1$，$y=x+c$，　$y=0$ 當 $x=7$

解：$y'=1$

又 $0=7+c$，　$c=-7$

故 $y=x-7$

17. $y'+2xy=0$，　$y=ce^{-x^2}$，　$y=0.5$ 當 $x=0$

解：$y'=-2cx\,e^{-x^2}$

$y'+2xy=-2cx\,e^{-x^2}+2cx\,e^{-x^2}=0$

又 $0.5=ce^0=c$，　$c=0.5$

故 $y=0.5\,e^{-x^2}$

18. $y'+y=2$，$y=ce^{-x}+2$，　$y=3.2$ 當 $x=0$

解：$y'=-ce^{-x}$

$\therefore y'+y=-ce^{-x}+ce^{-x}+2=2$

又 $3.2=ce^0+2=c+2$

$c=3.2-2=1.2$　故 $y=1.2\,e^{-x}+2$

19. $y'=3x^2$，　$y=x^3+c$，　$y=-1$ 當 $x=1$

解：$y'=3x^2$

又 $-1=1^3+c$　　$\therefore c=-2$，　故 $y=x^3-2$

求以所予函數爲其解，且包含 y′與 y 之一階微分方程式

20. $y=-e^{-2x}$

解：$y'=2e^{-2x}=-2y$，　故 $y'+2y=0$

21. $y = x^3 - 4$

解：$y' = 3x^2$

$xy' - 3y = 3x^3 - 3x^3 + 12 = 12$

故 $xy' - 3y = 12$

22. $y = \sin 2x$

解：$y' = 2\cos 2x$

$y'^2 + 4y^2 = 4\cos^2 2x + 4\sin^2 2x = 4$

故 $y'^2 + 4y^2 = 4$

23. 求在 xy 平面，通過點（2，2），且其在各點斜率爲 $-1/x^2$ 之曲線，並繪其圖。

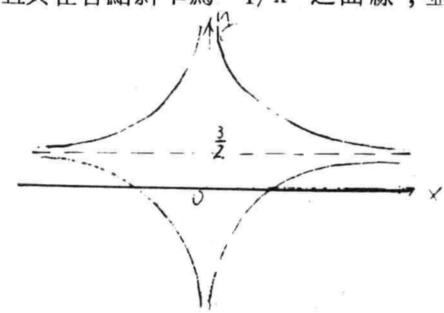
解：因 $y' = -\dfrac{1}{x^2}$，

故 $y = \dfrac{1}{x} + c$

通過（2, 2）$\therefore 2 = \dfrac{1}{2} + c$

$\therefore c = \dfrac{3}{2}$

故所求曲線爲 $y = \dfrac{1}{x} + \dfrac{3}{2}$



24. 若一物體在眞空受重力而下落，在 $t = 0$ 時，其初速爲 0，實驗顯示其速度與時間成正比，將此物理定律以一階微分方程式表示，並且由解此方程式而得定律 $S(t) = \dfrac{g}{2}t^2$，s 爲此物體與其起始點之距離，g 爲速度與時間之比例常數。

解：$\dfrac{ds}{dt} = gt$

$\therefore s(t) = \dfrac{1}{2}gt^2 + c$

$\because s(0) = 0 = c$，　$s(t) = \dfrac{1}{2}gt^2$

25. 如上題，若初速爲 v_0，求 s(t)。

解：$\dfrac{ds}{dt} = gt + v_0$

$\therefore s(t) = \dfrac{1}{2}gt^2 + v_0 t + c$

若以落體之初始位置爲0，則

$$s(0) = 0 = c, \text{得 } s(t) = \frac{1}{2}gt^2 + v_0 t$$

1-2

1. 證明 $y = ce^{x^2/2}$ 爲(3)式之解，c 爲任意常數，c 爲何值始得圖 6 中所示之解？

解：$\frac{dy}{dx} = xce^{x^2/2} = xy$ 故得證

如圖 6 知 $x = 1$ 時，$y = 1$

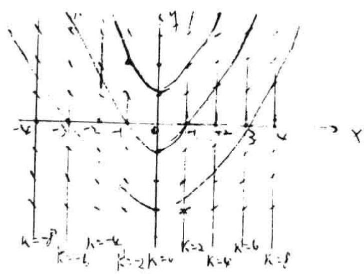
$\therefore 1 = ce^{1/2}$　$\therefore c = e^{-\frac{1}{2}}$　$\therefore y = e^{-\frac{1}{2}}e^{x^2/2} = e^{x^2-1/2}$

繪下列方程式之良好方向場，並繪數條近似解答曲線。

2. $y' = 2x$

解：令 $y' = 2x = k =$ 常數

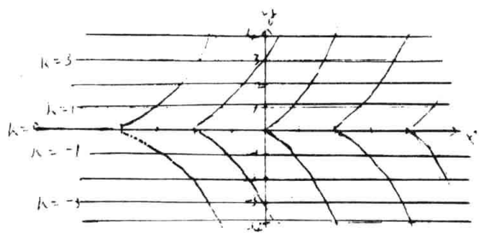
($y = x^2 + c$)



3. $y' = y$

解：令 $y' = y = k =$ 常數

($y = ce^x$)



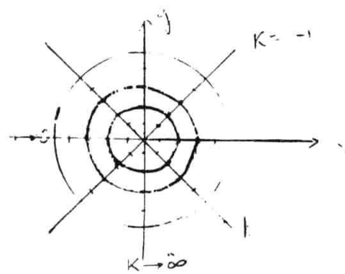
4. $yy' + x = 0$

解：即 $y' = -\frac{x}{y}$

令 $y' = -\frac{1}{y}x = k =$ 常數

$\therefore y = -\frac{1}{k}x$

($x^2 + y^2 = c$)

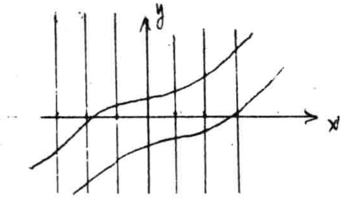


5. $y' = x^2$

解：令 $y' = x^2 = k =$ 常數

$\therefore x = \pm\sqrt{k}$

$\left(y = \dfrac{x^3}{3} + c \right)$

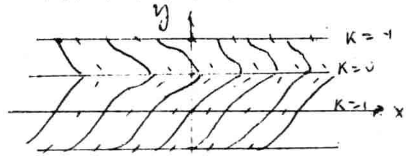


6. $y' = \cos y$

解：令 $y' = \cos y = k =$ 常數

即 $y = \cos^{-1} k$

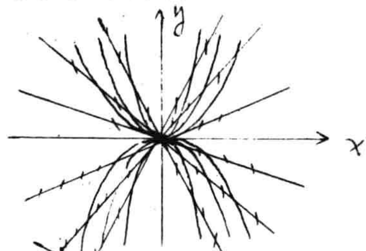
$(x = \ln(\sec y + \tan y) + c)$



7. $y' = \dfrac{2y}{x}$

解：令 $y' = \dfrac{2y}{x} = k =$ 常數

$\therefore y = \dfrac{k}{2} x \; (y = cx^2)$

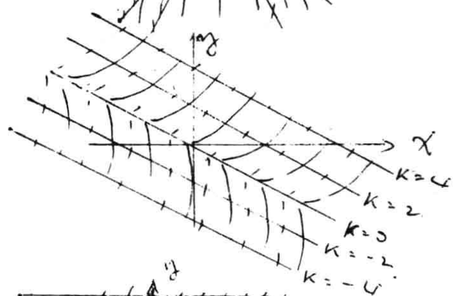


8. $y' = 2y + x$

解：令 $y' = 2y + x = k =$ 常數

$\therefore y = -\dfrac{x}{2} + \dfrac{k}{2}$

$\left(y = ce^{2x} - \dfrac{1}{2}x - \dfrac{1}{4} \right)$

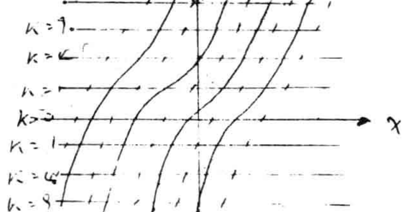


9. $y' = y^2$

解：令 $y' = y^2 = k =$ 常數

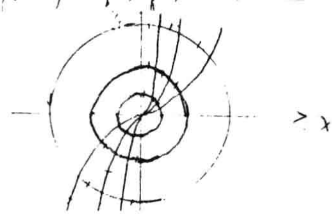
即 $y = \pm\sqrt{k}$

$\left(y = -\dfrac{1}{x + c} \right)$



10. $y' = x^2 + y^2$

解：令 $x^2 + y^2 = k =$ 常數

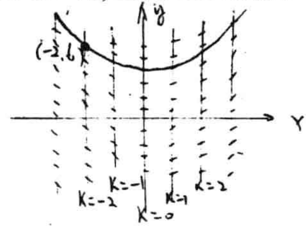


應用方向場法，繪已知方程式之近似解答曲線並滿足已知條件

11. $y' = x$,　$y(-2) = 6$

解：令 $y' = x = k =$ 常數

$y(-2) = 6$

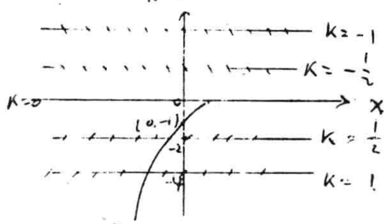


12. $4y' + y = 0$，$y(0) = -1$

解：令 $y' = -\dfrac{y}{4} = k =$ 常數

$y = -4k$

$y(0) = -1$

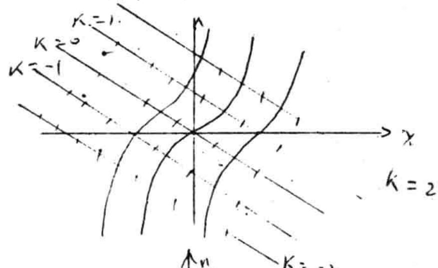


13. $y' = x + y$，$y(0) = 0$

解：令 $y' = x + y = k =$ 常數

$y = -x + k$

$y(0) = 0$

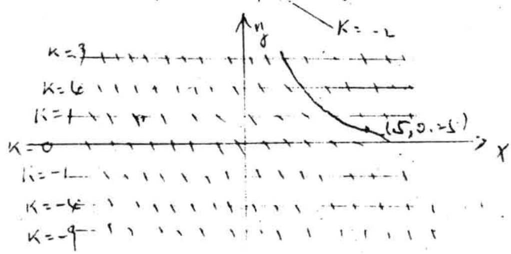


14. $y' + y^2 = 0$，$y(5) = 0.25$

解：令 $y' = -y^2 = k =$ 常數

$y = \pm\sqrt{-k}$,　$k \leq 0$

$y(5) = 0.25$



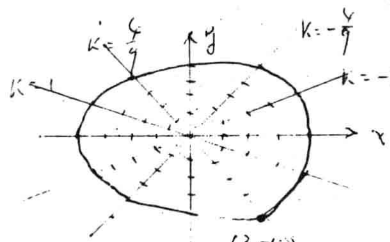
15. $9yy' + 4x = 0$，$y(3) = -4$

解：$\therefore y' = -\dfrac{4x}{9y}$

令 $y' = \dfrac{-4x}{9y} = k =$ 常數

$\therefore y = \dfrac{-4x}{9k}$

$y(3) = -4$



16.舉出微分方程式(2)之二例，其等斜線爲圓心位於原點之同心圓

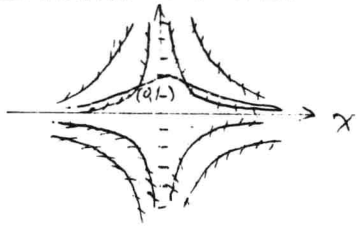
解：$y' = x^2 + y^2 + 1$， $y' = x^2 + y^2 + 2$等等

17.利用方向場法，繪通過點（0,1）並具有斜線$-2xy$之曲線

解：令 $y' = -2xy = k =$常數

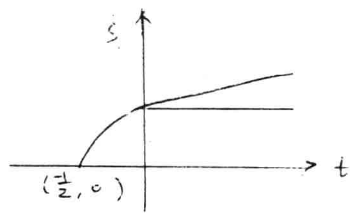
$\therefore xy = -\frac{k}{2}$

當 $x = 0$，$y = 1$



18.一物體 B 在一直線上運動，令 S(t) 爲在 L 上，物體與一定點 O 之距離，設每一瞬時 B 之速度等於 1/S(t)，且當 $t = 0$ 時，$S = 1$ 求對應之微分方程。

解：$\frac{ds}{dt} = \frac{1}{S}$， $S(0) = 1$



19.利用方向場法，求 $\sin(x^2)$ 由 0 至 1 之積分近似值，（此無法以初等方法算出，參見附錄 3 之 (38)）。

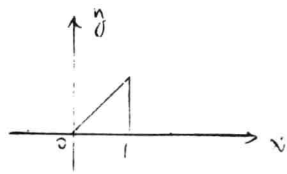
解：令 $y = \int_0^x \sin(t^2)\,dt$，

則 $y' = \sin(x^2)$，

且 $y(0) = 0$

由方向場法可求出 y 之曲線，

且 $y(1) \approx 0.42$



20.利用方向場法，求 x^2 由 0 至 $b = 0.2, 0.4, 0.6, 0.8, 1.0$ 之積分近似值，並將其與正合值比較。

解：令 $y = \int_0^x t^2\,dt$，

則 $y' = x^2$ 且 $y(0) = 0$，

由方向場法可求出 y 之曲線，

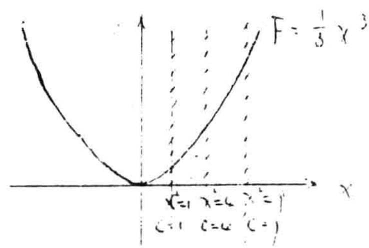
且 $y(0.2) = 0.00267$，

$y(0.4) = 0.02133$，

$y(0.6) = 0.07200$，

$y(0.8) = 0.17067$，

$y(1.0) = 0.33333$



節 1-3

1. 何以當完成積分後，須立即加入積分常數？

解：由初等微積分知不定積分乃是求反導函數，而此無限多反導數之間相差一常數。

2. 若 $yy' = (y^2)'/2$，如何利用此式解例 1. 之微分方程式？

解：$9yy' + 4x = 0$

$$\therefore yy' = -\frac{4}{9}x，\quad 得\ \frac{(y^2)'}{2} = -\frac{4}{9}x$$

$$\therefore (y^2)' = -\frac{8}{9}x$$

$$\therefore y^2 = -\frac{8}{9}\ \frac{x^2}{2} + c = -\frac{4}{9}x^2 + c$$

$$\therefore \frac{x^2}{9} + \frac{y^2}{4} = c$$

求下列各方程式之通解（ a, b, k, θ 與 ω 爲常數）

3. $y' = ky$

解：$\frac{dy}{dx} = ky \quad \frac{dy}{y} = k\,dx$

$$\therefore \ln y = kx + c$$

$\therefore y = Ae^{kx}$, 其中 $A = e^c$ 爲任意正常數

4. $y' = y^2$

解：$\therefore \frac{dy}{y^2} = dx \quad \therefore \int \frac{1}{y^2}dy = \int dx$

$$-\frac{1}{y} = x + c \qquad \therefore y = -\frac{1}{x+c}$$

5. $y' - 2y + a = 0$

解：$\frac{dy}{2y-a} = dx \qquad \therefore \frac{1}{2}\ln(2y-a) = x + c$

$$\therefore 2y - a = e^{2c}e^{2x} = Ae^{2x}$$

$$\therefore y = \frac{Ae^{2x} + a}{2}$$

6. $xy' + by = 0$

解：$x\frac{dy}{dx} = -by \qquad \frac{1}{y}dy = -\frac{b}{x}dx$

$$\ln y = -b\ln x + c \qquad \therefore y = Ax^{-b}$$

7. $(x-1)y' = 2x^3y$

解：$\frac{1}{y}dy = \frac{2x^3}{x-1}dx$

$$\int \frac{1}{y}dy = 2\int \frac{x^3}{x-1}dx$$

$$\ln y = 2\int [(x^2+x+1) + \frac{1}{x-1}]dx$$

$$= \frac{2}{3}x^3 + x^2 + 2x + 2\ln(x-1) + c$$

$$\therefore y = E_{xp}[c + 2\ln(x-1) + \frac{2}{3}x^3 + x^2 + 2x]$$

$$= (E_{xp}\,c)(x-1)^2 E_{xp}[\frac{2}{3}x^3 + x^2 + 2x]$$

$$= A(x-1)^2 E_{xp}[\frac{2}{3}x^3 + x^2 + 2x]$$

其中 $A = e^c$

8. $(x+2)y' - xy = 0$

解：$\frac{1}{y}dy = \frac{x}{x+2}dx = (1 - \frac{2}{x+2})dx$

$$\ln y = x - 2\ln(x+2) + c$$

$$= \ln\frac{e^x}{(x+2)^2} + c$$

$$y = A\frac{e^x}{(x+2)^2}$$

9. $y' = 2x^{-1}\sqrt{y-1}$

解：$\frac{1}{\sqrt{y-1}}dy = \frac{2}{x}dx$

$$2\sqrt{y-1} = 2\ln|x| + c$$

$$\sqrt{y-1} = \ln|x| + c$$

$$\therefore y = 1 + [\ln|x| + c]^2$$

10. $y' + e^{2x}y^2 = 0$

解：$\frac{1}{y^2}dy = -e^{2x}dx$

$$-\frac{1}{y}=-\frac{1}{2}e^{2x}+c$$

$$\frac{1}{y}=\frac{1}{2}e^{2x}-c=\frac{1}{2}(e^{2x}-2c)$$

$$\therefore y=\frac{2}{e^{2x}+A},\quad A=-2c$$ 爲任意常數

11. $y'=y\cot 2x$

解：$\frac{dy}{y}=\cot 2x\,dx$

$\ln y=\frac{1}{2}\ln|\sin 2x|+c \qquad \therefore y=A\sqrt{|\sin 2x|}$

12. $y'+\csc y=0$

解：$\frac{dy}{\csc y}=\sin y\,dy=-dx$

$\cos y=x+c$

$y=\cos^{-1}(x+c),\quad |x+c|\leq 1$

13. $y'=(1+x)(1+y^2)$

解：$\frac{dy}{1+y^2}=(1+x)\,dx$

$\tan^{-1}y=x+\frac{x^2}{2}+c$

$\therefore y=\tan\left(x+\frac{x^2}{2}+c\right)$

14. $yy'=0.5\sin^2\omega x$

解：$\frac{(y^2)'}{2}=0.5\sin^2\omega x \qquad \therefore (y^2)'=\sin^2\omega x$

$\therefore y^2=\frac{x}{2}-\frac{1}{4\omega}\sin 2\omega x+c$

15. $y'=y\tanh x$

解：$\frac{1}{y}\,dy=\tanh x\,dx$

$\ln y=\ln\cosh x+c$

$\therefore y=A\cosh x$

16. $y'+3y\sin\omega x=0$

解：$\frac{dy}{y}=-3\sin\omega x\,dx$

$\ln y=\frac{3}{\omega}\cos\omega x+c$

$y=Ae^{\frac{3}{\omega}\cos\omega x}$

17. $\sin 2x\,dy=y\cos 2x\,dx$

解：$\frac{dy}{y}=\cot 2x\,dx$ $\quad\therefore \ln y=\frac{1}{2}\ln|\sin 2x|+c$

$\therefore y=A\sqrt{|\sin 2x|}$

18. $y^2dy-\cos^2x\,dx=0$

解：$y^2\,dy=\cos^2x\,dx$

$\therefore \frac{1}{3}y^3=\frac{x}{2}+\frac{\sin 2x}{4}+c$

$\therefore y^3=\frac{3}{2}x+\frac{3}{4}\sin 2x+A$

19. $x\ln x\,dy-ydx=0$

解：$x\ln x\,dy=y\,dx$ $\quad \frac{1}{y}dy=\frac{1}{x\ln x}dx$

$\ln y=\ln\ \ln x+c$

$\therefore y=A\ln x$

20. $(1-\cos\theta)\,dr=r\sin\theta\,d\theta$

解：$\frac{dr}{r}=\frac{\sin\theta}{1-\cos\theta}d\theta$

$\ln r=\ln(1-\cos\theta)+c$

$\therefore r=A(1-\cos\theta)$

21. 應用式(3)，試證滿足式(5)之式(1)之解爲

$$\int_{y_0}^{y}g(y^*)\,dy^*=\int_{x_0}^{x}f(x^*)dx^*$$

證：由式(3)，得式(1)之解爲

$\int g(y)\,dy=\int f(x)\,dx+c$

或 $G(y)=F(x)+c$

由式(5)，得 $y(x_0)=y_0$，故

$$G(y_0)=F(x_0)+c \text{ 或 } c=G(y_0)-F(x_0)$$

故　$G(y)=F(x)+c$

$$=F(x)+G(y_0)-F(x_0)$$

或　$G(y)-G(y_0)=F(x)-F(x_0)$

即　$\int_{y_0}^{y} g(y^*)dy=\int_{x_0}^{x} f(x^*)dx^*$

22.利用(14)，求例 4 之微分方程式滿足 $y(0)=0$ 之解

解：$\dfrac{dy}{1+y^2}=-\dfrac{dx}{1+x^2}$，應用式(14)，得

$$\int_0^y \frac{dy^*}{1+y^{*2}}=-\int_0^x \frac{dx^*}{1+x^{*2}}$$

或　$\tan^{-1}y-\tan^{-1}0=-\tan^{-1}x+\tan^{-1}0$

$$\tan^{-1}y=\tan^{-1}(-x)$$

即　$y=-x$

23.應用(14)式，求例 4.之解答 y 。

解：應用式(14)，得

$$\int_1^y \frac{dy^*}{1+y^{*2}}=-\int_0^x \frac{dx^*}{1+x^{*2}}$$

故 $\tan^{-1}y-\tan^{-1}1=-\tan^{-1}x$

$$y=\tan(\tan^{-1}1-\tan^{-1}x)$$

$$=\frac{1-x}{1+x}$$

解下列各初值問題

24. $(x+1)y'=2y \quad y(0)=1$

解：$\dfrac{dy}{y}=\dfrac{2}{x+1}dx$

$\ln y=2\ln(x+1)+c'$

$\therefore y=c(x+1)^2 \quad \because y(0)=1 \qquad \therefore 1=c\times 1^2$

$\therefore c=1 \qquad y=(x+1)^2$

25. $y'=y\tan 2x$，　$y(0)=2$

解：$\dfrac{dy}{y}=\tan 2x\,dx$

$\therefore \ln y = \frac{1}{2}\ln(\sec 2x) + c'$

$\therefore y = c\sqrt{\sec 2x}$

$y(0) = 2 \quad \therefore 2 = c\sqrt{\sec 0} = c$

$\therefore y = 2\sqrt{\sec 2x}$

26. $y'x\ln x = y, \quad y(2) = \ln 4$

解：$\frac{dy}{y} = \frac{1}{x\ln x}dx$

$\therefore \ln y = \ln \ln x + c'$

$\therefore y = c\ln x$

$\therefore y(2) = \ln 4 \quad \therefore \ln 4 = c\ln 2 \quad \therefore c = 2$

$\therefore y = 2\ln x$

27. $2xy' = 3y, \quad y(1) = 4$

解：$\frac{dy}{y} = \frac{3}{2x}dx \quad \therefore \ln y = \frac{3}{2}\ln x + c'$

$\therefore y = cx^{\frac{3}{2}} \quad y(1) = 4 \quad \therefore 4 = c\cdot 1^{\frac{3}{2}}$

$\therefore c = 4 \quad y = 4x^{\frac{3}{2}} \quad x > 0$

28. $(x^2+1)yy' = 1, \quad y(0) = -3$

解：$ydy = \frac{1}{x^2+1}dx$

$\frac{y^2}{2} = \tan^{-1}x + c' \quad \therefore y^2 = 2\tan^{-1}x + c$

$\therefore y(0) = -3 \quad \therefore (-3)^2 = 2\tan^{-1}0 + c \quad c = 9$

$\therefore y^2 = 2\tan^{-1}x + 9$

29. $y' = 2e^x y^3, \quad y(0) = 0.5$

解：$\frac{dy}{y^3} = 2e^x dx \quad \therefore -\frac{1}{2y^2} = 2e^x + c'$

$\therefore \frac{1}{y^2} = -4e^x + c$

$\because y(0) = 0.5 \quad \frac{1}{(0.5)^2} = -4e^0 + c \quad \therefore c = 8$

$\frac{1}{y^2} = -4e^x + 8 \quad \therefore y^2 = \frac{1}{8-4e^x}$

30. $y' = 3x^2e^{-y}$　$y(-1) = 0$

解：$e^y dy = 3x^2\,dx$　　$e^y = x^3 + c$

$\therefore y = \ln|x^3 + c|$

$\because y(-1) = 0$　$\therefore 0 = \ln|-1 + c|$　　$c = 2$

$\therefore y = \ln|x^3 + 2|$

31. $dr/dt = -rt$，$r(0) = r_0$

解：$\dfrac{dr}{r} = -t\,dt$　　$\ln r = -\dfrac{t^2}{2} + c'$

$r = ce^{-t^2/2}$　　$r(0) = r_0$　　$r_0 = ce^0$

$\therefore c = r_0$　　$\therefore r = r_0 e^{\frac{-t^2}{2}}$

32. $y' = y^2 \sin x$，$y(\pi) = 0.2$

解：$\dfrac{dy}{y^2} = \sin x\,dx$　　$\therefore \dfrac{-1}{y} = -\cos x + c$

$\because y(\pi) = 0.2$　$\dfrac{-1}{0.2} = -\cos\pi + c$　　$\therefore c = -6$

$\dfrac{1}{y} = \cos x + 6$　　$\therefore y = \dfrac{1}{6 + \cos x}$

33. $y' = \sec y$，$y(0) = 0$

解：$\dfrac{dy}{\sec y} = dx$　$\therefore \sin y = x + c$

$\because y(0) = 0$　　$\therefore c = 0$　　$\therefore \sin y = x$

$\therefore y = \sin^{-1} x$

34. $yy' = xe^{y^2}$　$y(1) = 0$

解：$\displaystyle\int_0^y \frac{y\,dy}{e^{y^2}} = \int_1^x x\,dx$

$\therefore \dfrac{1}{2}[-e^{-y^2} + e^0] = \dfrac{1}{2}(x^2 - 1)$

$\therefore e^{-y^2} = 2 - x^2$，$-y^2 = \ln|2 - x^2|$

$\therefore y = \ln\dfrac{1}{\sqrt{2 - x^2}}$　　$-\sqrt{2} < x < \sqrt{2}$

35. $xyy' = y + 2$，$y(2) = 0$