科技用書

鋼筋混凝土設計習題詳解

劉嘉晃 編著

大行出版社 印行

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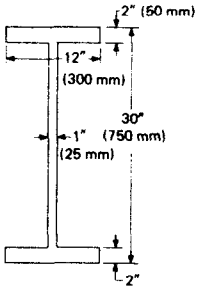
目　錄

第三章　受彎之矩形斷面強度

所有問題依照ACI規範的強度法解之（除問題3-1與3-2外），所有載重除非特別註明皆爲資用載重。解題應儘量用基本原理，避免直接用公式。

【材料力學問題】

3-1　一簡支梁跨徑20ft（6m），爲均質彈性且能承受張力與壓力，它承受均勻分佈荷重2klf（kips每ft）與距右端5ft（1.5m）的集中載重10kips（44KN）。

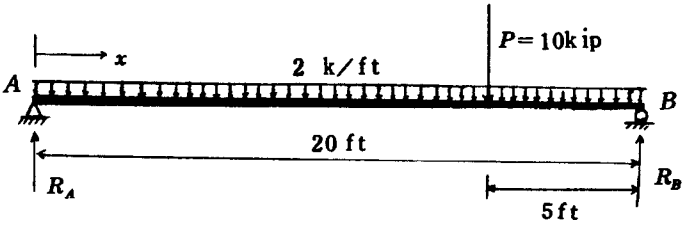


問題3－1

(a)　應用基本靜力學與內力偶法（internal-couple method），$M=$（C或T）（內力偶臂），計算如圖斷面的最大撓曲應力。

(b)　用撓曲公式，$f=\dfrac{MC}{I}$檢驗之。

解：(a)



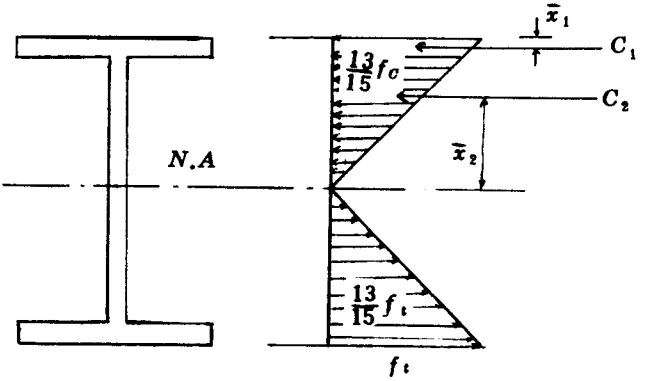
(1) $$R_A = \frac{\left[\left(\frac{2\times 20\times 20}{2}\right)+10\times 5\right]}{20} = 22.5\ \text{kip}$$

$$V_x = 22.5 - 2x$$

$$M_x = 22.5x - \frac{1}{2}\times 2x^2 = 22.5x - x^2$$

當$V_x = 0$時，$x = 11.25$ ft　　$M = M_{max}$

$$M_{max} = 22.5\times 11.25 - 11.25^2 = 126.56\ \text{ft-k}$$



(2) $$C_1 = \frac{1}{2}\left[f_c + \frac{13}{15}f_c\right]\cdot b\cdot t = \frac{336}{15}f_c = 22.4f_c$$

$$C_2 = \frac{1}{2}\times\frac{13}{15}f_c\times 13\times 1 = \frac{169}{30}f_c = 5.633f_c$$

$$\overline{x_1}=\frac{\frac{13}{15}f_c\times 2\times\frac{2}{2}+\frac{2}{15}f_c\times\frac{2}{2}\times\frac{1}{3}}{\frac{13}{15}f_c\times 2+\frac{1}{2}\times 2\times\frac{2}{15}f_c}=0.488\text{ in}$$

$$\overline{x_2}=\frac{2}{3}\times 13=\frac{26''}{3}=8.67\text{ in}$$

$$M=C_1(30-2x_1)+C_2(2x_2)$$

$$=22.4f_c(30-2\times 0.488)+5.633f_c\times 2\times 8.67$$

$$=725.9f_c$$

(3) $$f_c=\frac{126.56\times 12}{725.9}=2.09\text{ kip}=f_t\leftarrow$$

(b) $$I=\frac{1}{12}\times 1\times 26^3+\left[\frac{1}{12}\times 12\times 2^3+12\times 2\times 14^2\right]\times 2$$

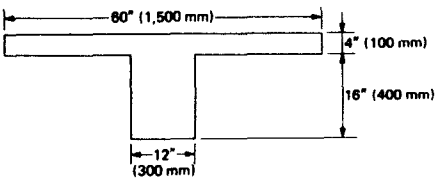
$$=10888.67\text{ in}^4$$

$$f=\frac{MC}{I}$$

$$=\frac{126.56\times 12\times 15}{10888.7}=2.09\text{ ksi}\leftarrow$$

以上證實兩者均相同。

3-2　如圖一彈性均質簡支梁，跨徑25ft（7.5m），承受均勻載重1 kip／ft（15KN／m）與距左端8ft（2.4m）



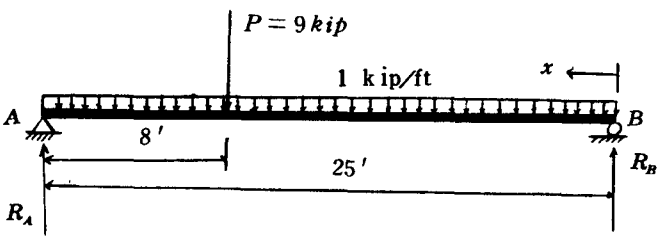
問題3－2

的集中載重9 kips（40 K N），試用：

(a) 基本靜力學與內力偶法，$M=$（C或T）（內力矩臂）

(b) 繞曲公式，$f=\dfrac{MC}{I}$，計算最大張與壓應力。

解：(a)



(1) $R_B=\dfrac{(\dfrac{1\times 25\times 25}{2}+9\times 8)}{25}=15.38\ kip$

$V_x=15.38-1\cdot x=0$

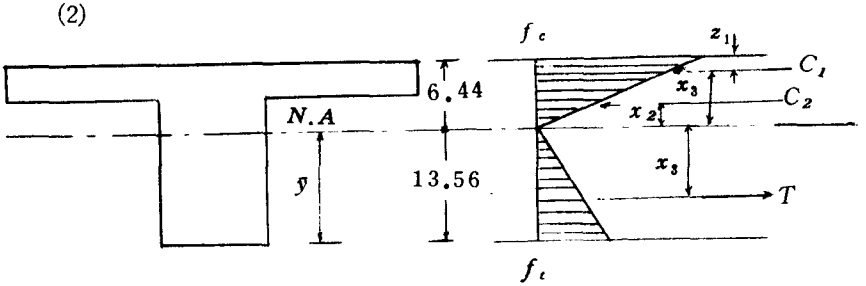
$x=15.38\ ft$

$M_x=15.38x-\dfrac{1}{2}x^2$

以 $x=15.38$ 代入

$M_{max}=15.38^2-\dfrac{1}{2}\times 15.38^2=118.27\ ft\text{-}k$

(2)



$$\overline{y}=\frac{12\times 16\times 8+60\times 4\times 18}{(12\times 16+60\times 4)}=13.56\text{ in}$$

$$C_1=\frac{1}{2}\left(f_c+\frac{2.44}{6.44}f_c\right)\times 4\times 60=165.47f_c$$

$$C_2=\frac{1}{2}\times\frac{2.44}{6.44}f_c\times 2.44\times 12=5.55f_c$$

$$T=\frac{1}{2}\text{ft}\times 13.56\times 12=81.36\text{ ft}$$

$$T=C_1+C_2$$

$$81.36\text{ ft}=165.47f_c+5.55f_c$$

$$f_t=2.102\,f_c\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots①$$

$$z_1=\frac{0.379f_c\times 60\times 4\times 2+0.621f_c\times\frac{4}{2}\times 60\times\frac{4}{3}}{0.379f_c\times 60\times 4+0.621f_c\times\frac{4}{2}\times 60}$$

$$=1.70\text{ in}$$

$$x_1=6.44-1.70=4.74\text{ in}$$

$$x_2=\frac{2}{3}\times 2.44=1.63\text{ in}$$

$$x_3=\frac{2}{3}\times 13.56=9.04\text{ in}$$

$$M=C_1x_1+C_2x_2+Tx_3$$

$$118.27\times 12=165.47f_c\times 4.74+5.55f_c\times 1.63+81.36f_t\times 9.04$$

$$1419.24=783.37f_c+735.5f_t\quad\cdots\cdots\cdots\cdots②$$

解①，②得　$f_c=0.61$ ksi ←

$f_t=1.28$ ksi ←

(b)　$\overline{y}=13.56\text{ in}$

$$I=\frac{1}{12}\times 12\times 16^3+\frac{1}{12}\times 60\times 4^3+12\times 16\times(13.56-8)^2+4\times 60\times(18-13.56)^2$$

$$=15082.7\ \text{in}^4$$

$$f_c=\frac{MC_1}{I}$$

$$=\frac{118.27\times 12\times 6.44}{15082.7}=0.61\ \text{ksi}\leftarrow$$

$$f_t=\frac{MC_2}{I}$$

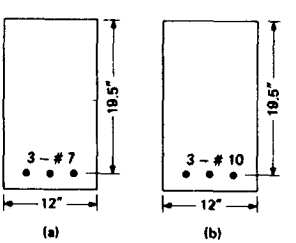
$$=\frac{118.27\times 12\times 13.56}{15082.7}=1.28\ \text{ksi}\leftarrow$$

以上證實兩者均相同。

【鋼筋混凝土問題】

3-3 所示各梁之$f_c'=3500$ psi及$f_y=60,000$ psi，試以內力偶方法與衛特尼矩形應力塊解之，決定標稱抗彎強度M_n，假如活載重佔60%，求各梁之資用彎矩容量（考慮超載因數）。

〔$f_c'=24\ \text{N}/\text{mm}^2$，$f_y=420\ \text{N}/\text{mm}^2$〕。



問題3－3

解：(a)　3 - # 7

$A_s = 3 \times 0.60 = 1.80\ \text{in}^2$

$T = A_s f_y = 1.8 \times 60000 = 108\ \text{kip}$

$C = 0.85 f_c' ba$

$$a = \frac{T}{0.85 f_c' b} = \frac{108000}{0.85 \times 3500 \times 12} = 3.03\ \text{in}$$

$$x = \frac{a}{\beta_1} = \frac{0.03}{0.85} = 3.56$$

$$\epsilon_s = \frac{d - x}{x} \times 0.003 = \frac{19.5 - 3.56}{3.56} \times 0.003$$

$$= 0.0134$$

$$\epsilon_y = \frac{f_y}{E_s} = \frac{60}{29000} = 0.00207 < \epsilon_s \qquad \text{OK}$$

$$M_n = T\left(d - \frac{a}{2}\right)$$

$$= \frac{108\left(19.5 - \frac{3.03}{2}\right)}{12}$$

$$= 161.87\ \ ft\text{-}k \ \leftarrow$$

而　$M_L = 0.6 M_w$, $M_D = 0.4 M_w$

$$M_n = \frac{M_u}{\phi} = \frac{1.4 M_D + 1.7 M_L}{0.9}$$

$$= \frac{1.4 \times 0.4 M_w + 1.7 \times 0.6 M_w}{0.9}$$

$$= 1.76 M_w$$

$$M_w = \frac{161.87}{1.76} = 92.2\ ft\text{-}k \ \leftarrow$$

M_w 爲安全的資用彎矩

(b) 3-# 10

$A_s = 3 \times 1.27 = 3.81\ \text{in}^2$

$T = A_s f_y = 3.81 \times 60 = 228.6\ \text{kip}$

$$a = \frac{T}{0.85 f_c' b} = \frac{228.6}{0.85 \times 3.5 \times 12} = 6.40\ \text{in}$$

$$x = \frac{a}{\beta_1} = \frac{6.4}{0.85} = 7.53\ \text{in}$$

$$\epsilon_s = \frac{d-x}{x} \times 0.003 = \frac{19.5 - 7.53}{7.53} \times 0.003$$

$$= 0.0048$$

$$= 0.0048 > \epsilon_y \quad \text{OK}$$

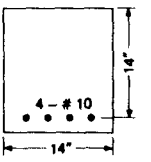
$$M_n = T\left(d - \frac{d}{2}\right)$$

$$= \frac{228.6\left(19.5 - \frac{6.4}{2}\right)}{12}$$

$$= 310.5^2\ \text{ft-k} \leftarrow$$

$$M_w = \frac{M_n}{1.76} = \frac{310.52}{1.76} = 176.43\ \text{ft-k} \leftarrow$$

3-4 使用基本原理，試證明所示斷面之鋼筋量爲不足，平衡或過量，並指出是否有某些方面與ACI規範相違背。$f_c' = 4000$ psi，$f_y = 58{,}000$ psi。($f_c' = 28\ \text{N/mm}^2$，$f_y = 400\ \text{N/mm}^2$)



問題3－4

解：(a)　$\dfrac{0.003}{f_y/E_s}=\dfrac{x_b}{d-x_b}$

$$\frac{87000}{f_y}=\frac{x_b}{d-x_b}$$

$$x_b=\frac{87000}{87000+f_y}\,d=\frac{87000}{87000+58000}\times 14 \quad =8.4\text{ in}$$

$\beta_1=0.85$　　$(f_c'=4000\text{ psi})$

$C_b=0.85f_c'\beta_1 x_b b=T_b=A_s b f_y$

$$A_s b=\frac{0.85f_c'\beta_1 x_b b}{f_y}$$

$$=\frac{0.85\times 4000\times 0.85\times 8.4\times 14}{58000}$$

$$=5.86\text{ in}^2$$

4 - # 10

$A_s=4\times 1.27=5.08\text{ in} < A_s b$

∴　斷面爲鋼筋不足斷面

(b)　比較實際使用鋼筋與ACI規範最大允許值

$_{max}A_s=0.75\,A_s b=0.75\times 5.86=4.40\text{ in}^2$

實際　$A_s=5.08 > 4.40\text{ in}^2$　　(N.G)

與ACI－10.33的規定相違背。

3-5　設有一跨徑24ft之簡支矩形梁受承1.0kips／ft之活載重及1.3kips／ft之呆載重（包括梁重），試決定適當之斷面尺寸b，d（取$\dfrac{d}{b}$大約1.75）及鋼筋面積A_s並求：

(a)　容許之最大尺寸？

(b)　容許之最小尺寸？

$f_c'=3000$ psi，$f_y=60,000$ psi（參看ACI-10-

3-2及10-5-1）（活載重＝15 KN／m，靜載重＝19 KN／m，$f_y = 420\ N/mm^2$，$f'_c = 21\ N/mm^2$，跨徑＝7.3 m。）

解：(1) $W_u = 1.4W_D + 1.7W_L = 1.4 \times 1.3 + 1.7 \times 1.0$

$= 352\ k/ft$

$$M_u = \frac{1}{8} W_u \ell^2 = \frac{1}{8} \times 3.52 \times 24^2 = 253.44\ ft\text{-}k$$

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y} \left(\frac{87000}{87000 + f_y}\right)$$

$$= \frac{0.85 \times 0.85 \times 3000}{60000} \left(\frac{87000}{87000 + 60000}\right)$$

$$= 0.0214$$

$$0.75\rho_b = 0.0160$$

(2) 假設 $\rho = 1.5\%$

$$m = \frac{f_y}{0.85 f'_c} = \frac{60000}{0.85 \times 3000} = 23.53$$

$$R_u = \rho f_y \left(1 - \frac{1}{2}\rho m\right)$$

$$= 0.015\ (\ 60000\ )\left(1 - \frac{1}{2} \times 0.015 \times 23.53\right)$$

$$= 741.2$$

$$M_n = \frac{M_u}{\phi} = \frac{253.44}{0.9} = 281.6\ ft\text{-}k$$

$$(db^2)_{req'd} = \frac{M_n}{R_u} = \frac{281.6 \times 12000}{741.2}$$

$$= 4560\ in^3$$

b	10	12	14	15
d	21.4	19.5	18.0	1.74
$\frac{d}{b}$	2.14	1.63	1.29	1.16

(3) 取 $b = 12\,\text{in}$ ，$d = 21.0\,\text{in}$ 計算，以使比 $\rho = 1.5\,\%$ 還大的有效深度

$$(R_u)_{\text{修}} = \frac{M_n}{bd^2} = \frac{281.6 \times 12000}{12 \times (21)^2} = 638.5\ \text{psi}$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_u}{f_y}}\right)$$

$$= \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 638.5}{60000}}\right)$$

$$= 0.0125$$

$$A_s = \rho bd = 0.0125 \times 12 \times 21 = 3.15\,\text{in}^2$$

$b = 12\,\text{in}$ ，$d = 21\,\text{in}$ ，$A_s = 3.15\,\text{in}^2$ ←

(a) 容許的最小尺寸：

取　$\rho = 0.75\,\rho_b = 0.016$

$$R_u = \rho f_y\left(1 - \frac{1}{2}\rho m\right)$$

$$= 0.016 \times 60000 \times \left(1 - \frac{1}{2} \times 0.016 \times 23.53\right)$$

$$= 779.3$$

$$(bd^2)_{req'd} = \frac{M_n}{R_u} = \frac{281.6 \times 12000}{779.3}$$

$$= 4336.2\ \text{in}^3$$

取　$b = 12\,\text{in}$，$d = \sqrt{\frac{4336.2}{b}} = 19.0\,\text{in}$

最小容許尺寸 $b = 12\,\text{in}$, $d = 19\,\text{in}$ ←

(b) 容許的最大尺寸

取 $\rho = \dfrac{200}{f_y} = \dfrac{200}{60000} = 0.00333$

$$R_u = \rho f_y \left(1 - \frac{1}{2}\rho m\right)$$

$$= 0.0033 \times 60000 \left(1 - \frac{1}{2} \times 0.00333 \times 23.53\right)$$

$$= 192$$

$$(bd^2)_{req'd} = \frac{M_u}{R_u} = \frac{281.6 \times 12000}{192}$$

$$= 17585.6\ \text{in}^3$$

b	12	14	16	18	20
d	38.3	35.4	33.0	31.3	29.6
$\dfrac{d}{b}$	3.19	2.53	2.1	1.74	1.48

取 $b = 18$, $d = 31.0$ 爲最大容許尺寸 ←

3-6 同問題 3-5 ，但 $f'_c = 4000\ \text{psi}$, $\dfrac{d}{b}$ 大約2.0 。($f'_c = 28\ \text{N}/\text{mm}^2$) 。

解：(1) $M_u = 253.44\ \text{ft-k}$, $M_n = 281.6\ \text{ft-k}$

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y}\left(\frac{87000}{87000 + f_y}\right)$$

$$= \frac{0.85 \times 0.85 \times 4000}{60000}\left(\frac{87000}{87000 + 60000}\right)$$

$$= 0.0285$$

$0.75\,\rho_b = 0.0214$

$$\rho_{min} = \frac{200}{f_y} = 0.0033$$

(2) 假設取　$\rho = 0.375\,\rho_b = 0.0157 = 1.57\ \%$

$$m = \frac{f_y}{0.85\,f'_c} = \frac{60000}{0.85 \times 4000} = 17.65$$

$$R_u = \rho f_y \left(1 - \frac{1}{2}\rho m\right)$$

$$= 0.0157 \times 60000 \left(1 - \frac{1}{2} \times 0.0157 \times 17.65\right)$$

$$= 814.5$$

$$(b d^2)_{req'd} = \frac{M_n}{R_u} = \frac{281.6 \times 12000}{814.5}$$

$$= 4148.8\ \text{in}^3$$

b	10	12
d	20.4	18.6
$\frac{d}{b}$	2.04	1.55

(3) 取　$b = 10\,\text{in}$ ，$d = 21\,\text{in}$ 計算

$$(R_u)_{修} = \frac{M_n}{b d^2} = \frac{281.6 \times 12000}{10 \times (21)^2}$$

$$= 766.26\ \text{psi}$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2 m R_u}{f_y}}\right)$$

$$= \frac{1}{176.5}\left(1 - \sqrt{1 - \frac{2 \times 17.65 \times 766.26}{60000}}\right)$$

$$= 0.0147$$

$A_s = \rho b d = 0.0147 \times 10 \times 21 = 3.08\ \text{in}^2$

採用 $b = 10\text{in}$, $d = 21\text{in}$, $A_s = 3.08\text{in}^2$ ←

(a) 最大容許尺寸：

取 $\rho = \rho_{min} = \dfrac{200}{f_y} = 0.0033$

$$R_u = \rho f_y \left(1 - \frac{1}{2}\rho m\right)$$

$$= 0.0033 \times 60000 \times \left(1 - \frac{1}{2} \times 0.0033 \times 17.65\right)$$

$$= 194.18$$

$$(bd^2)_{req'd} = \frac{M_n}{R_u} = \frac{281.6 \times 12000}{194.18}$$

$$= 17402.8$$

b	10	15	16	18
d	41.7	34.1	33.0	31.1
$\dfrac{d}{b}$	4.17	2.27	2.06	1.73

取 $b = 16\text{in}$, $d = 33.0\text{in}$ 爲最大容許尺寸 ←

(b) 最小容許尺寸

取 $\rho = 0.75\rho_b = 0.0214$

$$R_u = \rho f_y \left(1 - \frac{1}{2}\rho m\right)$$

$$= 0.0214 \times 60000 \times \left(1 - \frac{1}{2} \times 0.0214 \times 17.65\right)$$

$$= 1041.5\ \text{psi}$$