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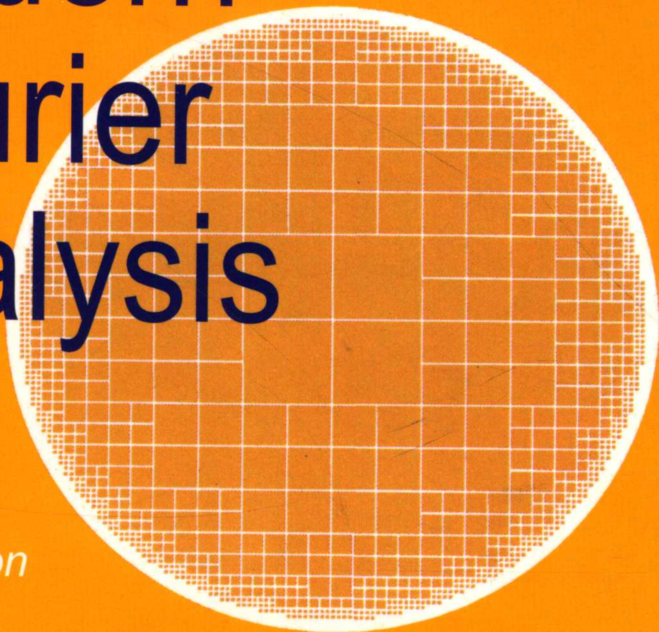
Loukas Grafakos

Modern Fourier Analysis



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Για την Ιωάννα, την Κωνσταντίνα,
και την Θεοδώρα

Preface

I am truly elated to have had the opportunity to write the present third edition of this book, which is a sequel to GTM 249 *Classical Fourier Analysis, 3rd Edition.* This edition was born from my desire to improve the exposition, to fix a few inaccuracies, and to add a new chapter on multilinear operators. I am very fortunate that diligent readers across the globe have shared with me numerous corrections and suggestions for improvements.

Based on my experience as a graduate student, I decided to include great detail in the proofs presented. I hope that this will not make the reading unwieldy. First time readers may prefer to skim through the technical aspects of the presentation and concentrate on the flow of ideas.

This second volume *Modern Fourier Analysis* is addressed to graduate students who wish to delve deeper into Fourier analysis. I believe that after completing a study of this text, a student will be prepared to begin research in the topics covered by the book. While there is more material than can be covered in a semester course, the list of sections that could be taught in a semester without affecting the logical coherence of the book is: 1.1, 1.2, 1.3, 2.1, 2.2, 3.1, 3.2, 3.3, 4.1, 4.2, 4.3, and 5.1.

In such a large piece of work, it is impossible to have no mistakes or omissions. I encourage you to send your corrections to me directly (grafakosl@missouri.edu). The website

`http://math.missouri.edu/~loukas/FourierAnalysis.html`

will be updated with any significant corrections. Solutions to all of the exercises for the present edition will be available to instructors who teach a course out of this book.

Athens, Greece, *Loukas Grafakos*
March 2014

Acknowledgments

I am extremely fortunate that several people have pointed out errors, misprints, and omissions in the previous three editions of the books in this series. All these individuals have provided me with invaluable help that resulted in the improved exposition of the text. For these reasons, I would like to express my deep appreciation and sincere gratitude to all the following people.

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Among all these people, I would like to give special thanks to an individual who has studied extensively the two books in the series and has helped me more than anyone else in the preparation of the third edition: Danqing He. I am indebted to him for all the valuable corrections, suggestions, and constructive help he has provided me with in this work. Without him, these books would have been a lot poorer.

Finally, I would also like to thank the University of Missouri for granting me a research leave during the academic year 2013-2014. This time off enabled me to finish the third edition of this book on time. I spent my leave in Greece.

Contents

Chapter 1
Smoothness and Function Spaces

We embark on the study of smoothness with a quick examination of differentiability properties of functions. There are several ways to measure differentiability and numerous ways to quantify smoothness. In this chapter we measure smoothness using the Laplacian, which is easily related to the Fourier transform. This relation becomes the foundation of a very crucial and deep connection between smoothness and Littlewood–Paley theory.

Certain spaces of functions are introduced to serve the purpose of measuring and fine-tuning smoothness. The main function spaces we study in this chapter are Sobolev and Lipschitz spaces. Before undertaking their study, we introduce relevant notation and we review basic facts about smooth functions and tempered distributions.

1.1 Smooth Functions and Tempered Distributions

We denote by $\mathbf{R}^n$ the Euclidean space of n tuples of real numbers. The magnitude of a point $x=(x_1,\dots,x_n)\in\mathbf{R}^n$ is $|x|=(x_1^2+\cdots+x_n^2)^{1/2}$. An open ball centered at $x_0\in\mathbf{R}^n$ of radius $R>0$ is denoted by $B(x_0,R)$. The partial derivative of a function f on $\mathbf{R}^n$ with respect to the jth variable x_j is denoted by $\partial_j f$. The mth partial derivative with respect to the jth variable is denoted by $\partial_j^m f$. The *gradient* of a function f is the vector $\nabla f=(\partial_1 f,\dots,\partial_n f)$. A *multi-index* α is an ordered n-tuple of nonnegative integers. Given α,β multi-indices, we write $\alpha\le\beta$ if $\alpha_j\le\beta_j$ for all $j=1,\dots,n$. For a multi-index $\alpha=(\alpha_1,\dots,\alpha_n)$, $\partial^\alpha f$ denotes the derivative $\partial_1^{\alpha_1}\cdots\partial_n^{\alpha_n} f$. If $\alpha=(\alpha_1,\dots,\alpha_n)$ is a multi-index, then the number $|\alpha|=\alpha_1+\cdots+\alpha_n$ is called the *size* of α and indicates the *total order of differentiation* of $\partial^\alpha f$. The space of functions in $\mathbf{R}^n$ all of whose derivatives of order at most $N\in\mathbf{Z}^+$ are continuous is denoted by $\mathscr{C}^N(\mathbf{R}^n)$ and the space of all *infinitely differentiable functions* on $\mathbf{R}^n$ by $\mathscr{C}^\infty(\mathbf{R}^n)$. The space of smooth functions with compact support on $\mathbf{R}^n$ is denoted by $\mathscr{C}_0^\infty(\mathbf{R}^n)$. The class of Schwartz functions $\mathscr{S}(\mathbf{R}^n)$ is the space of all $\mathscr{C}^\infty(\mathbf{R}^n)$ functions all of whose

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