Third Edition

Quantum Hall Effects

Recent Theoretical and Experimental Developments

Zyun F. Ezawa

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Third Edition

Quantum Hall Effects

Recent Theoretical and Experimental Developments

PREFACE TO THE THIRD EDITION

Enthusiasm for research on the quantum Hall (QH) effect is unbounded. The QH effect is one of the most fascinating and beautiful phenomena in all branches of physics. Tremendous theoretical and experimental developments are still being made in this sphere. For instance, the concept of topological insulator stems from the QH effect, and attracts much attention recently. The quantum anomalous Hall (QAH) effect is the QH effect without Landau levels. The quantum spin Hall (QSH) effect is an analogue of the QH effect, where spin currents flow instead of charge currents. Typical topological insulators are the QAH and QSH insulators. These phenomena are becoming feasible experimentally.

Composite bosons, composite fermions and anyons were among distinguishing ideas in the original edition. In the 2nd edition, fantastic phenomena associated with the interlayer phase coherence in the bilayer system were extensively described. The microscopic theory of the QH effect was formulated based on the noncommutative geometry. Furthermore, the unconventional QH effect in graphene was reviewed, where the electron dynamics can be treated as relativistic Dirac fermions and even the supersymmetric quantum mechanics plays a key role.

In this 3rd edition, all chapters are carefully reexamined and updated. One of the highlights of this edition is a new chapter on topological insulators. Graphene's silicon cousin named silicene would be a best candidate of topological insulators, where electrons obey the massive Dirac equation with the mass being controllable under external electric field. It presents a beautiful demonstration of topological phase transitions. Other new topics are recent prominent experimental discoveries in QH effects, which are reviewed in several chapters by the experimentalists themselves in Part V. This new edition provides an instructive and comprehensive overview of the QH effect. It is also suitable for an introduction to quantum field theory with vividly described applications. Only knowledge of quantum mechanics is assumed. This book is ideal for students and researchers in condensed matter physics, particle physics, theoretical physics and mathematical physics.

In Part I, two sections are added on the spin field and skyrmions in easy-axis as well as easy-plain ferromagnets, where the relation between a skyrmion and a bimeron (a pair of merons) is explained in details. In Part II, two chapters are added on QH effects in silicene and on topological insulators. Part III is updated substantially in view of recent experimental works on Josephson effects in the bilayer QH system. Furthermore, sections are added on the dynamics of Goldstone modes in the canted antiferromagnetic phase at $\nu = 2$: A gapless Goldstone mode may appear and induce a spin Josephson supercurrent. Part IV has undergone many minor improvements. Part V contains six new chapters for recent important experimental achievements on various aspects of QH effects.

To complete the third edition I have benefited from fruitful discussions with A. Sawada, Y. Hirayama, A. Fukuda, D. Terasawa, L. Tiemann, Y. Hama, G. Tsitsishvili and others. Special thanks are due to K. Hashimoto, I. Kukushkin, V. Volkov, Y. Hirayama, M. Zudov, L. Tiemann and M. Kawasaki for providing me with the contents of Part V. In designing this Part, the suggestions from K. von Klitzing were very helpful. Further, I am grateful to N. Shibata for providing me with the contents of Chapter 19 and to M. Ezawa for the contents of Chapters 20, 21 and 22. The illustration in the cover of this book, showing the band structure with edge modes of a silicene topological (QAH) insulator, was presented by M. Ezawa.

Zyun Francis Ezawa
Tokyo
January 2013

PREFACE TO THE SECOND EDITION

The quantum Hall (QH) effect is one of the most fascinating and beautiful phenomena in all branches of physics. Composite bosons, composite fermions and fractional charged excitations (anyons) were among the distinguishing ideas in the original edition. Seven years have passed since the original edition. Tremendous theoretical and experimental developments are still being made in this sphere. Many novel ideas have been proposed to understand various novel experimental results, which we have included in this new edition.

First, physics in higher Landau levels is quite different from that in the lowest Landau level because the effective Coulomb interactions are different. Charge density waves such as stripe states emerge, and indeed have been observed experimentally. Second, unconventional QH effects were discovered in graphene (a single atomic layer graphite), which immediately triggered enormous theoretical and experimental studies. It is remarkable that the electron dynamics is governed by the relativistic Dirac theory and that even supersymmetric quantum mechanics plays a key role. Third, intriguing phenomena associated with the interlayer phase coherence and SU(4) QH ferromagnets in the bilayer system have been fully revealed. They include the anomalous Hall resistivity in counter flow experiments and the anomalous diagonal resistivity near the commensurate-incommensurate phase transition point. The latter would signal the formation of a soliton lattice made of sine-Gordon solitons between the two layers. They also include an SU(4) skyrmion as a quasiparticle, which changes its shape from a pseudospin SU(2) texture to a spin SU(2) texture as the density imbalance is controlled between the two layers. Fourth, the microscopic theory of the QH effect is formulated entirely within the theory of noncommutative geometry. Thus, quasiparticles are noncommutative solitons in QH ferromagnets.

This new edition provides an instructive, comprehensive and self-contained overview of the QH effect including recent developments. It is also suitable for an introduction to quantum field theory with vivid applications. For instance, the Dirac theory of electrons and holes has a remarkable realization in QH effects in graphene. A fantastic world of noncommutative geometry together with noncommutative solitons has a concrete realization in QH systems, where various imaginative ideas can be explored theoretically and tested experimentally. QH effects have proved to be so special in condensed matter physics that they are deeply connected with fundamental principles of physics and mathematics. This book is ideal for students and researchers in condensed matter physics, particle physics, theoretical physics and mathematical physics.

In Part I, a new chapter is added for Dirac electrons, holes and supersymmetry. In Part II, two chapters are added for charge density wave states in higher Landau levels and unconventional QH effects in graphene. Part III is revised fully to meet up-to-date achievements in bilayer QH systems. Part IV is rewritten anew in view of noncommutative geometry. Furthermore, almost all parts are retouched for improvement, and a number of misprints have been corrected.

To complete the second edition I have benefited from fruitful discussions on the subject with A. Sawada, Y. Hirayama, N. Kumada, A. Fukuda, D. Terasawa, M. Morino, K. Iwata, S. Kozumi, K. Hasebe, S. Suzuki, K. Ishii, G. Tsitsishvili and others. In particular, the collaboration with A. Sawada and G. Tsitsishvili was indispensable to complete this revision. Special thanks are due to N. Shibata and M. Ezawa for providing me with the contents of Chapter 19 and 20, respectively. Further, I am grateful to N. Shibata for careful reading of the manuscript, and to M. Ezawa for allowing me to use an illustration of the Dirac cone in the cover of this book.

Zyun Francis Ezawa
Sendai
January 2007

PREFACE TO THE FIRST EDITION

Quantum Hall (QH) effects are remarkable macroscopic quantum phenomena observed in the 2-dimensional electron system. The integer QH effect was discovered in 1980 by K. von Klitzing, a century after the discovery of the classical Hall effect, for which he received the Nobel prize in 1985. The fractional QH effect was discovered in 1982 by D. Tsui, H. Störmer and A.C. Gossard. It was predicted by B. Laughlin that a quasiparticle is an anyon carrying electric charge e/m at the filling factor $\nu = 1/m$. In 1997 a direct observation of fractional charges was successfully carried out at $\nu = 1/3$ by measuring a back scattering current noise in Hall-bar experiments. B. Laughlin, D. Tsui and H. Störmer received the Nobel prize in 1998.

QH effects are so special in condensed matter physics that they are deeply connected with the fundamental principles of physics. Moreover, they present concrete realizations of various modern concepts related with topological investigations not only in physics but also in mathematics. The QH system provides us with a rare opportunity to enjoy the interplay between condensed matter physics and particle physics. It is worthwhile to make the subject as a part of the training for all graduate students in physics.

Many fancy ideas appear in QH effects. Composite particle (boson or fermion) is one of them. It is an electron bound to flux quanta. Laughlin started his seminal paper by saying that "The $\frac{1}{3}$ effect, recently discovered by Tsui, Störmer and Gossard, results from the condensation of the two-dimensional electron gas in a GaAs-Ga_xAl_{1-x} heterostructure into a new type of collective ground state". It is our present understanding that the QH state is a condensate of composite bosons. Intriguingly a single electron is converted into a boson by acquiring flux quanta in QH states. Such a statistical transmutation is allowed in the planar geometry due to its intrinsic topological structure. Despite their bosonic low-energy properties, composite bosons obey the fermionlike exclusion principle. The hierarchy of fractional QH states is understood by the use of composite fermions.

Topological solitons play the leading role in QH effects. Indeed, charged excitations (quasiparticles) are topological solitons in the QH condensate. When quantum coherence develops with the spin wave as a Goldstone mode, quasiparticles are skyrmion O(3)-spin textures. Skyrmions were originally proposed in nuclear physics, where they are O(4)-isospin textures to be identified with nucleons. Though their relevance is still unclear in nuclear physics, their existence is firmly established in the QH system.

Edge excitations are described by the chiral Tomonaga-Luttinger model. Electrons are topological excitations in this model, and obey a relativistic field equation. When tunneling interactions are allowed between the opposite edges, topological excitations turn into sine-Gordon solitons. Physics on the edge is "exactly solvable", due to the existence of infinitely many conservation rules. It can be a laboratory to test various results of conformal field theories.

The bilayer QH system consists of two quantum wells separated by a barrier with ~30 nm width. A phase transition has been observed between two distinguishable phases at a fixed filling factor by changing the electron density. When the tunneling gap is not too large, interlayer coherence develops, as is a reminiscence of the superconductor Josephson junction. Pseudoparticles are CP^3 skyrmions.

Physics confined to the lowest Landau level is curious, because the x and y coordinates of the electron position become noncommutative. It is a simplest physical system subject to noncommutative geometry. The system is characterized by the Moyal algebra or the W_∞ algebra. When restricted to the edge of the QH droplet, the Kac-Moody algebra emerges. The excitation spectrum of the QH system forms a representation of the algebra.

This book is intended to give a pedagogical and self-contained introduction to these new concepts in QH effects. It is accessible, and will be of interest, to students and researchers in condensed matter physics, particle and mathematical physics. It comprises four parts. Part I is a quick summary of quantum field theory, where I explain various concepts necessary to understand QH effects, namely, canonical quantization, quantum coherence, topological solitons, anyons and so on. I hope that this part is useful to those who are not familiar to these concepts. Readers may skip this part and come back to relevant places when necessary. Part II is devoted to monolayer QH systems, while Part III to bilayer QH systems. Some algebraic aspects of the QH system are reviewed in Part IV, where I derive some basic formulas used in Part II and Part III.

I have benefited from fruitful collaborations on the subject with A. Sawada, A. Iwazaki, H. Ohno, Y. Hirayama, K. Muraki, Y. Horikoshi, Y. Ohno, T. Saku, N. Kumada, M. Hotta, K. Sasaki, K. Hasebe, Y-S. Wu and others. I am grateful to H. Aoki, D. Yoshioka, A. MacDonald, S. Murphy and J. Eisenstein for valuable discussions. Special thanks are due to K. Shizuya, S. Katsumoto and K. Takahashi for careful reading of the manuscript. Their comments were very helpful for me to improve the manuscript.

Zyun Francis Ezawa
Sendai
January 2000

Quantum Hall Effect

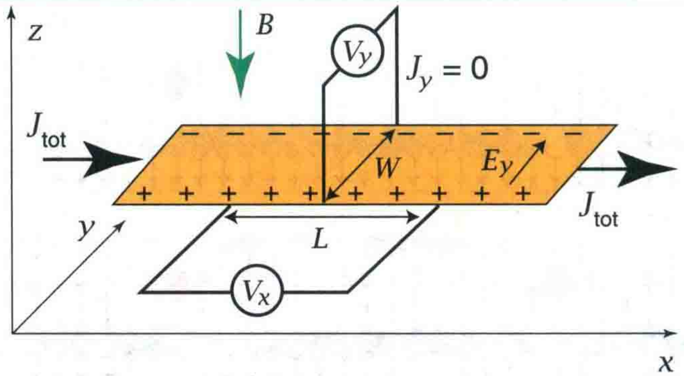

Fig. 1 Hall and diagonal resistivities R_{xy} and R_{xx} are independent of sample properties and given by $R_{xy} = V_y/J_{tot} \longrightarrow \nu^{-1}(2\pi\hbar/e^2)$, $R_{xx} = -WV_x/LJ_{tot} \longrightarrow 0$, in quantum Hall (QH) states. Here, ν is the filling factor.

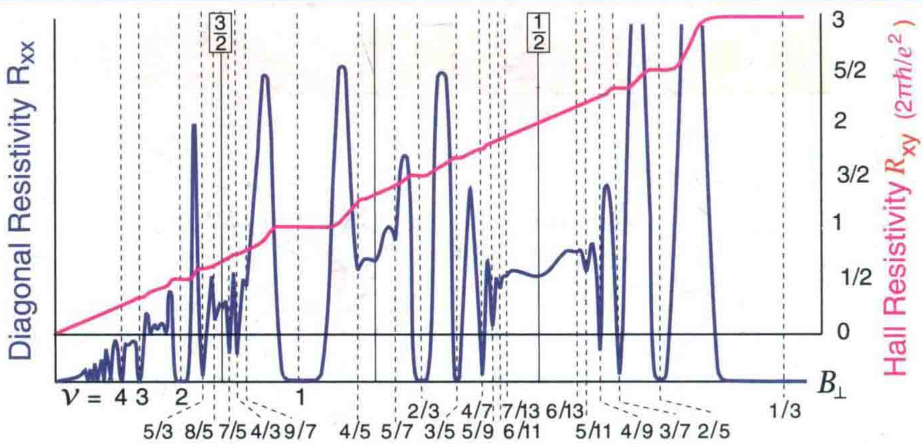

Fig. 2 QH states are detected by plateaux developed in the Hall resistivity R_{xy} or dips in the diagonal resistivity R_{xx}.

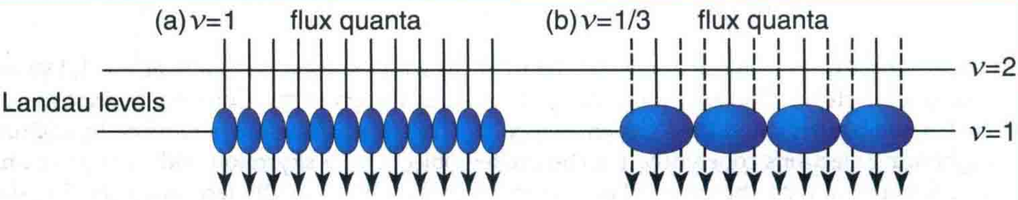

Fig. 3 There are m flux quanta (vertical lines) per electron at $\nu = 1/m$. A composite particle (fermion or boson) is an electron bound to (m–1 or m) flux quanta. It behaves as a "fat" electron in the fractional QH state. Here, (a) $m = 1$ and (b) $m = 3$.

Skyrmions

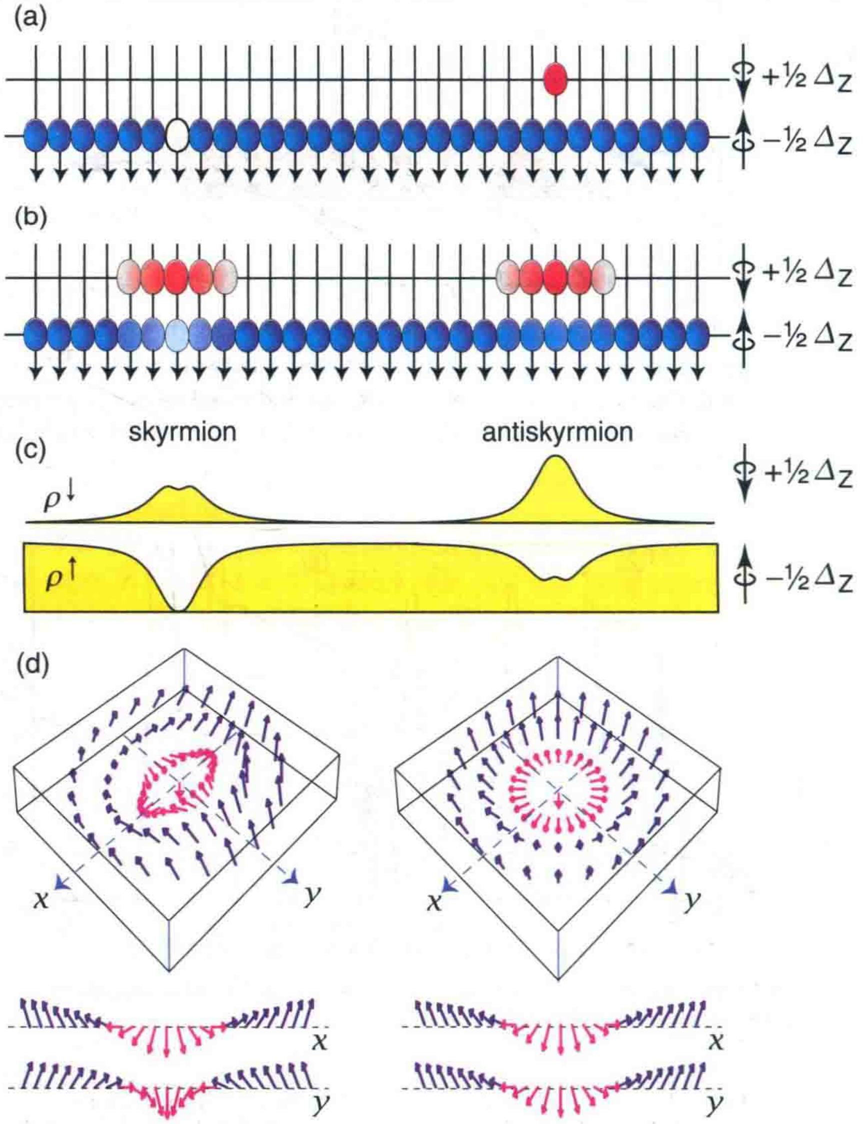

Fig. 4 (a) An up-spin electron is excited to the down-spin level thermally at $\nu = 1$, leaving a quasihole behind. The excitation energy consists of the exchange Coulomb energy, the direct Coulomb energy and the Zeeman energy. (b) The direct Coulomb is reduced by exciting neighboring electrons coherently. (c) The excited objects are a skyrmion with charge $+e$ and an antiskyrmion with charge $-e$. They are topological solitons in QH ferromagnets. The size is determined so as to optimize the Coulomb and Zeeman energies. (c) The electron densities $\rho^{\uparrow}(\boldsymbol{x})$ and $\rho^{\downarrow}(\boldsymbol{x})$ are illustrated around them. (d) The electron spin is reversed at the center of the skyrmion and the antiskyrmion. The normalized spin density is depicted.

Edge States

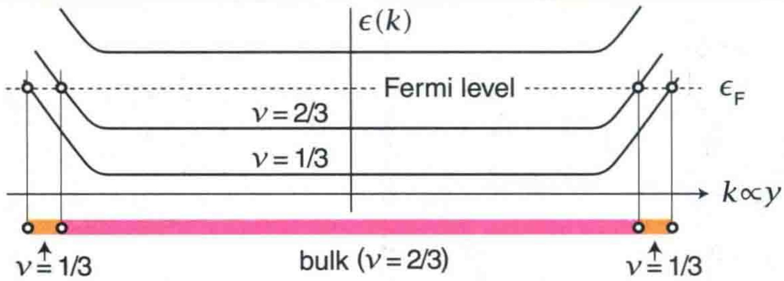

Fig. 5 Dispersion relation is illustrated in a QH bar as a function of the one-dimensional momentum $\hbar k$, related to the electron position as $y = \ell_B^2 k$ in the Landau gauge. Gapless edge channels appear when the Fermi level crosses energy levels. This is an example of the bulk-edge correspondence inherent to topological insulators.

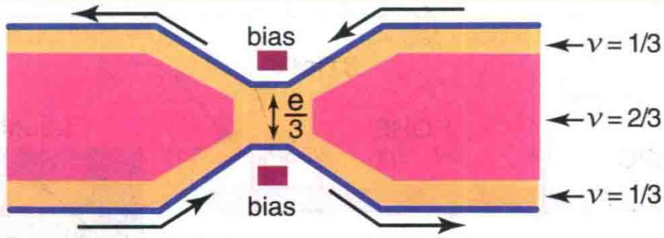

Fig. 6 Quasiparticles flow along the edge. With weak pinch-off by bias voltages, they tunnel between the top and bottom edges through the QH liquid.

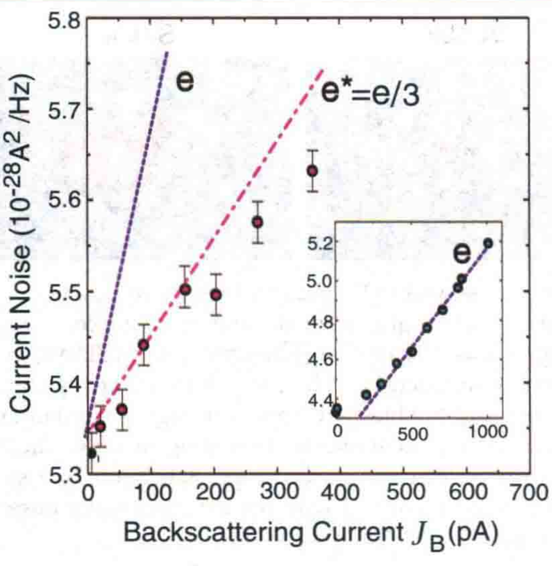

Fig. 7 The charge of quasiparticles can be determined by the Shchottky noise formula, when they tunnel one by one as in Fig.6. Tunneling noise was measured in the QH state at the bulk filling factor $\nu = 2/3$, and also at $\nu = 4$ (given in the inset). The charge $e/3$ of quasiparticles was clearly observed at $\nu = 2/3$. Data are taken from L. Saminadayar et al.[475]

Stripes and Bubbles

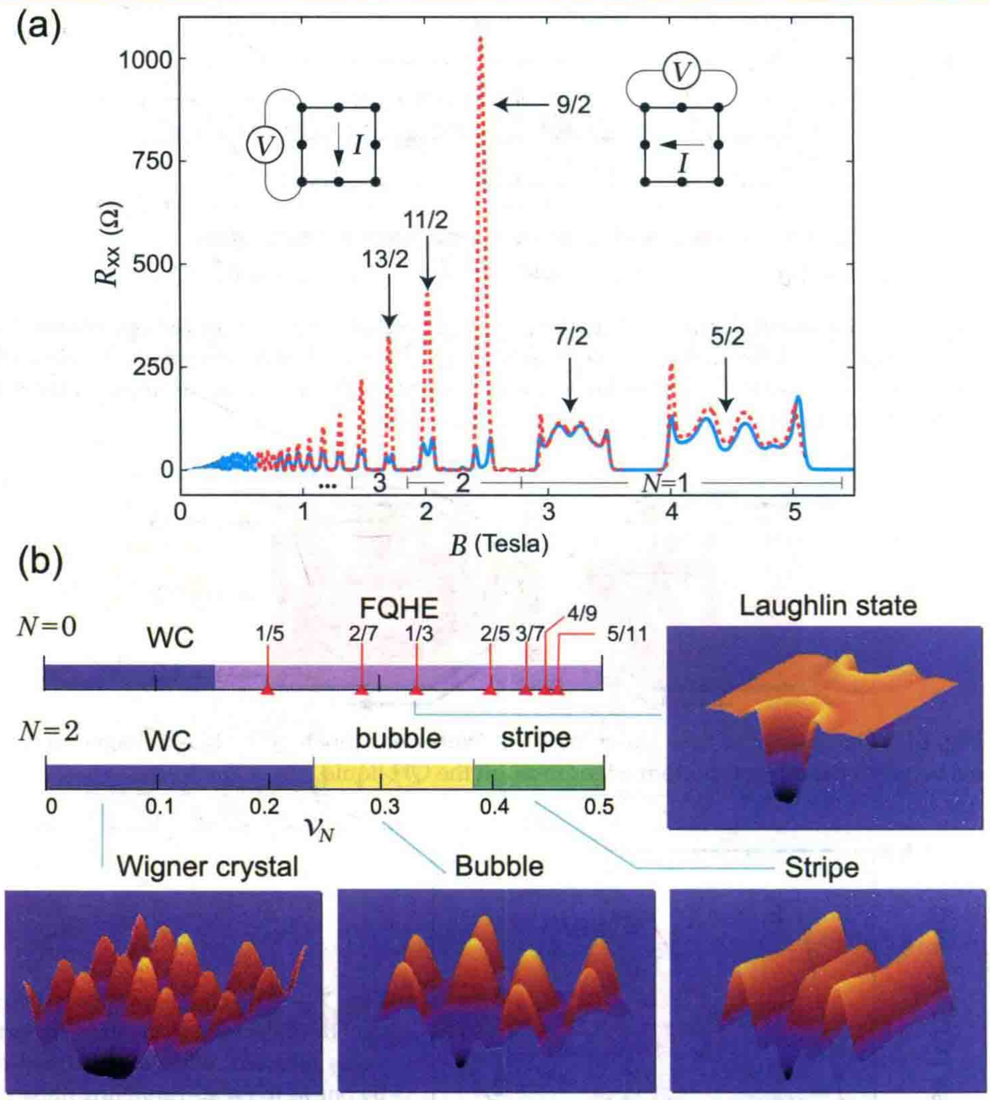

Fig. 8 (a) A strong anisotropy has been observed in the diagonal resistivity R_{xx} at $\nu = n/2$ with $n = 5, 7, 9, 11, \cdots$ by changing the direction of current through the sample. Data are taken from M. Lilly et al.[350] (b) We expect a variety of charge density waves (CDWs) in QH systems. Electrons form a Wigner crystal at low density. The Laughlin state emerges in the lowest Landau level ($N = 0$), while stripes and bubbles will appear in higher Landau levels ($N > 0$), where the Coulomb potential is not a monotonically decreasing function. In these CDW states electrons form clusters with other electrons. The stripe state leads to a strong anisotropy in the diagonal resistivity R_{xx} to be identified with the experimentally observed one. These figures are taken from N. Shibata et al.[494] See Chapter 19.

QH Effects in Graphene and Silicene

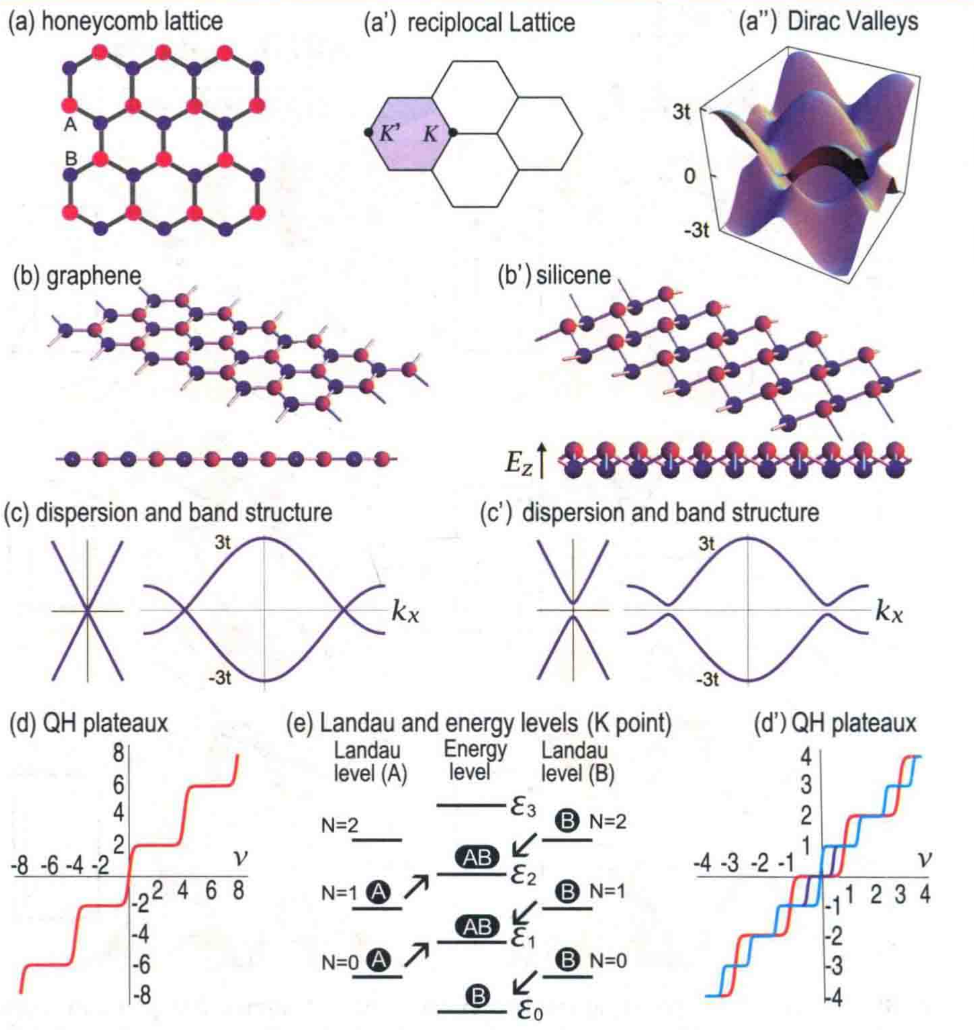

Fig. 9 (a) The honeycomb structure for graphene and silicene. (a′) The reciplocal lattice is also a honeycomb lattice with inequivalent K and K′ points. (a″) The energy spectrum has a valley-like structure near these points. (b) Graphene is purely 2-dimensional, while (b′) silicene is made of two sublattices for A and B sites with a tiny separation. The staggerd potential is generated between them in electric field E_z. Silicene undergoes a phase transition at $E_z = E_{cr}$, as E_z increases. (c) Graphene has a linear dispersion with massless Dirac electron, while (c′) silicene has a parabolic dispersion with massive Dirac electron. The Dirac mass is negative, zero and positive for $E_z < E_{cr}$, $E_z = E_{cr}$ and $E_z > E_{cr}$. (d) QH plateaux in graphene. (d′) QH planeaux in silicene for $E_z = 0$ (red), $E_z = E_{cr}$ (cyan) and elsewhere (cyan and dark blue instead of cyan around $\nu = 0$). (e) The mechanism of Landau-level mixing. Except for the zero-energy level, an electron in one energy level is a superposition of two electrons coming from different Landau levels for A and B sites. These illustrations are taken from M. Ezawa.[142,146]

Topological Insulators in Silicene

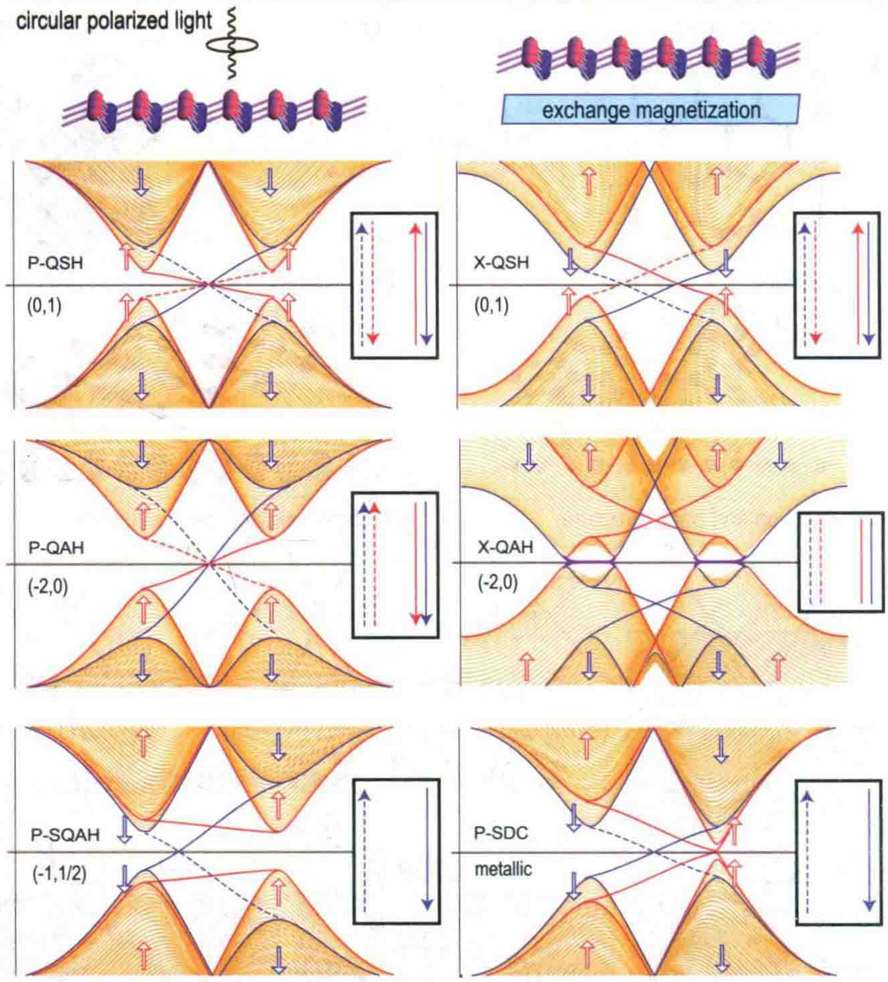

Fig. 10 Silicene is a topological insulator made of silicone atoms. The quantum anomalous Hall (QAH) effect is the QH effect without Landau levels. The quantum spin Hall (QSH) effect is the QH effect of spins instead of charges. The QSH effect is induced by internal magnetic field due to the spin-orbit coupling, while the QAH effect occurs (a) by irradiating coherent cycular polarized raser beam or (b) by introducing the exchange field. A topological insulator is characterized by the bulk property, that is, a set of the first Chern number C and the $\mathbb{Z}_2$ index C_s, or by the edge property, that is, gapless edge modes connecting the conduction and valence bands. The band structure and the edge modes are presented together with (C,C_s) for typical topological insulators realizable in silicene. Here, P- and X- are the abbreviations of photo-induced and exchange-induced: P-SQAH and P-SDC stand for the photo-induced spin-polarized QAH state and photo-induced single Dirac cone state. The gapless edge modes are flat bands in the X-QAH state. Only the P-SDC is a metalic state in the figures. These illustrations are taken from M. Ezawa.[145,148]