



Mathematical Models of Economic Growth and Crises

Alexei Krouglov

Mathematics Research Developments

NOVA

The main goal of this book is to present coherent mathematical models to describe economic growth and related economic issues. The book is a continuation of the author's previous book *Mathematical Dynamics of Economic Markets*, which presented mathematical models of economic forces acting on the markets.

In this current volume, the author looks into various aspects that savings and investment exert on the market. He discusses the models that examine economic growth in situations when savings and investment were done in the form of a one-time withdrawal of the product, constant-rate withdrawal of product, constant-accelerated withdrawal of product, and exponential withdrawal of product from the market.

The impact of four economic concepts on economic growth is also examined. These concepts are demand, supply, investment, and debt. This volume presents mathematical models exploring interconnections among these concepts and studies their mutual impacts on both economic growth and decline. A mathematical model is built in order to verify a hypothesis that weak recovery after the financial crisis could be attributed to the decline of investments that were not compensated by the decrease of an interest rate. In addition, the phenomenon of economic crises is analyzed and a few mathematical models are built.



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Mathematical Models of Economic Growth and Crises ■ Kruglov



MATHEMATICS RESEARCH DEVELOPMENTS

**MATHEMATICAL MODELS
OF ECONOMIC GROWTH
AND CRISES**

ALEXEI KROUGLOV



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Chapter 1

INTRODUCTION

This book is a continuation of my previous book (Krouglov, 2006) where I tried to present a coherent mathematical description of economic forces acting on economic markets. In (Krouglov, 2006) I also attempted to mathematically explain economic processes underlying an economic growth on markets. I tried to mathematically describe an economic growth in single-product economy. The explanation was briefly that markets participants withdraw part of product from markets in the form of savings and use the withdrawn product in consecutive production in the form of investment. The withdrawal drives product's supply on market down while at the same time it drives product's price on markets up, which in turn drives product's demand down. When an impact of the product's price increase exceeds an impact of the product's demand decrease we observe an economic growth whereas we observe an economic decline in the opposite situation.

In *Chapter 2* of this book I continue examining the phenomenon of an economic growth and present few mathematical models for this purpose. The work is based on research done in (Krouglov, 2006) where the system of ordinary differential equations was presented,

which linked economic forces behind the product's demand, product's supply and product's prices on the market. In this chapter I look into various effects that savings and investment exert on market of the single-product economy.

In *Chapter 3*, I examine an impact of four important economic concepts on the economic growth – demand, supply, investment, and debt. I present mathematical models to explore interconnections of these concepts and study their impacts on an economic growth and decline. I also build a mathematical model to verify hypothesis that weak recovery after the financial crisis could be attributed to a decline of investment that was not compensated by the decrease of interest rate.

In *Chapter 4*, I examine the phenomenon of economic crises and present few mathematical models for this purpose. The models for four economic crises are considered in this chapter. The first model is about the last financial crisis, which tries to explain how relatively small disturbances on financial markets have produced large effects in real economy. I build a model describing how fluctuations on two connected markets could amplify each other, which is known as *resonance phenomenon*. The second model is related to monetary part of the Japanese economic policy known as *Abenomics* where a “weird” result happened. Despite steady purchasing of the bonds by the Bank of Japan their price decreased and the yield increased. I construct a mathematical model of the financial market to investigate this phenomenon. The third model is devoted to the *secular stagnation* hypothesis, which was advanced by economist Lawrence Summers. Summers's ideas were close to my work where I was developing the model of an economic growth. I complemented the model with a situation where external supply of product comes on market. The results of model told the external supply of product provided with either constant rate or constant acceleration could cause either a restricted (secular stagnation) or an unrestricted (secular decline) economic decline. The fourth model is a simplified model describing four different stages of the Greek economic crisis (before the Eurozone, before the Euro crisis, after the Euro crisis,

and during the austerity period) and two possible stages of the recovery (with austere and benign economic transformations). Theoretical result is that study of different stages of the crisis was performed with mathematical accuracy.

