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## Analysis, Modeling and Control



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## Stochastic Dynamic Systems—Analysis, Modelling and Control

Jie Li

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Prof. Jie Li is currently a Chair Professor in the Structural Engineering at Tongji University, and the director of Shanghai Institute of Disaster Prevention and Relief. He currently serves as the president of International Association for Structural Safety and Reliability (IASSAR), a member of board directors of International Civil Engineering Risk and Reliability Association (CERRA), and a member of Joint Committee on Structural Safety (JCSS). Prof. Li also serves as the vice president of Chinese Society of Vibration Engineering, the chairman of the Random Vibration Committee



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# An Expanded System Method for the Stochastic Dynamic Analysis

Jie Li   J. B. Roberts

**Abstract:** By applying an orthogonal expansion technique in random space, a general formulation for an expanded system method, suitable for the dynamic analysis of linear multi-degree of freedom (MDOF) stochastic structural systems, is established. This can be used for various kinds of systems which possess combinations of random mass, damping and stiffness parameters. The proposed method is validated through an analysis of simulated data.

## 1 Introduction

The effect of uncertainty, with regard to material properties or structural geometry, on the dynamic response of structures to time-varying excitation is of major concern in the field of reliability design and in the probabilistic safety assessment of many engineering structures. Recent advances in computer technology have greatly enhanced the scope for work in this area and have intensified research efforts towards developing efficient methods for analysing the dynamic response of stochastic structural systems.

Previous studies, in the past two decades, have concentrated on the use of either simulation methods<sup>[1,2]</sup> or perturbation techniques<sup>[3-6]</sup>; these are two of the major tools available for stochastic structural analysis. Because exact solutions are impossible to obtain for most real engineering structures, the use of simulation methods is usually regarded as the most practical approach for design purposes. However, a basic problem with simulation is that it invariably requires an extensive computation, which is often very expensive to implement. On the other hand, perturbation methods usually suffer from problems with

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regard to their accuracy and to their convergence, which may be exacerbated when a dynamic response is sought. In particular, because of the effect of secular terms, generated in first and second order solutions, the accuracy of statistical results tends to deteriorate rapidly as time elapses<sup>[7, 8]</sup>.

In an attempt to avoid these difficulties alternative approaches, based on orthogonal decompositions of the structural response, have been developed recently and applied to the analysis of both static<sup>[9, 10]</sup> and dynamic systems<sup>[11, 12]</sup>. Although the specific examples which have been considered so far in the published literature have shown that a good agreement with simulation estimates can be obtained, further work is required to establish and validate a general methodology.

This paper focuses on the dynamic analysis of linear MDOF stochastic structural systems responding to deterministic time-varying excitation. On the basis of an orthogonal expansion technique in a multi-dimensional random space, a general formulation is established, leading to an expanded system method. A subjunctive structure technique is applied, akin to the normal finite element method, which can be used to generate a basic matrix for the expanded system. The general scheme developed here can be applied to various MDOF stochastic structural systems, possessing random mass, damping and stiffness properties. The method is validated through a comparison with simulation results.

## **2 Transformation from a Random Field to Independent Random Variables**

Randomness in material properties, or in structural geometry, may be modelled mathematically either in terms of a vector of independent random variables or as a continuous random field. Recent research has shown that a random field representation can be converted into a random vector representation by discretizing the field; the mid-point, local averaging and shape function methods offer three particular ways of achieving this objective<sup>[7, 13]</sup>. The resulting vector of (in general) correlated random variables can be further transformed into a vector of independent random variables through a spectral decomposition of the covariance matrix.

As an alternative approach, a random field model can be transformed into an independent random vector representation by means of the Karhunen-Loeve decomposition. This leads to an integral equation which involves the eigenvalues and eigenvectors of the correlation kernel which specifies the random field<sup>[10]</sup>.

By means of a discrete integration rule, or through the use a Galerkin-type approximation, the solution of the integral equation can be converted into a matrix eigenvalue problem. This leads to a result which is equivalent to that found by the two-step method described above. Thus, here only the two-step method is discussed, and local averaging is adopted as the discretization method.

Let  $B(\mathbf{u})$ ,  $\mathbf{u} \in \Omega$ , denote a multi-dimensional Gaussian random field defined within the domain  $\Omega$ . The field is completely described by its mean function,  $B_0(\mathbf{u})$ , the variance function  $\sigma^2(\mathbf{u})$  and the normalized covariance function  $\rho(\mathbf{u}, \mathbf{u}')$ .  $B(\mathbf{u})$  can be expressed as

$$B(\mathbf{u}) = B_0(\mathbf{u}) + B_\sigma(\mathbf{u}) \quad (1)$$

where  $B_\sigma(\mathbf{u})$  is a random field with zero mean and the same covariance function as  $B(\mathbf{u})$ .

If the domain  $\Omega$  is divided into  $N$  elements, the local average of the  $i$ th element of the field can be defined as

$$v_i = \frac{1}{\Omega_i} \int_{\Omega_i} B(\mathbf{u}) d\mathbf{u}, \quad \mathbf{u} \in \Omega_i \quad (2)$$

where  $\Omega_i$  is the volume of  $i$ th element.

The mean of  $v_i$  is given by

$$v_{0i} = E\{v_i\} = \frac{1}{\Omega_i} \int_{\Omega_i} B_0(\mathbf{u}) d\mathbf{u}, \quad \mathbf{u} \in \Omega_i \quad (3)$$

where  $E\{v_i\}$  is an expectation operator. The covariance of the  $i$ th element and the  $j$ th element is given by

$$R_{ij} = \text{Cov}(v_i, v_j) = \frac{1}{\Omega_i \Omega_j} \int_{\Omega_i} \int_{\Omega_j} \sigma(\mathbf{u}) \sigma(\mathbf{u}') \rho(\mathbf{u}, \mathbf{u}') d\mathbf{u} d\mathbf{u}' \quad (4)$$

The local average values,  $v_i$  now form a new vector,  $\mathbf{v}$  which can be written as

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_\sigma, \quad \mathbf{v}_0 = \{v_{0i}\} \quad (5)$$

Here  $\mathbf{v}_\sigma$  is a zero-mean random vector with a covariance matrix

$$\mathbf{R} = [R_{ij}] \quad (6)$$

the elements of which are defined by Eq. (4).

It is possible to use a spectral decomposition of the covariance matrix to obtain an alternative to Eq. (5), in terms of independent random variables;



i. e.

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{V}\mathbf{b} \quad (7)$$

$$\mathbf{V} = [\Phi_1 \sqrt{\theta_1}, \Phi_2 \sqrt{\theta_2}, \dots, \Phi_N \sqrt{\theta_N}], \quad \mathbf{b} = [b_i] \quad (8)$$

$b_i$  are independent standard random variables and  $\theta_i$  and  $\Phi_i$  are, respectively, the eigenvalues and eigenvectors of  $\mathbf{R}$ : The eigenvectors are normalized such that  $\Phi_i^T \Phi_j = \delta_{ij}$ , where  $\delta_{ij}$  is the Kronecker delta.

Using a subset,  $r < N$ , of the terms in Eq. (6), corresponding to the largest eigenvalues, the number of random variables describing the random field can be reduced. Then  $N$  can be replaced by  $r$  in Eq. (7).

It is worth pointing out that the above description is for a continuous system. On the other hand, for a discrete system which has been decomposed in advance, by using a finite element mesh, it can be assumed that the random variables (not necessarily Gaussian) in each element are independent each other or have the characteristic of independent identical probability distributions in a specific elements set. Both of these assumptions lead to an expression similar to Eq. (7) but with a diagonal nominal variance matrix

$$\mathbf{V} = \text{diag}(v_{a1}, v_{a2}, \dots, v_{ar}) \quad (9)$$

where  $r$  is the number of independent random variables and  $v_{aj}$  is a nominal variance of  $v_j$ . The relationship between the nominal variance  $v_{aj}$  and the variance  $v_{oj}$  is decided by the distribution of random variables. For example, when the distribution of random variables belongs to uniform distribution, there exists  $v_a = \sqrt{3} v_o$ .

In summary, there are two ways of describing the randomness of structural parameters. Both of them lead to a model in terms of a set of independent random variables. Thus, in the following parts of the paper, it is only necessary to discuss stochastic structural systems with independent random variables.

### 3 The Dynamic Matrix

According to finite element methodology, the stiffness matrix of general MDOF dynamic system can be written as

$$\mathbf{K} = \sum_{i=1}^N \mathbf{T}_i^T \mathbf{K}_i^e \mathbf{T}_i \quad (10)$$

where  $N$  is the finite element number and  $\mathbf{T}_i$  is a position matrix which depends

on the structural form,  $\mathbf{K}_i^e$  is the  $i$ th element stiffness matrix.

When the elastic modulus,  $E$ , or resistance moment of a section,  $EI$ , is considered as a random field, the independent random variable's expression such as Eq. (7) can be derived based on the discussion in former section. Then, by substituting these expressions into Eq. (10), the random stiffness matrix of stochastic MDOF system is of the form

$$\mathbf{K} = \mathbf{K}_0 + \sum_{j=1}^{N_k} \mathbf{K}_j b_j \quad (11)$$

where  $N_k$  is the number of independent stiffness parameters and

$$\mathbf{K}_0 = \sum_{i=1}^N \mathbf{T}_i^T \mathbf{K}_{0i}^e \mathbf{T}_i, \quad \mathbf{K}_j = \sum_{i=1}^N \mathbf{T}_i^T (\mathbf{K}_{aj}^e)_i \mathbf{T}_i \quad (12)$$

$\mathbf{K}_{0i}^e$  and  $\mathbf{K}_{aj}^e$  can be calculated for specific structures.

The random mass matrix,  $\mathbf{M}$ , and the random damping matrix,  $\mathbf{C}$ , can be expressed in a similar form to Eq. (11), with  $N_m$  and  $N_c$  as the number of independent mass and damping parameters, respectively.

## 4 Dynamic Response of Stochastic Structures

The general equation of motion for a linear MDOF stochastic dynamic system is as follows:

$$\mathbf{M}\ddot{\mathbf{Y}} + \mathbf{C}\dot{\mathbf{Y}} + \mathbf{K}\mathbf{Y} = \mathbf{F}(t) \quad (13)$$

Here  $\mathbf{F}(t)$  is deterministic external load vector and  $\mathbf{Y}$  is the displacement response vector.

Let

$$\mathbf{A}_{ms} = \begin{cases} \mathbf{M}_j, & s \leq N_m, \quad j = s \\ \mathbf{0}, & s > N_m \end{cases} \quad (14)$$

$$\mathbf{A}_{cs} = \begin{cases} \mathbf{0}, & s \leq N_m \\ \mathbf{C}_j, & N_m < s \leq N_m + N_c, \quad j = s - N_m \\ \mathbf{0}, & s > N_m + N_c \end{cases} \quad (15)$$

$$\mathbf{A}_{ks} = \begin{cases} \mathbf{0}, & s \leq N_m + N_c \\ \mathbf{K}_j, & s > N_m + N_c, \quad j = s - (N_m - N_c) \end{cases} \quad (16)$$

and

$$\mathbf{A}_{m0} = \mathbf{M}_0, \quad \mathbf{A}_{c0} = \mathbf{C}_0, \quad \mathbf{A}_{k0} = \mathbf{K}_0 \quad (17)$$

Then Eq. (13) can be written as

$$(\mathbf{A}_{m0} + \sum_{s=1}^R \mathbf{A}_{ms} b_s) \ddot{\mathbf{Y}} + (\mathbf{A}_{c0} + \sum_{s=1}^R \mathbf{A}_{cs} b_s) \dot{\mathbf{Y}} + (\mathbf{A}_{k0} + \sum_{s=1}^R \mathbf{A}_{ks} b_s) \mathbf{Y} = \mathbf{F}(t) \quad (18)$$

where  $R = N_m + N_c + N_k$ .

From the viewpoint of functional analysis, the dynamic response of a stochastic structural system can be regarded as a locus in the space of the random variables. Let  $\mathbf{H}$  denote a Hilbert space spanned by a set of the basic functions  $\{H_i(\mathbf{b})\}_{i=0}^{\infty}$ . Then the locus of  $\mathbf{Y}$  can be expanded as

$$\mathbf{Y}(\mathbf{b}, t) = \sum_{i=1}^{\infty} \mathbf{X}_i(t) H_i(\mathbf{b}) \quad (19)$$

where  $\mathbf{X}_i(t)$  is a deterministic time process vector.

For the multi-dimensional random space  $\mathbf{H}$ , it is convenient to define

$$H_i(\mathbf{b}) = \prod_{s=1}^R H_{l_s}(b_s) \quad (20)$$

where  $H_{l_s}(b_s)$  is a polynomial in the variables  $b_s$  of degree  $l_s$ ,  $l_s = 0, 1, 2, \dots$ . For different probability distribution functions of random variables, the polynomial choice is different. For example, weighted Hermite polynomials correspond to the standard normal distribution, Legendre polynomials to a uniform distribution, and so on.

The orthogonality of  $H_i(\mathbf{b}_s)$  can be expressed by

$$E \left\{ \prod_{s=1}^R H_{l_s}(b_s) \prod_{s=1}^R H_{k_s}(b_s) \right\} = \begin{cases} 1, & \text{When } l_s = k_s \\ 0, & \text{others} \end{cases} \quad (21)$$

The solution of the stochastic structural system Eq. (18), say,  $\mathbf{Y}$ , now can be approximated by the following series

$$\mathbf{Y}(\mathbf{b}, t) = \sum_{\substack{0 \leq l_s \leq N_s \\ 1 \leq s \leq R}} \mathbf{X}_{l_1 l_2 \dots l_R}(t) \prod_{s=1}^R H_{l_s}(b_s) \quad (22)$$

where  $N_s$  is the expanded order for the variables  $b_s$ ,  $\mathbf{X}_{l_1 l_2 \dots l_R}(t)$  are deterministic functions of time corresponding to  $\mathbf{X}_i(t)$  in Eq. (19).

Substituting Eq. (22) into Eq. (18), multiplying the resulting equation by  $\prod_{s=1}^R H_{k_s}(b_s)$  and then taking an expectation with respect to the random variables,  $\mathbf{b}$ , we obtain the following equations:

The global equation is

$$\mathbf{A}_m \ddot{\mathbf{X}} + \mathbf{A}_c \dot{\mathbf{X}} + \mathbf{A}_k \mathbf{X} = \mathbf{F}(t) \quad (23)$$

and the corresponding elements expression is

$$\sum_{p=1}^{MN} [(a_m)_{l,p} \ddot{x}_p + (a_c)_{l,p} \dot{x}_p + (a_k)_{l,p} x_p] = f_l(t) \quad (24)$$

where, omitting the indices  $m, c, k$ ,

$$a_{l,p} = \mathbf{A}_0 \delta_{l,p} + \sum_{s=1}^R \mathbf{A}_s (\gamma_{k_s-1} \delta_{l-\zeta_s,p} + \beta_{k_s} \delta_{l,p} + \alpha_{k_s+1} \delta_{l+\zeta_s,p}) \quad (0 \leq k_s \leq N_s) \quad (25)$$

$$MN = \prod_{s=1}^R (N_s + 1) \quad (26)$$

$$l = 1 + \sum_{s=1}^R k_s \prod_{j=s+1}^R (N_j + 1) \quad (27)$$

$$\zeta_s = \begin{cases} 1, & s = R \\ \prod_{j=1}^{R-j} (N_{R-j} + 1), & s < R \end{cases} \quad (28)$$

$$f_l = f_{k_1 k_2 \dots k_R} = \mathbf{F}(t) \prod_{s=1}^R \delta_{0 k_s} \quad (29)$$

$\alpha, \beta$  and  $\gamma$ , appearing in Eq. (25), come from the following recurrence relation for the orthogonal polynomials

$$b_s H_{l_s}(b_s) = \alpha_{l_s} H_{l_s-1}(b_s) + \beta_{l_s} H_{l_s}(b_s) + \gamma_{l_s} H_{l_s+1}(b_s) \quad (30)$$

It is worth pointing out that the above formulae differ completely from other corresponding results in the literature<sup>[11, 12]</sup>.

Obviously, Eq. (23) is similar to Eq. (18), but the order of the system has been expanded. The dynamic system governed by Eq. (23) is called here the “expanded-order system” for the original stochastic system. The method of solving the stochastic structural system, using Eqs. (23) to (29), is referred to as the expanded system method.

According to Eq. (22), the expansion order for the different variables may be varied. An analysis of the expanded matrices  $\mathbf{A}_m$ ,  $\mathbf{A}_c$  and  $\mathbf{A}_k$  shows that, for the same order of expansion, the symmetric property of the matrices is retained. However, if the orders in the expansion vary, symmetry may be lost in some cases.

If the initial conditions for Eq. (18) are taken as  $\mathbf{Y}(0) = \mathbf{Y}_0$ ,  $\dot{\mathbf{Y}}(0) = \dot{\mathbf{Y}}_0$ , where  $\mathbf{Y}_0$  and  $\dot{\mathbf{Y}}_0$  are deterministic vectors, then the initial conditions for Eq. (23)

are

$$\mathbf{X}_l(0) = \mathbf{X}_{k_1 k_2 \dots k_R}(0) = \mathbf{Y}_0 \prod_{s=1}^R \delta_{0k_s}, \quad \dot{\mathbf{X}}_l(0) = \dot{\mathbf{X}}_{k_1 k_2 \dots k_R}(0) = \dot{\mathbf{Y}}_0 \prod_{s=1}^R \delta_{0k_s} \quad (31)$$

The expanded system Eq. (23), with initial conditions given by Eq. (31), can be solved by using any of the analytical methods available for deterministic dynamic structural systems. Once the solutions of Eq. (21) have been obtained, the mean and variance of the stochastic system response can be calculated by using the following formulae:

$$\mathbf{E}\{\mathbf{Y}(t)\} = \mathbf{X}_{0 \dots 0}(t), \quad \text{Var}\{\mathbf{Y}(t)\} = \sum_{\substack{1 \leq l_s \leq N_s \\ 1 \leq s \leq R}} \mathbf{X}_{l_1, l_2 \dots l_R}^T(t) \mathbf{X}_{l_1, l_2 \dots l_R}(t) \quad (32)$$

The statistics of other kinds of response, such as velocity, acceleration, etc. can also be obtained by using expressions analogous to Eqs. (32).

## 5 Examples

To verify the validity of the proposed method, a number of specific examples have been analysed. As an example, results for a two degree-of-freedom system with random mass and stiffness properties are presented here. The damping variability is not considered here because its effect is negligible for a non-resonant response. The system was assumed to be a shear-type story structure with the following element parameters

$$m_1 = m_2 = 1, \quad k_1 = k_2 = 39.48, \quad \delta_{m_1} = \delta_{m_2} = 0.1, \quad \delta_{k_1} = \delta_{k_2} = 0.2$$

Here  $\delta$  is defined as ratio of the variance to the mean of the parameters. The probability distribution of random variables is assumed to be normal, and the damping coefficient of the system is 0.05.

Two kinds of input were chosen. One was a sinusoidal load acting on each mass element with the form

$$f_1(t) = f_2(t) = \sin(\omega_s t), \quad \omega_s = 3.1416$$

The another was an irregular ground motion input to the system. Actually, an earthquake record (the El Centro NS record) was selected for the latter input. First and fourth order expansions were chosen for forming the expanded system. At the same time, corresponding results were also generated directly from simulated data using 5 000 samples in each case. Fig. 1 and Fig. 2 depict the

mean response and variance response results for the case of a sinusoidal force. It can be seen that a first order expansion is sufficient for estimating the mean response of the system. However, an accurate estimation of the response variance may need an expanded system of higher order. Similar trends may be observed in the case of the irregular base motion excitation input (Fig. 3 and Fig. 4).

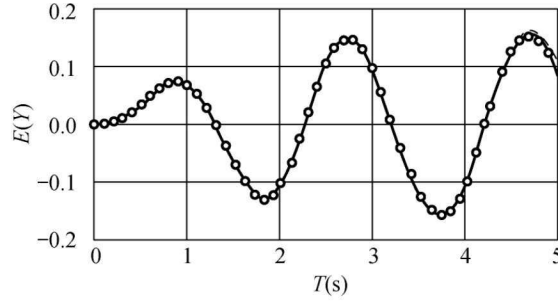


Fig. 1 Mean response Comparison between the expanded order system method and simulation method; Sine wave load. (Circle points for 5 000 simulations, Solid line for 4th orders expansion and dashed line for 1st order expansion)

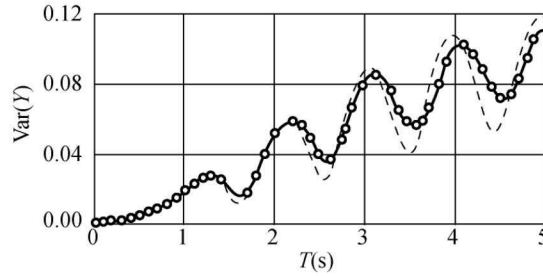


Fig. 2 Variance, response Comparison between the expanded order system method and simulation method; Sine wave load. (Circle points for 5 000 simulations, Solid line for 4th orders expansion and dashed line for 1st order expansion)

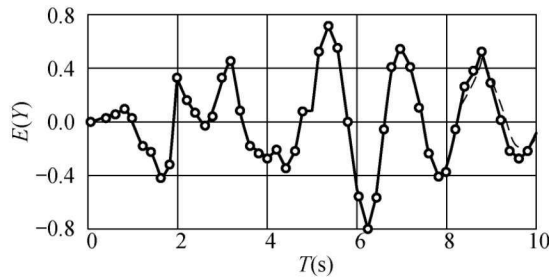


Fig. 3 Mean response Comparison between the expanded order system method and simulation method; Earthquake input. (Circle points for 5 000 simulations, Solid line for 4th orders expansion and dashed line for 1st order expansion)

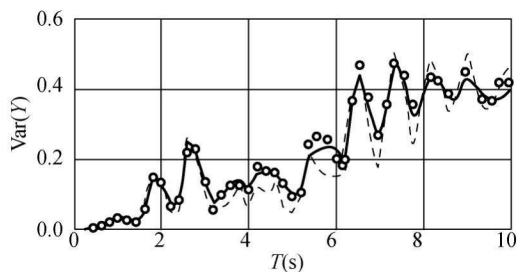


Fig. 4 Variance response Comparison between the expanded order system method and simulation method; Earthquake input. (Circle points for 5 000 simulations, Solid line for 4th orders expansion and dashed line for 1st order expansion)

## 6 Conclusions

An expanded system method has been presented for the dynamic analysis of stochastic structure. The general formulation established in the paper can be used for various MDOF stochastic structural systems which possess random mass, damping or stiffness parameters. On the basis of the discussion in section 2, the proposed method can be used for both continuous systems and discrete systems. For the former case, a two-step method is suggested for converting the random field into a representation in terms of independent random variables. The proposed scheme is believed to afford a new approach to probability design or safety assessment of complex engineering structures.

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# Stochastic Structural System Identification

## Part 1: Mean Parameter Estimation

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**Abstract:** This paper presents a new scheme for the estimation of the mean values of parameters in multi-degree-of-freedom structural systems. It is based on a combination of a differential operator transform of the measured data with the extended Kalman filter method. The proposed method can deal with a wide variety of estimation problems including those which are of the non-linear-in-the parameter type. On combining this method with a technique for estimating the variance of the parameters, discussed detailly in part two of this paper, a complete stochastic structural system identification technique can be formulated. Results from simulation studies indicate that the new method can yield reliable estimates of the system parameters even when the noise level in the measurement records is significant.

## 1 Introduction

Current research on the identification of structural systems aims at developing methods for estimating those parameters which represent the main structural properties, in some sense. Inherent structural uncertainties are usually ignored. However, in the safety assessment of many engineering structures, randomness of various structural properties can be a crucial factor, especially when dynamic responses are of concern. In these circumstances it is necessary to incorporate probabilistic information on interior structural properties into an overall model and to identify the statistics of the parameters, using both the model and available experimental data. The most important statistics are the means and variances of the parameters.

This paper focuses on the estimation of the means of parameters in multi-

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