Larsen & Keller

Control Theory Fundamentals

Camden Howard

Control Theory Fundamentals

Edited by
Camden Howard

Larsen & Keller
www.larsen-keller.com

Control Theory Fundamentals
Edited by Camden Howard
ISBN: 978-1-63549-077-0 (Hardback)

Larsen & Keller
Published by Larsen and Keller Education,
5 Penn Plaza,
19th Floor,
New York, NY 10001, USA

Cataloging-in-Publication Data

Control theory fundamentals / edited by Camden Howard.
p. cm.
Includes bibliographical references and index.
ISBN 978-1-63549-077-0
1. Automatic control. 2. Control theory. 3. Electronic control. 4. Electric controllers. I. Howard, Camden.
TJ213 .C66 2017
629.8--dc23

Printed and bound in China.

For more information regarding Larsen and Keller Education and its products, please visit the publisher's website www.larsen-keller.com

Control Theory Fundamentals

Preface

This book explores all the important aspects of control theory in the present day scenario. It provides thorough insights into this field and explains in detail the uses and methods of the subject. Control theory deals particularly with control systems or any other systems that produce a desired output through the input of references or values. It studies the behavior and inputs of dynamical systems. The book studies, analyses and uphold the pillars of this field and its utmost significance in modern times. It picks up individual branches and explains their need and contribution in the context of the growth of this subject and technology. The topics covered in this extensive book deal with the core subjects of this area. For all those who are interested in control theory, this textbook can prove to be an essential guide.

Given below is the chapter wise description of the book:

Chapter 1- Control theory combines the principles of engineering and mathematics in order to deal with the behavior of changeable systems and how the change is brought about. It is profoundly connected to control systems. This chapter explains to the reader the significance of control theory.

Chapter 2- This chapter gives an overview on control systems. A control system is a device that manages, commands or regulates the behavior of other systems or devices. Control systems can be applied to manual operations or machines that require or can facilitate an operator. The content on control systems offers an insightful focus, keeping in mind the complex subject matter.

Chapter 3- Control systems can best be understood in confluence with the major topics listed in the following chapter. The major categories of control systems are dealt with great detail in the chapter. Industrial control system, PID controller, fly by wire are some of the control systems analyzed in this chapter. The topics discussed are of great importance to broaden the existing knowledge on control systems.

Chapter 4- Tools and methods are an important component of any field of study. The following chapter elucidates the various tools and methods of control systems. Some of the tools, like a transducer, are explained in this chapter. The following text also explains to the reader the relevance of control systems.

Chapter 5- The application of control theory to design systems with desired behavior is certified as control engineering. This chapter is a compilation of the important topics related to control engineering, such as senor and acutuator. It offers an insightful focus, keeping in mind the complex subject matter.

Chapter 6- System architecture deals with the design of systems that store content or data. It may use different computer languages or architecture description languages to access its content. This chapter gives an in-depth understanding of system architecture, and provides the reader with an elucidated knowledge on the subject matter.

Chapter 7- The following chapter elucidates the applications that are related to control system. It discusses the functions of control systems in a critical manner providing key analysis to the subject matter. The applications explained are electrical network, digital signal processor, microcontroller and cruise control.

At the end, I would like to thank all those who dedicated their time and efforts for the successful completion of this book. I also wish to convey my gratitude towards my friends and family who supported me at every step.

Editor

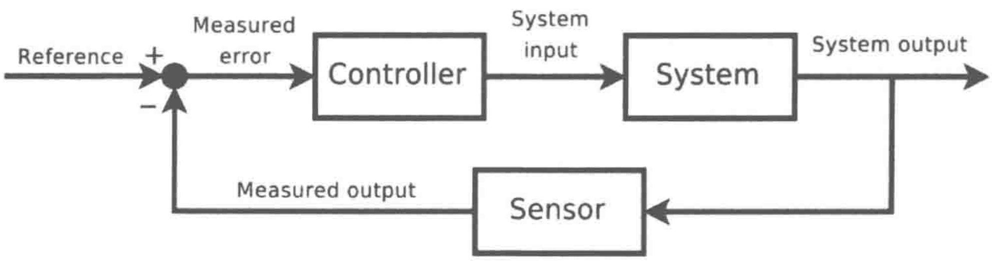
Table of Contents

Introduction to Control Theory

Control theory combines the principles of engineering and mathematics in order to deal with the behavior of changeable systems and how the change is brought about. It is profoundly connected to control systems. This chapter explains to the reader the significance of control theory.

Control Theory

Control theory is an interdisciplinary branch of engineering and mathematics that deals with the behavior of dynamical systems with inputs, and how their behavior is modified by feedback. The usual objective of control theory is to control a system, often called the *plant*, so its output follows a desired control signal, called the *reference*, which may be a fixed or changing value. To do this a *controller* is designed, which monitors the output and compares it with the reference. The difference between actual and desired output, called the *error* signal, is applied as feedback to the input of the system, to bring the actual output closer to the reference. Some topics studied in control theory are stability (whether the output will converge to the reference value or oscillate about it), controllability and observability.

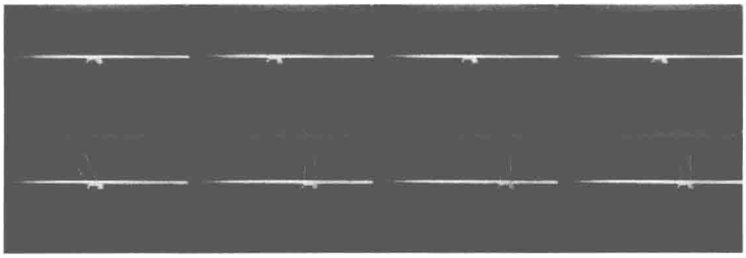


A block diagram of a negative feedback control system. Illustrates the concept of using a feedback loop to control the behavior of a system by comparing its output to a desired value, and applying the difference as an error signal to dynamically change the output so it is closer to the desired output

Extensive use is usually made of a diagrammatic style known as the block diagram. The transfer function, also known as the system function or network function, is a mathematical representation of the relation between the input and output based on the differential equations describing the system.

Although a major application of control theory is in control systems engineering, which deals with the design of process control systems for industry, other applications range far beyond this. As the general theory of feedback systems, control theory is useful wherever feedback occurs. A few examples are in physiology, electronics, climate modeling, machine design, ecosystems, navigation, neural networks, predator–prey interaction, gene expression, and production theory.

Overview



Smooth nonlinear trajectory planning with linear quadratic Gaussian feedback (LQR) control on a dual pendula system.

Control theory is

- a theory that deals with influencing the behavior of dynamical systems
- an interdisciplinary subfield of science, which originated in engineering and mathematics, and evolved into use by the social sciences, such as economics, psychology, sociology, criminology and in the financial system.

Control systems may be thought of as having four functions: measure, compare, compute and correct. These four functions are completed by five elements: detector, transducer, transmitter, controller and final control element. The measuring function is completed by the detector, transducer and transmitter. In practical applications these three elements are typically contained in one unit. A standard example of a measuring unit is a resistance thermometer. The compare and compute functions are completed within the controller, which may be implemented electronically by proportional control, a PI controller, PID controller, bistable, hysteretic control or programmable logic controller. Older controller units have been mechanical, as in a centrifugal governor or a carburetor. The correct function is completed with a final control element. The final control element changes an input or output in the control system that affects the manipulated or controlled variable.

An Example

An example of a control system is a car's cruise control, which is a device designed to maintain vehicle speed at a constant *desired* or *reference* speed provided by the driver. The *controller* is the cruise control, the *plant* is the car, and the *system* is the car and the cruise control. The system output is the car's speed, and the control itself is the engine's throttle position which determines how much power the engine delivers.

A primitive way to implement cruise control is simply to lock the throttle position when the driver engages cruise control. However, if the cruise control is engaged on a stretch of flat road, then the car will travel slower going uphill and faster when going downhill. This type of controller is called an *open-loop controller* because there is no feedback; no measurement of the system output (the car's speed) is used to alter the control (the throttle position.) As a result, the controller cannot compensate for changes acting on the car, like a change in the slope of the road.

In a *closed-loop control system*, data from a sensor monitoring the car's speed (the system output) enters a controller which continuously subtracts the quantity representing the speed from the reference quantity representing the desired speed. The difference, called the error, determines the throttle position (the control). The result is to match the car's speed to the reference speed (maintain the desired system output). Now, when the car goes uphill, the difference between the input (the sensed speed) and the reference continuously determines the throttle position. As the sensed speed drops below the reference, the difference increases, the throttle opens, and engine power increases, speeding up the vehicle. In this way, the controller dynamically counteracts changes to the car's speed. The central idea of these control systems is the *feedback loop*, the controller affects the system output, which in turn is measured and fed back to the controller.

Classification

Linear Versus Nonlinear Control Theory

The field of control theory can be divided into two branches:

- *Linear control theory* – This applies to systems made of devices which obey the superposition principle, which means roughly that the output is proportional to the input. They are governed by linear differential equations. A major subclass is systems which in addition have parameters which do not change with time, called *linear time invariant* (LTI) systems. These systems are amenable to powerful frequency domain mathematical techniques of great generality, such as the Laplace transform, Fourier transform, Z transform, Bode plot, root locus, and Nyquist stability criterion. These lead to a description of the system using terms like bandwidth, frequency response, eigenvalues, gain, resonant frequencies, poles, and zeros, which give solutions for system response and design techniques for most systems of interest.

- *Nonlinear control theory* – This covers a wider class of systems that do not obey the superposition principle, and applies to more real-world systems, because all real control systems are nonlinear. These systems are often governed by nonlinear differential equations. The few mathematical techniques which have been developed to handle them are more difficult and much less general, often applying only to narrow categories of systems. These include limit cycle theory, Poincaré maps, Lyapunov stability theorem, and describing functions. Nonlinear systems are often analyzed using numerical methods on computers, for example by simulating their operation using a simulation language. If only solutions near a stable point are of interest, nonlinear systems can often be linearized by approximating them by a linear system using perturbation theory, and linear techniques can be used.

Frequency Domain Versus Time Domain

Mathematical techniques for analyzing and designing control systems fall into two different categories:

- *Frequency domain* – In this type the values of the state variables, the mathematical variables representing the system's input, output and feedback are represented as functions of frequency. The input signal and the system's transfer function are converted from time functions to functions of frequency by a transform such as the Fourier transform, Laplace transform, or Z transform. The advantage of this technique is that it results in a simplification of the mathematics; the *differential equations* that represent the system are replaced by *algebraic equations* in the frequency domain which are much simpler to solve. However, frequency domain techniques can only be used with linear systems, as mentioned above.

- *Time-domain state space representation* – In this type the values of the state variables are represented as functions of time. With this model the system being analyzed is represented by one or more differential equations. Since frequency domain techniques are limited to linear systems, time domain is widely used to analyze real-world nonlinear systems. Although these are more difficult to solve, modern computer simulation techniques such as simulation languages have made their analysis routine.

Siso Vs Mimo

Control systems can be divided into different categories depending on the number of inputs and outputs.

- Single-input single-output (SISO) – This is the simplest and most common type, in which one output is controlled by one control signal. Examples are the cruise control example above, or an audio system, in which the control input is the input audio signal and the output is the sound waves from the speaker.
- Multiple-input multiple-output (MIMO) – These are found in more complicated systems. For example, modern large telescopes such as the Keck and MMT have mirrors composed of many separate segments each controlled by an actuator. The shape of the entire mirror is constantly adjusted by a MIMO active optics control system using input from multiple sensors at the focal plane, to compensate for changes in the mirror shape due to thermal expansion, contraction, stresses as it is rotated and distortion of the wavefront due to turbulence in the atmosphere. Complicated systems such as nuclear reactors and human cells are simulated by computer as large MIMO control systems.

History

Centrifugal governor in a Boulton & Watt engine of 1788

Although control systems of various types date back to antiquity, a more formal analysis of the field began with a dynamics analysis of the centrifugal governor, conducted by the physicist James Clerk Maxwell in 1868, entitled *On Governors*. This described and analyzed the phenomenon of self-oscillation, in which lags in the system may lead to overcompensation and unstable behavior. This generated a flurry of interest in the topic, during which Maxwell's classmate, Edward John Routh, abstracted Maxwell's results for the general class of linear systems. Independently, Adolf Hurwitz analyzed system stability using differential equations in 1877, resulting in what is now known as the Routh–Hurwitz theorem.

A notable application of dynamic control was in the area of manned flight. The Wright brothers made their first successful test flights on December 17, 1903 and were distinguished by their ability to control their flights for substantial periods (more so than the ability to produce lift from an airfoil, which was known). Continuous, reliable control of the airplane was necessary for flights lasting longer than a few seconds.

By World War II, control theory was an important part of fire-control systems, guidance systems and electronics.

Sometimes, mechanical methods are used to improve the stability of systems. For example, ship stabilizers are fins mounted beneath the waterline and emerging laterally. In contemporary vessels, they may be gyroscopically controlled active fins, which have the capacity to change their angle of attack to counteract roll caused by wind or waves acting on the ship.

The Sidewinder missile uses small control surfaces placed at the rear of the missile with spinning disks on their outer surfaces and these are known as rollerons. Airflow over the disks spins them to a high speed. If the missile starts to roll, the gyroscopic force of the disks drives the control surface into the airflow, cancelling the motion. Thus, the Sidewinder team replaced a potentially complex control system with a simple mechanical solution.

The Space Race also depended on accurate spacecraft control, and control theory has also seen an increasing use in fields such as economics.

People in Systems and Control

Many active and historical figures made significant contribution to control theory including

- Pierre-Simon Laplace (1749–1827) invented the Z-transform in his work on probability theory, now used to solve discrete-time control theory problems. The Z-transform is a discrete-time equivalent of the Laplace transform which is named after him.
- Alexander Lyapunov (1857–1918) in the 1890s marks the beginning of stability theory.
- Harold S. Black (1898–1983), invented the concept of negative feedback amplifiers in 1927. He managed to develop stable negative feedback amplifiers in the 1930s.
- Harry Nyquist (1889–1976) developed the Nyquist stability criterion for feedback systems in the 1930s.
- Richard Bellman (1920–1984) developed dynamic programming since the 1940s.

- Andrey Kolmogorov (1903–1987) co-developed the Wiener–Kolmogorov filter in 1941.
- Norbert Wiener (1894–1964) co-developed the Wiener–Kolmogorov filter and coined the term cybernetics in the 1940s.
- John R. Ragazzini (1912–1988) introduced digital control and the use of Z-transform in control theory (invented by Laplace) in the 1950s.
- Lev Pontryagin (1908–1988) introduced the maximum principle and the bang-bang principle.
- Pierre-Louis Lions (1956) developed viscosity solutions into stochastic control and optimal control methods.

Classical Control Theory

To overcome the limitations of the open-loop controller, control theory introduces feedback. A closed-loop controller uses feedback to control states or outputs of a dynamical system. Its name comes from the information path in the system: process inputs (e.g., voltage applied to an electric motor) have an effect on the process outputs (e.g., speed or torque of the motor), which is measured with sensors and processed by the controller; the result (the control signal) is "fed back" as input to the process, closing the loop.

Closed-loop controllers have the following advantages over open-loop controllers:

- disturbance rejection (such as hills in the cruise control example above)
- guaranteed performance even with model uncertainties, when the model structure does not match perfectly the real process and the model parameters are not exact
- unstable processes can be stabilized
- reduced sensitivity to parameter variations
- improved reference tracking performance

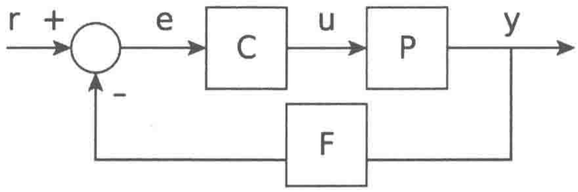
In some systems, closed-loop and open-loop control are used simultaneously. In such systems, the open-loop control is termed feedforward and serves to further improve reference tracking performance.

A common closed-loop controller architecture is the PID controller.

Closed-loop Transfer Function

The output of the system $y(t)$ is fed back through a sensor measurement F to a comparison with the reference value $r(t)$. The controller C then takes the error e (difference) between the reference and the output to change the inputs u to the system under control P. This is shown in the figure. This kind of controller is a closed-loop controller or feedback controller.

This is called a single-input-single-output (*SISO*) control system; *MIMO* (i.e., Multi-Input-Multi-Output) systems, with more than one input/output, are common. In such cases variables are represented through vectors instead of simple scalar values. For some distributed parameter systems the vectors may be infinite-dimensional (typically functions).



If we assume the controller *C*, the plant *P*, and the sensor *F* are linear and time-invariant (i.e., elements of their transfer function *C(s)*, *P(s)*, and *F(s)* do not depend on time), the systems above can be analysed using the Laplace transform on the variables. This gives the following relations:

$$Y(s) = P(s)U(s)$$

$$U(s) = C(s)E(s)$$

$$E(s) = R(s) - F(s)Y(s).$$

Solving for *Y(s)* in terms of *R(s)* gives

$$Y(s) = \left(\frac{P(s)C(s)}{1 + F(s)P(s)C(s)} \right) R(s) = H(s)R(s).$$

The expression $H(s) = \frac{P(s)C(s)}{1 + F(s)P(s)C(s)}$

is referred to as the *closed-loop transfer function* of the system. The numerator is the forward (open-loop) gain from *r* to *y*, and the denominator is one plus the gain in going around the feedback loop, the so-called loop gain. If $| P(s)C(s) | \gg 1$, , i.e., it has a large norm with each value of *s*, and if $| F(s) | \approx 1$, , then *Y(s)* is approximately equal to *R(s)* and the output closely tracks the reference input.

PID Controller

The PID controller is probably the most-used feedback control design. *PID* is an initialism for *Proportional-Integral-Derivative*, referring to the three terms operating on the error signal to produce a control signal. If *u(t)* is the control signal sent to the system, *y(t)* is the measured output and *r(t)* is the desired output, and tracking error $e(t) = r(t) - y(t)$, a PID controller has the general form

$$u(t) = K_P e(t) + K_I \int e(t) \mathrm{d}t + K_D \frac{\mathrm{d}}{\mathrm{d}t} e(t).$$

The desired closed loop dynamics is obtained by adjusting the three parameters K_P , K_I and K_D , often iteratively by "tuning" and without specific knowledge of a plant model. Stability can often be ensured using only the proportional term. The integral term permits the rejection of a step disturbance (often a striking specification in process control). The derivative term is used to provide damping or shaping of the response. PID controllers are the most well established class of control systems: however, they cannot be used in several more complicated cases, especially if MIMO systems are considered.

Applying Laplace transformation results in the transformed PID controller equation

$$u(s) = K_P e(s) + K_I \frac{1}{s} e(s) + K_D s e(s)$$

$$u(s) = \left(K_P + K_I \frac{1}{s} + K_D s \right) e(s)$$

with the PID controller transfer function

$$C(s) = \left(K_P + K_I \frac{1}{s} + K_D s \right).$$

There exists a nice example of the closed-loop system discussed above. If we take

PID controller transfer function in series form

$$C(s) = K \left(1 + \frac{1}{sT_i} \right) (1 + sT_d)$$

1st order filter in feedback loop

$$F(s) = \frac{1}{1 + sT_f}$$

linear actuator with filtered input

$$P(s) = \frac{A}{1 + sT_p}, \text{ A = const}$$

and insert all this into expression for closed-loop transfer function H(s), then tuning is very easy: simply put

$$K = \frac{1}{A}, T_i = T_f, T_d = T_p$$

and get H(s) = 1 identically.

For practical PID controllers, a pure differentiator is neither physically realisable nor desirable due to amplification of noise and resonant modes in the system. Therefore, a phase-lead compensator type approach is used instead, or a differentiator with low-pass roll-off.

Modern Control Theory

In contrast to the frequency domain analysis of the classical control theory, modern control theory utilizes the time-domain state space representation, a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors and the differential and algebraic equations are written in matrix form (the latter only being possible when the dynamical system is linear). The state space representation (also known as the "time-domain approach") provides a convenient and compact way to model and analyze systems with multiple inputs and outputs. With inputs and outputs, we would otherwise have to write down Laplace transforms to encode all the information about a system. Unlike the frequency domain approach, the use of the state-space representation is not limited to systems with linear components and zero initial conditions. "State space" refers to the space whose axes are the state variables. The state of the system can be represented as a point within that space.