Carlo Rainieri · Giovanni Fabbrocino

Operational Modal Analysis of Civil Engineering Structures

An Introduction and Guide for Applications

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Preface

Operational Modal Analysis is the testing procedure yielding experimental estimates of the modal parameters from measurements of the structural response only.

This book reports relevant information and established research results about Operational Modal Analysis in a unified framework. Most of the material in the book is currently disseminated throughout several books and journal papers. An effort has been made to organize this material in a book entirely focused on Operational Modal Analysis. The different aspects of output-only modal testing, from test design to post-processing of results, are analyzed. The book is intended to provide a fundamental theoretical and applicative resource for professional engineers and researchers involved in modal testing of civil structures.

It has been conceived as a guide through the most relevant theoretical and practical concepts in view of the development of a customized system for output-only modal testing based on programmable hardware. The illustrated essential theory provides a general framework to acquire the ability and understanding of the techniques. On the other hand, the large attention devoted to the implementation details provides a valuable stimulus in approaching the study. The applicative perspective makes learning easy and the book suitable for a wide range of readers. In order to simplify the practical implementation of concepts and methods, the use of LabVIEW for software and system development is recommended, because it is characterized by an advantageous learning curve. Moreover, it is very powerful and versatile, making possible the integration of measurements and data processing in a single platform.

Under this premise, it is possible to understand the choice of focusing the attention on implementation details rather than heavy mathematical proofs. The mathematics is kept as simple as each topic allows; most of the equations are functional to the prompt implementation of algorithms and methods by the reader. The basic software accompanying the book is oriented to fit the needs of both the modal analysts on one hand, and undergraduate/graduate students, researchers and developers on the other hand. The latters, in fact, are usually interested in writing their own code for further developments or business opportunities, and the accompanying software serves as a reference. Test engineers, instead, can find here the tools and the fundamental information to promptly start the modal tests and properly interpret the results.

The material is presented at a level suitable for upper-level undergraduate or post-graduate students and professional engineers. In fact, all the material in the book and the organization of the topics are based on the courses given by the authors at undergraduate and graduate students of the University of Molise and the University of Naples Federico II as well as the field experience made in the context of the spin-off company S2X s.r.l. An attempt has been made to produce a self-contained book, with basics of structural dynamics and modal analysis as the only prerequisite to understand most of the presented material. Sufficient details are given in the chapters to cover the necessary multidisciplinary skills that are required to the modal analyst. Several references are also provided at the end of each chapter for the reader who is interested in more details about the various topics.

A number of explanatory applications will help the reader in gaining confidence with the concepts and understanding the potential of output-only modal testing. Most of the analyzed case studies are applications to real structures. This circumstance permits to highlight issues and challenges of output-only modal testing that are often encountered in the practice.

The last part of the book is focused on automated Operational Modal Analysis, providing an outlook on its promising applicative perspectives in the field of vibration-based Structural Health Monitoring. An overview of the latest developments in the field of automated Operational Modal Analysis is presented. It basically represents a particular viewpoint about the matter, since a wide consensus in the definition of the "best methods" for automated output-only modal identification has not been reached, yet. However, the analysis of the main issues related to automation, together with the attention devoted throughout the book to relevant aspects of data acquisition and storage (including storage in MySQL relational databases), aims at linking the material in this book with the wider area of civil Structural Health Monitoring, that is currently a very active research field.

Since this is a new book, instructors, students, and professional engineers are invited to write us (carlo.rainieri@unimol.it, giovanni.fabbrocino@unimol.it) if they have questions, suggestions, or if they identify errors or relevant issues.

We thank you in advance for the time you will spend for this.

Termoli, Italy

Carlo Rainieri
Giovanni Fabbrocino

List of Abbreviations

AC	Alternate current
ADC	Analog digital converter
ADO	ActiveX Data Object
AFDD-T	Automated frequency domain decomposition-tracking
AMUSE	Algorithm for multiple unknown signal extraction
AR	Auto regressive
ARES	Automated modal parameter extraction system
ARMA	Auto-regressive moving average
ARMAV	Auto-regressive moving average vector
BFD	Basic frequency domain
BMID	Blind modal identification
BR	Balanced realization
BSS	Blind source separation
CMRR	Common mode rejection ratio
COMAC	Coordinate modal assurance criterion
Cov-SSI	Covariance-driven stochastic subspace identification
CPU	Central processing unit
CVA	Canonical variate analysis
DC	Direct current
DD-SSI	Data-driven stochastic subspace identification
DDT	Dynamic data type
DFT	Discrete Fourier transform
DOF	Degree of freedom
DR	Dynamic range
DSN	Data source name
ECOMAC	Enhanced coordinate modal assurance criterion
EFDD	Enhanced frequency domain decomposition
EMA	Experimental modal analysis
ERA	Eigensystem realization algorithm
EVD	Eigenvalue decomposition
FDD	Frequency domain decomposition
FE	Finite element
FEM	Finite element model
FFT	Fast Fourier transform

FIR	Finite impulse response (filter)
FOH	First-order hold
FRF	Frequency response function
FSDD	Frequency spatial domain decomposition
ICA	Independent component analysis
IEPE	Integrated electronics piezoelectric
IIR	Infinite impulse response (filter)
IMAC	International Modal Analysis Conference
IOMAC	International Operational Modal Analysis Conference
IRF	Impulse response function
ISMA	International Conference on Noise and Vibration Engineering
ITD	Ibrahim time domain
IV	Instrumental variable
JAD	Joint approximate diagonalization
LMFD	Left matrix fraction description
LR	Lower residual (term)
LSB	Least significant bit
LSCE	Least squares complex exponential
LSCF	Least squares complex frequency
LSFD	Least squares frequency domain
LTI	Linear time invariant
MA	Moving average
MAC	Modal assurance criterion
MAX	Measurement and automation explorer
MDOF	Multi-degree of freedom
MFD	Matrix fraction description
MIMO	Multiple input multiple output
MISO	Multiple input single output
MLE	Maximum likelihood estimator
MOF	Modal overlap factor
MPC	Modal phase collinearity
MPD	Mean phase deviation
MSF	Modal scale factor
NExT	Natural excitation techniques
NMD	Normalized modal difference
ODBC	Open database connectivity
ODS	Operational deflection shape
OMA	Operational modal analysis
p-LSCF	Poly-reference least squares complex frequency
PC	Principal component
PCA	Principal component analysis
PEM	Prediction error method
PMR	Participating mass ratio
PSD	Power spectral density
r.c.	Reinforced concrete

RD	Random decrement
RMFD	Right matrix fraction description
rms	Root mean square
SDOF	Single degree of freedom
SERP	Stationary and ergodic random process
SHM	Structural health monitoring
SIMO	Single input multiple output
SISO	Single input single output
SNR	Signal-to-noise ratio
SOBI	Second-order blind identification
SRP	Stationary random process
SSI	Stochastic subspace identification
SVD	Singular value decomposition
UMPA	Unified matrix polynomial approach
UPC	Unweighted principal component
UR	Upper residual (term)
VI	Virtual instrument
ZOH	Zero order hold

List of Symbols

dB	Decibel
$h(\tau)$	IRF
$H(\omega)$	FRF
$[\cdot]$	Matrix
$\langle \cdot \rangle$	Row vector
$\{\cdot\}$	Column vector
i	Imaginary unit/number of block rows
f_s	Sampling frequency
T	Period/duration
N	Number of samples
*	Complex conjugate
^{H}	Hermitian adjoint
^{T}	Transpose
$^+$	Pseudoinverse
N_{DOF}	Number of DOFs
N_m	Number of modes
λ	Continuous-time pole/eigenvalue
μ	Discrete-time pole/mean
ε	Error
n	Order of state space model or polynomial model
p	Order of AR/ARMA model/number of time lags in SOBI
l	Number of output time series
$\hat{}$	Estimated quantity
$\gamma_{xy}^2(\omega)$	Coherence function
Δf	Frequency resolution
Δf_n	Relative scatter between natural frequency estimates
Δt	Sampling interval
t_k	Discrete time instant
$Re(\cdot)$	Real part of a complex number
$Im(\cdot)$	Imaginary part of a complex number
$\|\cdot\|$	L2-norm
$\|[\cdot]\|$	Determinant
$adj([\cdot])$	Adjoint matrix
$f_{d,r}$	Damped frequency

$\omega_{d,r}$	Damped circular frequency
f_r	Natural frequency
ω_r	Natural circular frequency
ξ_r	Damping ratio
σ_r	Real part of the pole associated to the r-th mode
$sgn(\cdot)$	Signum function
n_b	Number of data segments
$S_{YY}(\omega)$	Two-sided output power spectrum
$S_{YY}^{+}(\omega)$	Positive power spectrum
$G_{YY}(\omega)$	One-sided output power spectrum
$R_{yy}(\tau)$	Output correlation function
$C_{yy}(\tau)$	Output covariance function
$T_{ij}^{a}(\omega)$	Transmissibility function
$D_{yy}(\tau)$	RD signature
$p_r(t)$	Modal coordinate
α_j	Mode shape scaling factor
N_f	Number of frequency lines
Ω_f	Generalized transform variable at discrete frequency ω_f
z_f	z-domain polynomial basis function at discrete frequency ω_f
$tr(\cdot)$	Trace
$\otimes$	Kronecker product
$\ell(\cdot)$	Cost function
n_α	AR order of ARMA model
n_γ	MA order of ARMA model
N_c	Dimension of matrix coefficient
σ^2	Variance
t	Threshold to stop JAD
$\{\psi\}$	Mass normalized mode shape/eigenvector
$\{\phi\}$	Unscaled mode shape
$\{y(t)\}$	Displacement response in time domain/measured output
$\{f(t)\}$	Forcing vector
$\{\gamma_r\}$	Operational reference vector
$\{s(t)\}$	State vector/source
$\{z(t)\}$	Whitened data
$\{s_k\}$	Discrete-time state vector
$\{u_k\}$	Sampled input
$\{y_k\}$	Sampled output
$\{w_k\}$	Vector of process noise in state-space models
$\{v_k\}$	Vector of measurement noise in state-space models
$\{e_k\}$	Innovation
$\{u_1\}$	First singular vector
$\{\theta\}$	Unknown polynomial coefficients in LSCF
$\{\theta_{N_k}\}$	k-th vector of the numerator coefficients in LSCF
$\{\theta_d\}$	Vector of the denominator coefficients in LSCF

$\{\hat{s}_k\}$	Kalman state estimate at time t_k
$\{n(t)\}$	Additive noise
$\{x(t)\}$	Signal part of the observed data in SOBI
$[M]$	Mass matrix
$[C]$	Damping matrix
$[K]$	Stiffness matrix
$[Z(\omega)]$	Dynamic stiffness matrix
$[R_r]$	Residue matrix
$[A_c]$	State matrix (continuous time)
$[B_c]$	Input influence matrix (continuous time)
$[C_a]$	Output location matrix (acceleration)
$[C_v]$	Output location matrix (velocity)
$[C_d]$	Output location matrix (displacement)
$[C_c]$	Output influence matrix (continuous time)
$[D_c]$	Direct transmission matrix (continuous time)
$[A]$	Discrete-time state matrix/mixing matrix
$[B]$	Discrete-time input matrix
$[C]$	Discrete-time output matrix
$[D]$	Discrete-time direct transmission matrix
$\begin{bmatrix} [Q^{ww}] & [S^{wv}] \\ [S^{wv}]^T & [R^{vv}] \end{bmatrix}$	Covariance matrices of process noise and measurement noise
$[R_i]$	Output covariance matrix at time lag i
$[\Sigma]$	State covariance matrix/diagonal matrix of singular values
$[G]$	Next state-output covariance matrix
$[K_k]$	Non-steady state Kalman gain
$[P_k]$	Covariance of the Kalman state estimates in non-steady state
$[J]$	Jacobian matrix
$[\theta]$	Matrix of the unknown parameters in p-LSCF
$[\beta_o]$	Matrix of the unknown numerator coefficients for output o in p-LSCF
$[\alpha]$	Matrix of the unknown denominator coefficients in p-LSCF
$[H_{i\|j}]$	Hankel matrix (subfixes i and j denote the time instants of the first and the last entry in the first column of the matrix)
$[P]$	Kalman state covariance matrix in steady-state
$[\alpha_i]$	Matrix of the AR coefficients
$[\gamma_i]$	Matrix of the MA coefficients
$[O_i]$	Observability matrix
$[\Phi]$	Modal matrix
$[U]$	Matrix of the left singular vectors/demixing matrix in SOBI
$[V]$	Matrix of the right singular vectors
$[T_{i\|j}]$	Block Toeplitz matrix whose entries on the first row are correlations evaluated at time lags from i to j
$[\Gamma_i]$	Reversed controllability matrix
$[U_1]$	Matrix of the left singular vectors corresponding to nonzero singular values

$[V_1]$	Matrix of the right singular vectors corresponding to nonzero singular values
$[\Sigma_1]$	Submatrix of $[\Sigma]$ holding the nonzero singular values
$[\hat{S}_i]$	Kalman filter state sequence
$[Y_p]$	Matrix of the past outputs
$[Y_f]$	Matrix of the future outputs
$[Y_p^+]$	Matrix of the past outputs with one block row added
$[Y_f^-]$	Matrix of the future outputs with one block row removed
$[\Pi_E]$	Orthogonal projection on the row space of [E]
$[P_i]$	Projection matrix
$[O_i^{\uparrow}]$	Observability matrix with the last l rows deleted
$[O_i^{\downarrow}]$	Observability matrix with the first l rows deleted
$[\rho_w]$, $[\rho_v]$	Kalman filter residuals
$[W]$	Whitening matrix in SOBI

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Introduction 1

1.1 Operational Modal Analysis: A New Discipline?

The use of experimental tests to gain knowledge about the dynamic response of civil structures is a well-established practice. In particular, the experimental identification of the modal parameters can be dated back to the middle of the Twentieth Century (Ewins 2000). Assuming that the dynamic behavior of the structure can be expressed as a combination of modes, each one characterized by a set of parameters (natural frequency, damping ratio, mode shape) whose values depend on geometry, material properties, and boundary conditions, Experimental Modal Analysis (EMA) identifies those parameters from measurements of the applied force and the vibration response.

In the last decades the principles of system identification and the experimental estimation of the modal parameters have provided innovative tools for the understanding and control of vibrations, the optimization of design, and the assessment of performance and health state of structures. In fact, even if the Finite Element (FE) method and the fast progress in computing technologies have made excellent analysis tools available to the technical community, the development of new high-performance materials and the increasing complexity of structures have required powerful tools to support and validate the numerical analyses. In this context the experimental identification of the modal properties definitely supports the engineers to get more physical insight about the dynamic behavior of the structure and to discriminate between the errors due to discretization and those due to simplified or even wrong modeling assumptions. Moreover, since the vibration response originates from the modes, which are inherent properties of the structure, forces exciting the structure at resonant frequencies yield large vibration responses that can result in discomfort or even damage. Regular identification of modal parameters and analysis of their variation can support the assessment of structural performance and integrity.

Since the origin of EMA, testing equipment and data processing algorithms have significantly evolved. EMA is currently a well-established field, based on a sound

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theoretical background. An extensive illustration of EMA techniques can be found in a number of books (Ewins 2000, Heylen et al. 1998, Maia et al. 1997) widely adopted as references by the scientific and technical community.

EMA has been applied in different fields, such as automotive engineering, aerospace engineering, industrial machinery, and civil engineering. The identification of the modal parameters by EMA techniques becomes more challenging in the case of civil engineering structures because of their large size and low frequency range. The application of controlled and measurable excitation is often a complex task that requires expensive and heavy devices. For this reason the community of civil engineers has more recently focused the attention on the opportunities provided by *Operational Modal Analysis* (OMA). OMA can be defined as the modal testing procedure that allows the experimental estimation of the modal parameters of the structure from measurements of the vibration response only. The idea behind OMA is to take advantage of the natural and freely available excitation due to ambient forces and operational loads (wind, traffic, micro-tremors, etc.) to replace the artificial excitation. So, they are no more considered as disturbance but, on the contrary, they make possible the dynamic identification of large civil structures. Since OMA requires only measurements of the dynamic response of the structure in operational conditions, when it is subjected to the ambient excitation, it is also known under different names, such as ambient vibration modal identification or output-only modal analysis.

Over the years, OMA has evolved as an autonomous discipline. However, most of the OMA methods have been derived from EMA procedures, so they share a common theoretical background with input–output procedures. The main difference is in the formulation of the input, which is known in EMA while it is random and not measured in OMA. Thus, while EMA procedures are developed in a deterministic framework, OMA methods can be seen as their stochastic counterpart.

In the civil engineering field, OMA is very attractive because tests are cheap and fast, and they do not interfere with the normal use of the structure. Moreover, the identified modal parameters are representative of the actual behavior of the structure in its operational conditions, since they refer to levels of vibration actually present in the structure and not to artificially generated vibrations. On the other hand, the low amplitude of vibrations in operational conditions requires very sensitive, low-noise sensors and a high performance measurement chain. Additional limitations come from the assumption about the input, as mentioned, for instance, in Sect. 1.3. Nevertheless, it represents an attractive alternative to input–output modal analysis and it shares with EMA most of the fields of application of modal identification results. In some cases, such as testing of historical structures (where it reduces the invasiveness of tests and the risk of damage) or vibration-based health assessment and monitoring (where the replacement of the artificial excitation with ambient vibrations makes it especially suitable for automation), OMA outperforms EMA, and this justifies its increasing popularity in the civil engineering community.

Research findings and several successful applications of OMA in different fields are documented in a number of Journals and proceedings of international conferences such as the annual IMAC conference (http://www.sem.org/CONF-IMAC-TOP.asp) organized by the Society of Experimental Mechanics, or the