

# GPS SATELLITE SURVEYING

FOURTH EDITION

ALFRED LEICK  
LEV RAPOPORT  
DMITRY TATARNIKOV



WILEY

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# GPS SATELLITE SURVEYING Fourth Edition

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# PREFACE

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*GPS Satellite Surveying* has undergone a major revision in order to keep abreast with new developments in GNSS and yet maintain its focus on geodesy and surveying. All chapters have been reorganized in a more logical fashion. Because the GNSS systems have developed significantly since the last edition of the book, we have added new material on the GLONASS, Beidou, and Galileo systems, as well as on the ongoing modernization of GPS. A separate chapter was included on recursive least squares. Another chapter on RTK implementation was added that uses these recursive least-squares algorithms to process across-receiver observation differences and is capable of accepting observations from all GNSS systems. Examples are supported by real data processing. A third new chapter was added on GNSS user antennas. This chapter was prepared by an antenna expert to provide the necessary background information and details to allow practicing engineers to select the right antenna for a project. As to GNSS processing approaches, major new sections were added on PPP-RTK and TCAR. Six new additional appendices were added containing in-depth mathematical supplements for those readers who enjoy the mathematical rigor.

The original author of *GPS Satellite Surveying*, Alfred Leick, appreciates the contributions of Lev Rapoport and Dmitry Tatarnikov and most cordially welcomes these very qualified individuals as co-authors. All three of us wish to thank our families for their outstanding support throughout our professional careers. Lev Rapoport wishes to thank Javad GNSS for permission to use their receivers Triumph-1, Delta TRE-G3T, and Delta Duo-G2D for data collection, and Dr. Javad Ashjaee for the opportunity to get acquainted with GNSS technologies and observe its history through the eyes of a company employee. Dmitry Tatarnikov wishes to thank his colleagues at the Moscow Technology Center of Topcon for their contributions to the research,

development, and production of antennas, and the management of Topcon Corporation for support of this work. Alfred Leick expresses his sincere appreciation to anybody contributing to this and any of the previous revisions of *GPS Satellite Surveying*. We appreciate Tamrah Brown's assistance in editing the draft in such a short period of time.

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Alfred Leick received a Ph.D. from Ohio State University, Department of Geodetic Science, in 1977. He is the Editor-in-Chief of the peer-reviewed journal *GPS Solutions* (Springer), and author of numerous technical publications. His teaching career at the University of Maine in the area of GPS (Global Positioning System), geodesy, and estimation spans 34 years. Other teaching assignments included photogrammetry and remote sensing, digital image processing, linear algebra, and differential equations. He was the creator of the online GPS-GAP (GPS, Geodesy and Application Program) program at the University of Maine, which now continues to be available via Michigan Technological University ([www.onlineGNSS.edu](http://www.onlineGNSS.edu)) in modified form. Dr. Leick launched his GPS research in 1982 when testing the Macrometer satellite receiver prototype at M.I.T. He continued GPS research throughout the years, including while on sabbatical leave at the Air Force Geophysics Laboratory (Cambridge, MA) in 1984, 3S-Navigation (Irvine, CA) in 1996, Jet Propulsion Laboratory (Pasadena, CA) in 2002, as an Alexander von Humboldt Research Associate at the University of Stuttgart in 1985, a Fulbright Scholar at the University of Sao Paulo during the summers of 1991 and 1992, and a GPS Project Specialist on behalf of World Bank and NRC (National Research Council) at Wuhan Technical University of Surveying and Mapping (P.R. China) in the Spring of 1990. He is a Fellow of ACSM (American Congress on Surveying and Mapping).

Dmitry V. Tatarnikov holds a Masters in EE (1983), Ph.D. (1990), and Doctor of Science (the highest scientific degree in Russia, 2009) degrees, all in applied electromagnetics and antenna theory and technique from the Moscow Aviation Institute—Technical University (MAI), Moscow, Russia. He joined the Antenna and Microwave Research Dept. of MAI in 1979, and is currently a professor of Radiophysics, Antennas and Microwave Dept. at MAI. In 1979–1994, he was involved in microstrip-phased array antenna research and development. In 1994, he joined Ashtech R&D Center in Moscow as an Antenna Research Fellow in the high-precision GNSS area. In 1997–2001, he was with Javad Positioning Systems as a senior scientist in the antenna area, and since 2001 he has been leading the Antenna Design with Topcon Technology Center, Moscow, Russia. Prof. Tatarnikov has authored and co-authored more than 70 publications in this area, including a book, research papers, conference presentations, and 12 patents. He has developed student courses in applied electromagnetics, numerical electromagnetics, and receiver GNSS antennas. He is a member of IEEE and the Institute of Navigation (ION), USA.

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# ABBREVIATIONS

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## COMMONLY USED GNSS ABBREVIATIONS

ARNS	Aeronautical Radio Navigation Service
ARP	Antenna reference point
AS	Antispoofing
ASK	Amplitude shift keying
B1	B1 Beidou carrier (1561.098 MHz)
B2	B2 Beidou carrier (1207.14 MHz)
B3	B3 Beidou carrier (1268.52MHz)
BOC	Binary offset carrier
BPSK	Binary phase shift keying
C/A-code	Coarse/acquisition code (1.023 MHz)
CDMA	Code division multiple access
CIO	Celestial intermediary origin
CCRF	Conventional celestial reference frame
CEP	Celestial ephemeris pole
CORS	Continuously operating reference stations
CTP	Conventional terrestrial pole
CTRS	Conventional terrestrial reference system
DGPS	Differential GPS
DOD	Department of Defense
DOP	Dilution of precision
DOY	Day of year
E6	E6 Galileo carrier (1278.75 MHz)
ECEF	Earth-centered earth-fixed coordinate system



EOP	Earth orientation parameter
FAA	Federal Aviation Administration
FBSR	Feedback shift register
FDMA	Frequency division multiple access
FSK	Frequency shift keying
FOC	Full operational capability
GAST	Greenwich apparent sidereal time
GDOP	Geometric dilution of precision
GEO	Geostationary earth orbit
GIF	Geometry-free and ionospheric-free solution
GIM	Global ionospheric model
GLONASS	Global'naya Navigatsionnaya Sputnikovaya Sistema
GNSS	Global navigation satellite system
GML	Gauss midlatitude functions
GMST	Greenwich mean sidereal time
GPS	Global positioning system
GPSIC	GPS Information Center
GPST	GPS time
GRS80	Geodetic reference system of 1980
HDOP	Horizontal dilution of precision
HMW	Hatch/Melbourne/Wübbena function
HOW	Handover word
IAG	International Association of Geodesy
IAU	International Astronomical Union
ICRF	International celestial reference frame
IERS	International Earth Rotation Service
IGDG	Internet-based dual-frequency global differential GPS
IGS	International GNSS Service
IGSO	Inclined geosynchronous satellite orbit
ISC	Intersignal Correction
ITRF	International terrestrial reference frame
IOC	Initial operational capability
ION	Institute of Navigation
IWV	Integrated water vapor
JD	Julian date
JPL	Jet Propulsion Laboratory
L1	L1 carrier (1575.42 MHz)
L2	L2 carrier (1227.6 MHz)
L5	L5 carrier (1176.45 MHz)
LAMBDA	Least-squares ambiguity decorrelation adjustment
LC	Lambert conformal mapping
LEO	Low-earth orbiting satellite
LHCP	Left-hand circular polarization
MEO	Medium earth orbit
NAD83	North American datum of 1983

NAVSTAR	Navigation Satellite Timing and Ranging
NEP	North ecliptic pole
NGS	National Geodetic Survey
NIST	National Institute of Standards and Technology
NOAA	National Oceanic and Atmospheric Administration
PCO	Phase center offset
OPUS	Online processing user service
OTF	On-the-fly ambiguity resolution
P-code	Precision code (10.23 MHz)
PCV	Phase center variation
PDOP	Positional dilution of precision
ppb	parts per billion
ppm	parts per million
PPP	Precise point positioning
PPS	Precise positioning service
PRN	Pseudorandom noise
PSK	Phase shift keying
PWV	Precipitable water vapor
QPSK	Quadrature phase shift keying
RHCP	Right-hand circular polarization
RINEX	Receiver independent exchange format
RNSS	Radio navigation satellite services
RTCM	Radio Technical Commission for Maritime Services
RTK	Real-time kinematic positioning
SA	Selective availability
SBAS	Satellite-based augmentation system
SINEX	Solution independent exchange format
SLR	Satellite laser ranging
SNR	Signal-to-noise ratio
SP3	Standard product #3 for ECEF orbital files
SPC	State plane coordinate system
SPS	Standard positioning service
SRP	Solar radiation pressure
SVN	Space vehicle launch number
SWD	Slant wet delay
TAI	International atomic time
TDOP	Time dilution of precision
TEC	Total electron content
TECU	TEC unit
TIO	Terrestrial intermediary origin
TLM	Telemetry word
TM	Transverse Mercator mapping
TOW	Time of week
TRANSIT	Navy navigation satellite system
URE	User range error

USNO	U.S. Naval Observatory
UT1	Universal time corrected for polar motion
UTC	Coordinate universal time
VDOP	Vertical dilution of precision
VLBI	Very long baseline interferometry
VRS	Virtual reference station
VSWR	Voltage standing wave ratio
WAAS	Wide area augmentation service
WADGPS	Wide area differential GPS
WGS84	World Geodetic System of 1984
WVR	Water vapor radiometer
Y-code	Encrypted P-code
ZHD	Zenith hydrostatic delay
ZWD	Zenith wet delay

# GPS SATELLITE SURVEYING

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