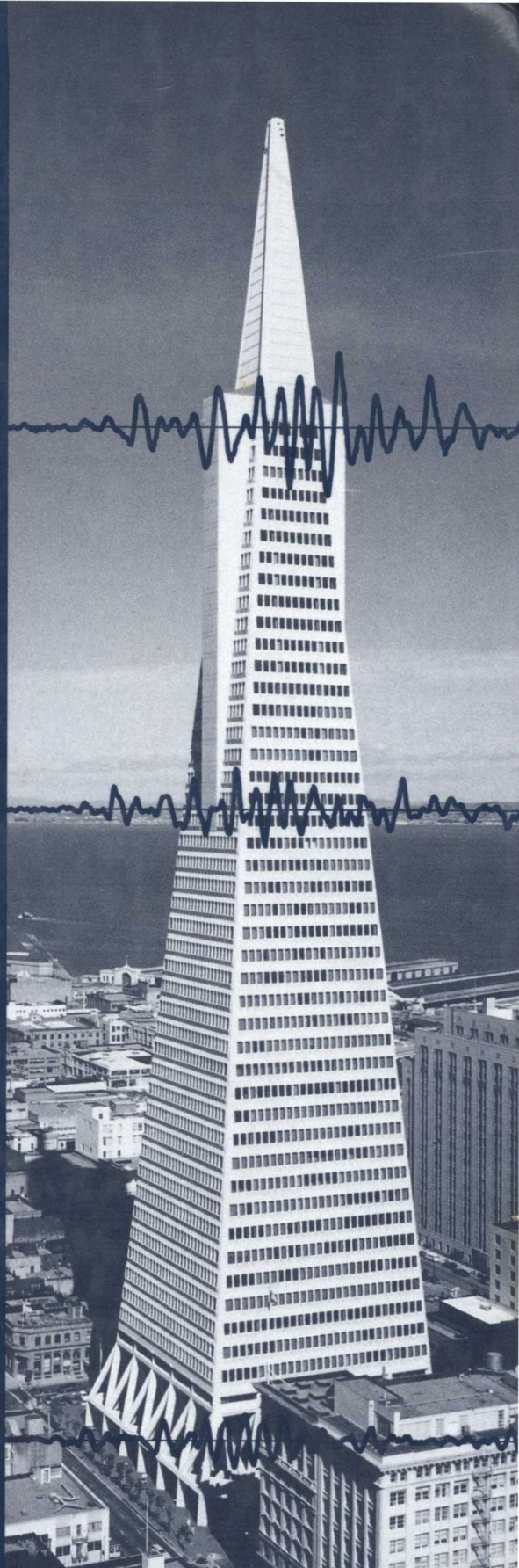


DYNAMICS OF STRUCTURES

THEORY AND APPLICATIONS
TO EARTHQUAKE ENGINEERING

ANIL K. CHOPRA



DYNAMICS OF STRUCTURES

Theory and Applications to Earthquake Engineering

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