TEXTBOOKS IN MATHEMATICS

ESSENTIALS OF MATHEMATICAL THINKING

Steven G. Krantz

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

TEXTBOOKS IN MATHEMATICS

ESSENTIALS OF MATHEMATICAL THINKING

Essentials of Mathematical Thinking addresses the growing need to better comprehend mathematics today. Increasingly, our world is driven by mathematics in all aspects of life. The book is an excellent introduction to the world of mathematics for students not majoring in mathematical studies.

The author has written this book in an enticing, rich manner with helpful graphics that will engage students and introduce new paradigms of thought. Careful readers will develop critical thinking skills which will help them compete in today's world.

The book explains:

- What goes behind a Google search algorithm
- How to calculate the odds in a lottery
- The value of Big Data
- How the nefarious Ponzi scheme operates

Instructors will treasure the book for its ability to make the field of mathematics more accessible and alluring with relevant topics and helpful graphics. The author encourages readers to see the beauty of mathematics and how it relates to their lives in meaningful ways.

CRC Press
Taylor & Francis Group
an informa business
www.crcpress.com

6000 Broken Sound Parkway, NW
Suite 300, Boca Raton, FL 33487
711 Third Avenue
New York, NY 10017
2 Park Square, Milton Park
Abingdon, Oxon OX14 4RN, UK

K31302
ISBN: 978-1-138-19770-1
90000
9 781138 197701
www.crcpress.com

ESSENTIALS OF MATHEMATICAL THINKING

Krantz

CRC

TEXTBOOKS in MATHEMATICS

ESSENTIALS OF MATHEMATICAL THINKING

Steven G. Krantz


CRC Press
Taylor & Francis Group
Boca Raton London New York

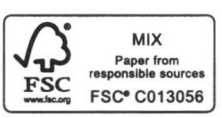
CRC Press is an imprint of the
Taylor & Francis Group an **informa** business
A CHAPMAN & HALL BOOK

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

International Standard Book Number-13: 978-1-1380-4257-5 (Hardback)
International Standard Book Number-13: 978-1-1381-9770-1 (Paperback)

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com



Printed and bound in Great Britain by
TJ International Ltd, Padstow, Cornwall

ESSENTIALS OF MATHEMATICAL THINKING

TEXTBOOKS in MATHEMATICS

Series Editors: Al Boggess and Ken Rosen

PUBLISHED TITLES

ABSTRACT ALGEBRA: A GENTLE INTRODUCTION
Gary L. Mullen and James A. Sellers

ABSTRACT ALGEBRA: AN INTERACTIVE APPROACH, SECOND EDITION
William Paulsen

ABSTRACT ALGEBRA: AN INQUIRY-BASED APPROACH
Jonathan K. Hodge, Steven Schlicker, and Ted Sundstrom

ADVANCED LINEAR ALGEBRA
Hugo Woerdeman

ADVANCED LINEAR ALGEBRA
Nicholas Loehr

ADVANCED LINEAR ALGEBRA, SECOND EDITION
Bruce Cooperstein

APPLIED ABSTRACT ALGEBRA WITH MAPLE™ AND MATLAB®, THIRD EDITION
Richard Klima, Neil Sigmon, and Ernest Stitzinger

APPLIED DIFFERENTIAL EQUATIONS: THE PRIMARY COURSE
Vladimir Dobrushkin

APPLIED DIFFERENTIAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS
Vladimir Dobrushkin

APPLIED FUNCTIONAL ANALYSIS, THIRD EDITION
J. Tinsley Oden and Leszek Demkowicz

A BRIDGE TO HIGHER MATHEMATICS
Valentin Deaconu and Donald C. Pfaff

COMPUTATIONAL MATHEMATICS: MODELS, METHODS, AND ANALYSIS WITH MATLAB® AND MPI, SECOND EDITION
Robert E. White

A COURSE IN DIFFERENTIAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS, SECOND EDITION
Stephen A. Wirkus, Randall J. Swift, and Ryan Szypowski

A COURSE IN ORDINARY DIFFERENTIAL EQUATIONS, SECOND EDITION
Stephen A. Wirkus and Randall J. Swift

PUBLISHED TITLES CONTINUED

DIFFERENTIAL EQUATIONS: THEORY, TECHNIQUE, AND PRACTICE, SECOND EDITION
Steven G. Krantz

DIFFERENTIAL EQUATIONS: THEORY, TECHNIQUE, AND PRACTICE WITH BOUNDARY VALUE PROBLEMS
Steven G. Krantz

DIFFERENTIAL EQUATIONS WITH APPLICATIONS AND HISTORICAL NOTES, THIRD EDITION
George F. Simmons

DIFFERENTIAL EQUATIONS WITH MATLAB®: EXPLORATION, APPLICATIONS, AND THEORY
Mark A. McKibben and Micah D. Webster

DISCOVERING GROUP THEORY: A TRANSITION TO ADVANCED MATHEMATICS
Tony Barnard and Hugh Neill

DISCRETE MATHEMATICS, SECOND EDITION
Kevin Ferland

ELEMENTARY NUMBER THEORY
James S. Kraft and Lawrence C. Washington

EXPLORING CALCULUS: LABS AND PROJECTS WITH MATHEMATICA®
Crista Arangala and Karen A. Yokley

EXPLORING GEOMETRY, SECOND EDITION
Michael Hvidsten

EXPLORING LINEAR ALGEBRA: LABS AND PROJECTS WITH MATHEMATICA®
Crista Arangala

EXPLORING THE INFINITE: AN INTRODUCTION TO PROOF AND ANALYSIS
Jennifer Brooks

GRAPHS & DIGRAPHS, SIXTH EDITION
Gary Chartrand, Linda Lesniak, and Ping Zhang

INTRODUCTION TO ABSTRACT ALGEBRA, SECOND EDITION
Jonathan D. H. Smith

INTRODUCTION TO ANALYSIS
Corey M. Dunn

INTRODUCTION TO MATHEMATICAL PROOFS: A TRANSITION TO ADVANCED MATHEMATICS, SECOND EDITION
Charles E. Roberts, Jr.

INTRODUCTION TO NUMBER THEORY, SECOND EDITION
Marty Erickson, Anthony Vazzana, and David Garth

PUBLISHED TITLES CONTINUED

INVITATION TO LINEAR ALGEBRA
David C. Mello

LINEAR ALGEBRA, GEOMETRY AND TRANSFORMATION
Bruce Solomon

MATHEMATICAL MODELLING WITH CASE STUDIES: USING MAPLE™ AND MATLAB®, THIRD EDITION
B. Barnes and G. R. Fulford

MATHEMATICS IN GAMES, SPORTS, AND GAMBLING–THE GAMES PEOPLE PLAY, SECOND EDITION
Ronald J. Gould

THE MATHEMATICS OF GAMES: AN INTRODUCTION TO PROBABILITY
David G. Taylor

A MATLAB® COMPANION TO COMPLEX VARIABLES
A. David Wunsch

MEASURE AND INTEGRAL: AN INTRODUCTION TO REAL ANALYSIS, SECOND EDITION
Richard L. Wheeden

MEASURE THEORY AND FINE PROPERTIES OF FUNCTIONS, REVISED EDITION
Lawrence C. Evans and Ronald F. Gariepy

NUMERICAL ANALYSIS FOR ENGINEERS: METHODS AND APPLICATIONS, SECOND EDITION
Bilal Ayyub and Richard H. McCuen

ORDINARY DIFFERENTIAL EQUATIONS: AN INTRODUCTION TO THE FUNDAMENTALS
Kenneth B. Howell

PRINCIPLES OF FOURIER ANALYSIS, SECOND EDITION
Kenneth B. Howell

REAL ANALYSIS AND FOUNDATIONS, FOURTH EDITION
Steven G. Krantz

RISK ANALYSIS IN ENGINEERING AND ECONOMICS, SECOND EDITION
Bilal M. Ayyub

SPORTS MATH: AN INTRODUCTORY COURSE IN THE MATHEMATICS OF SPORTS SCIENCE AND SPORTS ANALYTICS
Roland B. Minton

TRANSFORMATIONAL PLANE GEOMETRY
Ronald N. Umble and Zhigang Han

To the memory of Ed Nelson.

Preface

It is becoming increasingly clear that mathematical thinking is essential to understanding the world around us. Police tell us that most crimes these days are committed with electronic devices (usually computers). Many if not most medical procedures are now robotically assisted. The carburation system of a modern automobile is all electronic. Geometric visualization is one of the most important aspects of modern technology.

And it is plain that the thinking behind all of the technology described in the last paragraph is mathematical. The `Google` search engine is based on a mathematical algorithm. Not just logic, but analysis and combinatorics and geometry and many other ideas from the mathematical sciences are critical parts of our world.

A college education consists of learning different modes of discourse. Every modern student should thus have some exposure to mathematical discourse. Whether or not the student has any intention of working in the mathematical sciences, he/she will need these modes of thought to survive in the modern marketplace.

And learning mathematical thought does not have be dry, desultory, and boring. It can be fun and exciting and challenging (in a productive fashion). It can relate to things that are meaningful to students—such as the paradigm behind a Google search, or the means by which a music CD is encoded, or the RSA encryption algorithm. In a course on mathematical thinking the student will obviously learn some mathematical skills, but he/she will also gain *critical thinking skills*. These are valuable life tools.

This book may be thought of as a book for nonmathematicians taking

a mathematics course that is a breadth requirement for an undergraduate degree. Thus it has minimal mathematical prerequisites. It has very few equations. It has lots of pictures and lots of explanation. It is driven by examples. But those examples will lead students down new paths, and acquaint them with new paradigms of thought. They will stimulate and provoke and encourage. At the end of this journey, the student should feel that he/she has a new set of tools for attacking a variety of problems and situations. There should be a profound feeling of satisfaction.

A very important part of learning mathematical thinking is learning to write mathematics. The process of internalizing an idea and the process of figuring out how to write it down are closely linked. We exploit this observation by supplying on-the-fly exercises that force the student to *write out* an answer. Sometimes, when it is appropriate and not burdensome, we ask the student to provide a demonstration or a reason for a mathematical idea. We also ask the student to draw pictures.

The exercises in this book are of several types: **(a)** there are drill exercises, just as a means of getting students started in a topic, **(b)** there are more challenging problems that will require the student to really digest the key ideas, and **(c)** each section ends with an open-ended problem that will encourage students to talk to each other and collaborate.

The material in the first nine chapters of the book should be accessible to a broad audience with minimal background in mathematics. The last three chapters are more sophisticated. All readers will want to dip into these chapters, but only readers with some mathematical background will be able to work through the examples in any detail. In this way, we have been able to make the book more open-ended and appealing to a diverse group of readers with many needs and interests.

It is a pleasure to thank Lynn Apfel and Tim Davis for helpful remarks and suggestions for many different parts of this book. I thank the many fine reviewers that Taylor & Francis engaged to criticize my book. As always, my editor Robert Ross was supportive and proactive in the production of this book.

Steven G. Krantz
St. Louis, Missouri

Table of Contents