Igor Emri · Arkady Voloshin

Statics

Learning from Engineering Examples

Springer

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Statics

To Ilana and Vesna with love, you made it possible and worthwhile.

Arkady and Igor

Preface

Mathematical and physical theories cannot be used directly to solve real-life problems because models (theories) take into account only the most important physical quantities and cannot account for each particular characteristic of the given problem. The real-life engineering problems usually deal with objects of complicated geometrical configuration that cannot be easily modeled. Thus, simplifications are always required in order to apply the theory. The ability to simplify the real-life problems and represent them as solvable models is the most important skill of an engineer. Simplifications are commonly applied to the geometry of a real structure and to the selection of most important physical quantities that are of major importance to achieve an engineering solution. Because of these simplifications it is obvious that the analytical solution based on the assigned physical model does not represent the exact solution for the real problem, but at the best is a good engineering approximation.

This textbook of mechanics aims to teach the engineering students the ability to consider any problem and approach it in a systematic way that will allow creating a physical model of a real-life problem and arrive at the solution by writing and solving equations of equilibrium. In this process, the necessary simplifications will take place, and the complicated, real-life problem will be reduced to a manageable simple system that may be easily represented by its free body diagram and corresponding set of equilibrium equations.

Even though the field of mechanics of rigid bodies is well established and did not have any new developments in the last 100 years, we still have to develop better and easier ways for students to understand these basic elements and be capable to observe, understand, and simplify the existing problem. Thus, we enforce the concept of taking the real-world engineering examples and simplify them to become solvable by relatively simple means of the equations of equilibrium.

A set of equilibrium equations may be solved by "hand" or by any of the available computer tools. These tools rapidly change with new developments in computer science and engineering, which has nothing to do with the subject of this book. We therefore leave to the discretion of the instructor and student which of the available tools they may use for solving equations. These may be EXCEL, MATLAB, MATHEMATICA, or any other programs that may appear in the future.

Many of the currently available new texts are introducing so-called "computer" problems that are nothing more than using the analytical solution and substituting a range of variables to calculate the result. Our philosophy is that such an approach distracts students from comprehending the problem and leads them to rely on a numerical approach before developing a clear understanding of the mechanics. Therefore, we try not to emphasize such exercises.

Today's students are well versed in using computational software and thus just running a "do loop" to run the calculation through a range of values that does not contribute to the deeper understanding of the mechanics. Of course, we do not want to preclude students from using any computational software capable to ease the calculations necessary to get the result. For this purpose, a number of MATLAB routines are provided on the Springer website (http://extras.springer.com) that students may use to solve the linear system of equations of equilibrium. However, these routines still require from students the deep understanding of a given problem and ability to create the correct free body diagram. The MATLAB routines help to solve the system of equations, but do not solve the problem by themselves.

The implemented approach here will allow students to build a better understanding of the physical reality and ways to simplify it in order to create an acceptable engineering solution.

Ljubljana, Slovenia — Igor Emri
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Contents

1	**Introduction**	1
1.1	General Approach	1
1.2	Review of the Contents	2
1.3	Conventions on Notations	3
2	**Laws of Nature and Fundamental Concepts**	5
2.1	Forces and Newton's Laws	8
2.1.1	Newton's Laws	8
2.1.2	Internal and External Forces	11
2.1.3	Principle of Transmissibility	11
2.2	Two Forces Acting Upon a Body	12
2.2.1	Equilibrium Pair of Forces	12
2.2.2	Parallelogram of Forces	13
2.2.3	Couple of Forces	16
2.3	System of Units	24
2.4	Numerical Calculations	26
2.5	Problems	27
3	**From Reality to a Free Body Diagram**	29
3.1	Mechanical Systems	30
3.2	Loads	33
3.3	Supports and Free Body Diagrams	34
3.3.1	Two-Dimensional Systems	36
3.3.2	Three-Dimensional Systems	40
3.4	Problems	48
4	**Resultant and Equilibrium of Forces Acting at a Point**	59
4.1	Resultant and Equilibrium of In-Plane Forces	60
4.1.1	Resultant of Forces	61
4.1.2	Equilibrium of Forces	63
4.1.3	Problems	71
4.2	Resultant and Equilibrium of Forces Acting in Space	87
4.2.1	Resultant of Forces	88
4.2.2	Equilibrium of Forces	91

4.2.3 Problems . . . 94
4.3 Review Problems . . . 104

5 Equilibrium of Rigid Bodies . . . 109
5.1 Force–Moment Systems . . . 110
5.1.1 Moving a Force to an Arbitrary Point . . . 110
5.1.2 Reduction of a System of Forces . . . 114
5.1.3 Special Case of a Force-Couple System . . . 117
5.1.4 General Case of a Force-Couple System . . . 120
5.1.5 Moment of a Force About an Axis . . . 122
5.1.6 Problems . . . 124
5.2 Equilibrium in Two Dimensions . . . 129
5.2.1 Two-Force Members . . . 137
5.2.2 Three-Force Members . . . 138
5.2.3 Problems . . . 143
5.3 Equilibrium in Three Dimensions . . . 170
5.3.1 Problems . . . 174

6 Distributed Forces: Center of Gravity and Centroids . . . 183
6.1 Distributed Forces and Rigid Body . . . 184
6.2 Center of Gravity of a Flat Plate . . . 185
6.3 Centroids . . . 189
6.3.1 Centroids by Integration . . . 191
6.3.2 Centroids of Composite Bodies . . . 196
6.3.3 Problems . . . 201
6.4 Distributed Loads . . . 216
6.4.1 Effect of the Fluid Pressure . . . 217
6.4.2 Problems . . . 221
6.5 Center of Gravity and Centroid of Bodies . . . 230
6.5.1 Center of Gravity . . . 231
6.5.2 Centroids . . . 232
6.5.3 Theorems of Pappus . . . 236
6.5.4 Problems . . . 241

7 Classification of Structural Elements . . . 249
7.1 Types of Structural Elements . . . 250
7.1.1 Truss Members . . . 251
7.1.2 Beams . . . 252
7.1.3 Cables . . . 253
7.2 Internal Forces . . . 254
7.2.1 Problems . . . 260

8 Analysis of Truss Structures . . . 265
8.1 Method of Joints . . . 269
8.1.1 Problems . . . 284
8.2 Method of Sections . . . 293
8.2.1 Problems . . . 296

8.3 Compound Trusses 306
8.4 Space Trusses 308
8.4.1 Problems 313

9 Beams 321
9.1 Selection of Coordinate System for the Internal Forces and Moments and Sign Convention 323
9.2 Straight Beams 324
9.2.1 Diagrams of Internal Forces and Moments 329
9.2.2 Relationship Between the Distributed Load, Shear Force, and Bending Moment – Rules for Drawing Diagrams of Internal Forces and Moment Intuitively 337
9.2.3 Intuitive Drawing of Internal Forces and Moment Diagrams 341
9.2.4 Problems 346
9.3 Curved Beams 355
9.4 Piece-Wise Straight and Curved Beams 359
9.4.1 Problems 365
9.5 Superposition Principle 370
9.6 Problems 383

10 Cables 389
10.1 Cables Loaded by Concentrated Forces 390
10.2 Cables with Distributed Loads 403
10.2.1 Parabolic Solution 407
10.2.2 Hyperbolic Solution 423
10.3 Problems 450

11 Compound Structures 457
11.1 Introduction 458
11.1.1 Problems 462
11.2 Frames 471
11.2.1 Problems 476
11.3 Mechanisms 479
11.3.1 Problems 482

12 Friction 489
12.1 Introduction 490
12.2 Friction Between Solid Bodies 491
12.2.1 Stick-Slip Effect 495
12.3 Angles of Friction 499
12.3.1 Angles of Static and Kinetic Friction 501
12.4 Wedges 510
12.5 Rolling and Rolling Resistance 516
12.6 Plain Bearings 523
12.6.1 Radial or Journal Bearings 524
12.6.2 Axial or Trust Bearings 527

12.7 Belts and Ropes Friction 533
12.7.1 V-Shaped Belts 536
12.8 Problems 545

Appendix 563

Index 565

Introduction

1

Contents

1.1 General Approach 1
1.2 Review of the Contents 2
1.3 Conventions on Notations 3

Nearly, each "Statics" book starts with the statement similar to this one: "the main objective of the book is to provide a student with a clear understanding of mechanics and develop the ability to analyze the problem in a simple and logical manner." It is indeed hard to argue with such a statement. And if any of the "Statics" books would "provide a clear understanding and ability to analyze the problem," this book would be unnecessary.

The underlying basic mechanics laws stay the same, but the way they are presented and interpreted is changing with time. The structures built 2–3 thousand years ago demonstrate that humans understood the underlying laws of mechanics even though they were not stated in the forms as they were postulated by Newton in seventeenth century. This clearly demonstrates that comprehending laws of nature is a continuing process, and this book represents one of the small steps in evolution of our understanding of nature.

1.1 General Approach

The intent of this book is to introduce and explain the basic concepts on which "Statics" is based utilizing real-life engineering examples. This is why you will see a large number of photographs of the real machinery and structures. The traditional "teaching" is substituted here by a "learning" approach. We show you a real problem, analyze it, simplify it, and develop a way to solve it. We cannot solve each problem exactly, but we will show you that thinking and

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simplification can help you to deal with many structural problems in a simple and reasonable way.

1.2 Review of the Contents

The scientific approach is empirical, based solely on observation and experiment. The book is built on the basis of fundamental laws of nature as were developed by Sir Isaac Newton, knowledge of vector algebra and common sense. Chapter 2 introduces the basic concepts and definitions; it explains the system of units used in engineering. Chapter 3 introduces a student to the very essential task—how to represent physical reality in a way that one may solve problems and get meaningful results. It teaches you how to identify the important features of the structure that should be included in your model and what may be omitted. This is extremely an important step since the obtained results will be useful only to the extent you understand and justify the simplifications introduced in the process of creating physical model and free body diagram.

Chapter 4 discusses how to find a resultant force replacing number of forces acting on a point in two and three dimensions. After we are comfortable with finding resultant, the main focus point of statics is discussed—how to analyze equilibrium of forces acting at a point on the plane and in the space.

Equilibrium of rigid bodies is the topic of Chap. 5. We introduce procedures to use the fundamental laws of nature in order to find the unknown parameters for a system in equilibrium. Two- and three-dimensional cases are discussed.

Chapter 6 introduces concepts of the center of gravity and centroids. In the same chapter, we discuss the problems of the water pressure on submerged surfaces. In Chap. 7, we will introduce concepts, assumptions, and rules necessary to classify structural elements. The following classes of structures are discussed: trusses, beams, frames, machines, and cables. We also introduce a procedure to calculate internal forces in various structural elements.

The following three Chaps. 8–10 introduce and discuss detailed procedures to solve the corresponding problems. There we develop a set of approaches one may use to calculate the internal forces and moments in a variety of structural elements introduced in Chap. 7.

Chapter 11 discusses how to deal with the problems that do not fall in any of simple classes of structures: trusses, beams, or cables. Here, we develop ways to solve such problems by disassembling a structure and to solve each constituent using the methods introduced in previous chapters.

Chapter 12 deals with the frictional forces that prevent relative movement of solid bodies in contact.

The home problems are divided into two distinct classes: real-life problems and problems represented via physical models, created mainly for methodological reasons. The first group of problem will mainly teach how to study a real-life problem, simplify it, and represent it as a "physical model," which will allow using the solution techniques explained in the book. The second group of problems is mainly for learning the mathematical procedures and other tools necessary to solve the real-life problems represented by physical models. This process of modeling introduces inevitable errors; however, good engineering practice allows minimizing them and still getting solutions that will satisfy the real-life requirements. Some of the problems use the internationals system of units (SI units), whereas the others use the US customary units. Such approach will allow students to get familiar with both of the systems of units.

Problems identified by "*" are considered to be "challenging." They may be solved using the methods described in this book, but it will require extra effort from the student, thus they may be assigned for "extra" credit.

Appendix contains the basic information about the vectors, matrices, and the ways to manipulate them. It should be noted that the main purpose of this Appendix is to provide a refreshment of the rules on vectors and matrix algebra; it cannot serve as a tutorial.

Several routines written for MATLAB are available on the *extras.springer.com*. They allow students to simplify the process of calculations, but they still leave the burden of creating correct free body diagrams and writing appropriate equations of equilibrium. Their routines may be easily modified to solve a range of problems; they will decrease the calculation errors, but they will not solve a problem for you! You should always consider the MATLAB and other numerical procedures as a tool (e.g., pencil, calculator) and not as a problem solver.

Both authors are avid scuba divers and went to many scuba trips together (and still do). Since one cannot dive more than couple of hours per day, they used the rest of the time to discuss various topics of common interest. One of these was the way the course of "Statics" is taught at different universities. This was the start of a long and tedious work on this approach to comprehend "Statics."

We decided to deal with real problems as one may encounter in the real life and not with models and idealizations only, as it is customary in the many of the existing text books.

1.3 Conventions on Notations

In the course of this book, we usually use bold letters to define external forces, and we use capital letters to indicate the supports. For example, if A indicates support then we would use A, A_x, A_y, and A_z to define the components of the reaction forces. Distributed forces are usually defined by lower case q. Every time we use a summation sign $\sum$, it is assumed summation by all forces and/or moments. If we need to specify the details, $\sum_{i=1}^{N}$ is used.

Vectors are denoted in bold font and scalars in italics. Greek letters are used to define angles.

Vector or cross product is defined by x and scalar product is defined by a dot (·).

List of Symbols

a	Distance
b	Distance
c	Distance
A, B, C, D, E	Points
A, B, C, D, E, F, P, Q	Force reactions, magnitudes
A, B, C, D, E, F, P, Q, R	Force reactions, vectors
G	Center of gravity
q	Distributed forces
i, j, k	Unit vectors
M	Moment
O	Origin of coordinates
r	Position vector
x, y, z	Rectangular coordinates
$\alpha, \beta, \gamma, \theta\ldots$	Angles

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Laws of Nature and Fundamental Concepts 2

Contents

2.1 Forces and Newton's Laws 8
2.1.1 Newton's Laws 8
2.1.2 Internal and External Forces 11
2.1.3 Principle of Transmissibility 11
2.2 Two Forces Acting Upon a Body 12
2.2.1 Equilibrium Pair of Forces 12
2.2.2 Parallelogram of Forces 13
2.2.3 Couple of Forces 16
2.3 System of Units 24
2.4 Numerical Calculations 26
2.5 Problems 27

In theory, there is no difference between theory and practice. But, in practice, there is.

Jan L.A. van de Snepscheut

In this chapter you will learn:

- Basic concepts and definitions
- Fundamental laws of nature, as defined by Sir Isaac Newton
- Procedures to find a resultant force
- Mathematical definition of the moment
- How to calculate the moment of a couple
- How to find the projection of the moment of the axis
- System of units

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Everyday experience with objects and forces acting upon them shows that there is a definite relationship between the motion and the force. These relations were studied by Newton in the seventeenth century.[1] Based on the observation of the nature, he postulated basic relations describing interactions between forces, matter, space, and time. These physical quantities are fundamental quantities; they have to be accepted intuitively as *facts of nature*. In the scientific literature, there are many different ways to describe these quantities. Here, we are summarizing those that are appropriate for the purpose of this book.

Time is a quantity used to separate different stages of a process. In principle, evolution of any phenomenon may be used to measure time, providing that certain conditions are fulfilled. In order to introduce a unit of time, we have to consider a well-defined reference process, for example, swing of a pendulum, rotation of the Earth about its axis, and decay of a radioactive material. There are two commonly accepted independent approaches to define the time scale. The first one is based on the regularity of the celestial bodies' motions. The second one is based on the characteristic frequency of the electromagnetic radiation emitted or absorbed in quantum transitions between internal energy states of atoms.

Space can be considered as a boundless, three-dimensional extent in which objects and events occur and have relative positions and directions. The perception of space allows the concepts of position and geometry of a body. The fundamental elements of geometry are length, area, and volume. We have to define an appropriate unit to measure length, area, and volume. In principle, any distance may serve as the unit of length. Through the history, people used many different definitions for the unit of length. According to an international agreement, today we are using the unit of length called meter. Originally, the length unit called *meter* was defined as one ten millions of the distance from the North Pole to the equator on the meridian running through Paris. The latest definition of a *meter* is the length of the path traveled by light in a vacuum during a time interval of 1/299,792,458 of a second. Until recently, the English units of lengths were defined in terms of the imperial standard yard, which was the distance between the two lines on a bronze bar made in 1845. Because the imperial standard yard was shrinking at the rate of 1.5 millionths of an inch per year the United States adopted a copy of the international prototype meter as the national standard of length in 1889.

Matter is a substance that constitutes the observable universe and, together with energy, forms the basis of all objective phenomena. The main building blocks of matter are atoms. Matter has several states: gas, liquid, solid states, and plasma. Each state exhibits properties that distinguish it from the others. Moreover, these general states can be subdivided into groups according to particular types of properties listed in the periodic table. Matter exists in and occupies the space. Matter that occupies a specific space (volume) is called *body*. When distribution of matter in a given volume and its shape does not change in time, we talk about rigid bodies. Quantity of matter in a unit volume is called *density*. Inherent and

[1] Sir Isaac Newton, **Principia,** Vol. I The Motion of Bodies, University of California Press, 1962.