Anil K. Bera · Sergey Ivliev
Fabrizio Lillo *Editors*

Financial Econometrics and Empirical Market Microstructure

Springer

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Editors
Anil K. Bera
Department of Economics
University of Illinois
Urbana
Illinois
USA

Sergey Ivliev
Perm State University
Perm
Russia

Fabrizio Lillo
Scuola Normale Superiore
Pisa
Italy

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Contents

Mathematical Models of Price Impact and Optimal Portfolio Management in Illiquid Markets

Nikolay Andreev

Abstract The problem of optimal portfolio liquidation under transaction costs has been widely researched recently, producing several approaches to problem formulation and solving. Obtained results can be used for decision making during portfolio selection or automatic trading on high-frequency electronic markets. This work gives a review of modern studies in this field, comparing models and tracking their evolution. The paper also presents results of applying the most recent findings in this field to real MICEX shares with high-frequency data and gives an interpretation of the results.

Keywords Market liquidity • Optimal portfolio selection • Portfolio liquidation • Price impact

JEL Classification C61, G11

1 Introduction

With the development of electronic trading platforms, the importance of high-frequency trading has become obvious. This requires the need of automatic trading algorithms or decision-making systems to help portfolio managers in choosing the best portfolios in volatile high-frequency markets. Another actual problem in portfolio management field is optimal liquidation of a position under constrained liquidity during a predefined period of time.

Mathematical theory of dynamic portfolio management has received much attention since the pioneering work of Merton (1969), who obtained a closed-form solution for optimal strategy in continuous time for a portfolio of stocks where the market consisted of risk-free bank accounts and a stock with Bachelier–Samuelson dynamics of price. Optimal criterion had the following form:

N. Andreev (✉)
Financial Engineering and Risk Management Laboratory, National Research University Higher School of Economics, Moscow, Russia
e-mail: nandreev@hse.ru

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$$(C_t, X_t, Y_t) \in \text{Argmax}\, E\left(\int_0^T e^{-\rho t} U(C_t)\, dt + B(W_T, T)\right),$$

where C_t is consumption rate, X_t, Y_t—portfolio wealth in riskless asset and stocks respectively, $W_t = X_t + Y_t$ is total value of portfolio and $U(C) = \frac{C^\gamma}{\gamma}, \gamma < 1$, or $\log C$—a constant relative risk-aversion (CRRA) utility function, $B(W_t, t)$ is a function, increasing with wealth. This criterion formulates optimality as maximization of consumption and portfolio value at the end of a period. Merton asserted that it is optimal to keep assets in constant proportion for the whole period, that is $\pi_t = \frac{Y_t}{W_t} \equiv const$. This result is known as the Merton line due to the strategy's linear representation in (X_t, Y_t) plane.

2 Contemporary Price Impact Modeling

The ideal frictionless market of Merton (1969) does not adequately simulate the more complex real market. First of all, price dynamics obviously depend on an agent's actions in the market; moreover, there is no single characteristic of an asset's market value (price). Since the 1990s, electronic trading through limit order books (LOB) has been gaining popularity, providing the market with a set of orders with different volumes and prices during any trading period. Inability to close a deal at an estimated price led to the necessity of including transaction costs in portfolio management models and price impact modeling. For the past two decades, research in this field has provided complex models that allow for time varying forms of LOBs, temporary and permanent price impact, resilience etc.

The most sophisticated and yet also fundamental way of estimating transaction costs is estimating the whole structure of LOB. Usually the market is represented as a complex Poisson process where each event is interpreted as the arrival/liquidation/cancellation of orders at specific depth levels. Large (2007) considers the arrival of ten kinds of market events (market bid/ask order limit bid/ask order, cancellation of bid/ask order, etc.) according to a multivariate Hawkes process with intensity depending on the past trajectory. Intensity in Large's model does not depend on order depth (distance from best quote).

Cont and Larrard (2012) introduced a complex Poisson model with time and depth-varying intensity and obtained theoretical results on the subject. Unfortunately, due to the extreme complexity of the general approach, it is extremely difficult to calibrate the parameters. Thus, some simplifying assumptions, based on empirical observations of a particular market, are necessary. On the other hand, the Poisson model must be flexible enough to reflect dynamics of real events, otherwise forecast errors will make the result useless for practice.

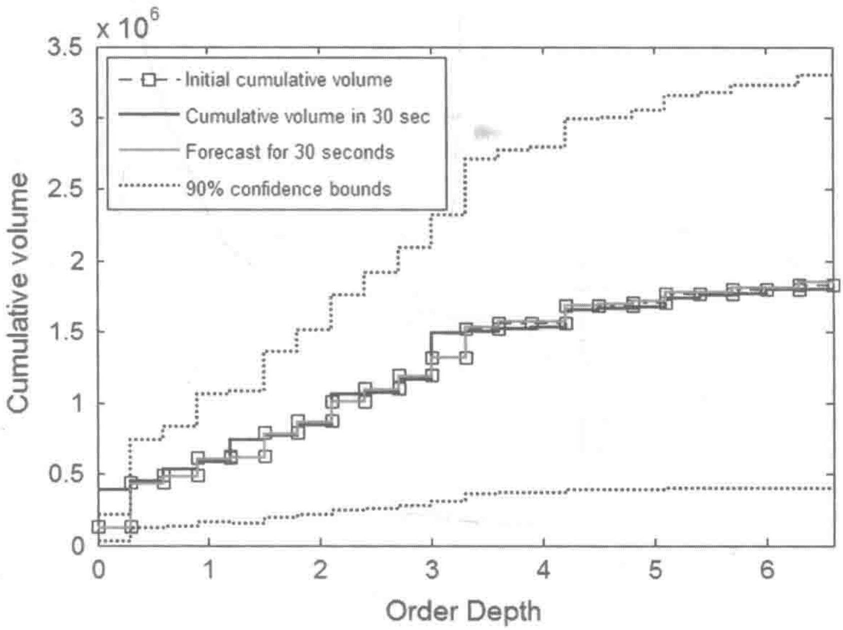

Fig. 1 LOB forecast in terms of cumulative volume as a function of depth for MICEX RTKM shares, 18 January 2006

Consider a simple LOB model with only two types of events: arrival and cancellation of limit order at one side of the book. Intensities are stationary and independent but depend on depth. Volume of each order is a random variable with *a priori* given parametric distribution with unknown parameters depending on depth. Thus, LOB is modelled via compound homogeneous space–time Poisson process. We calibrated the following model to real MICEX data, assuming from empirical observations that

1. event volume distribution is a mixture of discrete and lognormal;
2. intensities as functions of depth are power-law functions.

We estimate parameters θ of the model from order flow history using maximum likelihood and Bayesian methods. Then, using LOB structure L_{t_0} as initial state of the system we model $L_{t_0+T}\,|L_{t_0}, \theta$ and take $\widehat{L}_{t_0+T} = E\left(L_{t_0+T}\,|L_{t_0}, \theta\right)$ as a forecast. Results of forecasting structure for 30 s horizon and 90 % confidence bounds are presented in Fig. 1. We see that even for small horizon confidence interval is too wide for any practical use of such forecast. This is partly explained by presence of discrete part in volume mixture distribution, which is usually difficult to estimate from training sample. Atoms of volume distribution stand for volume values preferred by participants (100, 1,000, 5,000 lot etc.), orders with preferred volumes can amount up to 50 % of total number of orders.

Due to technical difficulties and intention to integrate an LOB model into portfolio optimization, a simple *a priori* form of the book is usually considered.

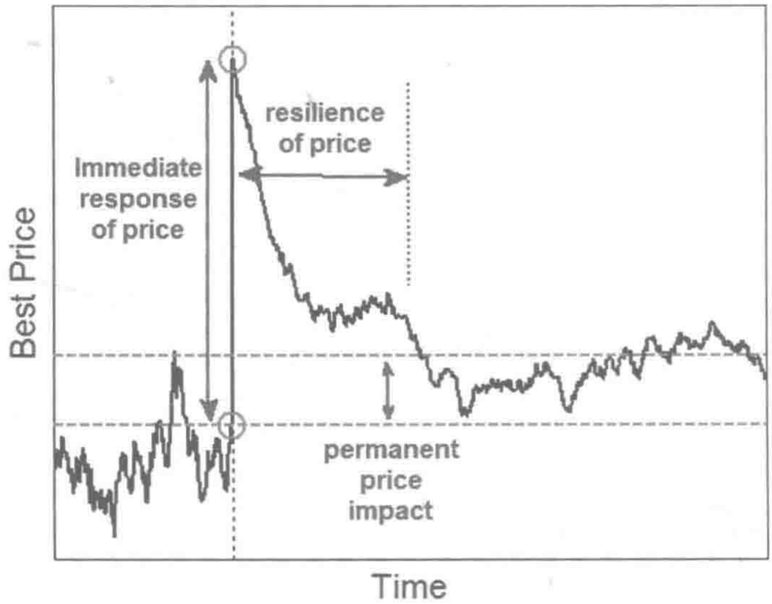

Fig. 2 Price impact aspects

Accent in modeling is made on the price impact function itself. Three main aspects are considered in such an approach:

- Immediate response of best price after a trade, which affects future costs until book replenishes.
- Resilience of LOB, i.e. ability to replenish after a trade; together with immediate response, this is often called temporary price impact. Infinite resiliency means that LOB replenishes instantaneously.
- Permanent price impact, or the effect of replenishment to a level other than pre-trade value; this effect describes the incorporation of information from the trade, which affects market expectations about 'fundamental price' of the asset (Fig. 2).

Permanent price impact is not considered in many classical models of optimal portfolio selection. For a particular case—optimal liquidation—many works assume the simplest dependence, where impact is a linear function of trade volume (i.e., Kyle 1985). Linear approximation can be considered appropriate in most practical cases because of difficulty in calibration of a more complex function in the presence of many agents.

Immediate response function is usually considered linear in volume, which is equivalent to the assumption of the flat structure of LOB (Obizhaeva and Wang 2012), or the assumption that trade volumes *a priori* are less than current market depth. Andreev et al. (2011) consider a polynomial form of immediate response function with stochastic coefficients. Fruth (2011) presents the most general law of immediate response in the form of a diffusion process under several mild conditions.

Resilience has been recently included in impact models and is usually described in exponential form with *a priori* given intensity: Suppose that K_{t_0} is immediate response after a trade at time t_0, then

$$\text{Temporary Impact}_t = K_{t_0} e^{-\int_{t_0}^{t} \rho(u)du}.$$

Almgren and Chriss (1999) considered instantaneous replenishment: $\rho_u = \infty$; Obizhaeva and Wang (2012), Gatheral et al. (2011) and others assumed exponential resilience with constant intensity: $\rho_u \equiv \text{const}$. General law of deterministic resilience rate has been presented in recent papers of Gatheral (2010), Gatheral et al. (2012), Alfonsi et al. (2009), and Fruth et al. (2011).

3 Overview of Contemporary Portfolio Management Models and Their Evolution

Davis and Norman (1990) introduced a consumption–investment problem for a CRRA agent with proportional transaction costs and obtained a closed-form solution for it. Another advantage of the model was allowing for discontinuous strategy. For this purpose, the original Merton framework had to be upgraded to semimartingale dynamics. Portfolio value in each of the assets is described by the following equations:

$$\begin{aligned} dX_t &= (r_t X_t - C_t)\,dt - (1+\lambda)\,dL_t + (1-\mu)\,dM_t, \qquad X_0 = x,\\ dY_t &= \alpha Y_t dt + \sigma Y_t dw_t + dL_t - dM_t, \qquad Y_0 = y, \end{aligned}$$

where coefficients λ, μ define proportional transaction costs, L_t, M_t are cumulative amounts of bought and sold risky asset respectively. Results demonstrated the existence of three behavioral regions for portfolio managers, and are presented in Fig. 3.

Unlike Merton's case, the so-called wait region appears due to transaction costs. That is, it is suboptimal to trade while in the area. Leaving the area leads to immediate buy or sell to get to the wait region's border. Analogous results were also obtained for the infinite horizon problem by Shreve and Soner (1994).

Another extension of the Merton model was presented by Framstad et al. (2001) for jump diffusion price dynamics. It was shown that wait region is absent in this case. That is, this strategy's structure is the same as for Merton's continuous diffusion market.

A number of papers considered a price impact model instead of unrealistic 'fundamental price' dynamics. For example, Vath et al. (2007) presented the following complex price impact function, depending on current price and volume

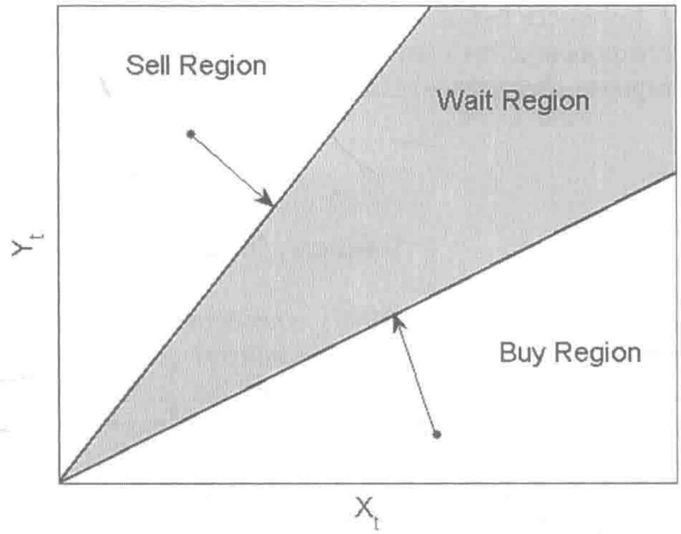

Fig. 3 Buy, Sell and Wait region in a model with proportional transaction costs

of a triggering trade. Around that time, Zakamouline (2002) took another step toward a realistic market model that allowed both proportional and fixed transaction costs. The proportional component described costs due to insufficient liquidity of the market, while the fixed component represented the participation fee for each transaction. Both papers considered discrete trading and produced interesting results. Buy and sell borders were no longer straight lines, as seen in Fig. 3, but still could be obtained beforehand and then used for decision making during trading sessions.

Neither of the abovementioned models considered the form and dynamics of the limit book itself—only the dynamic of an aggregated of a deal, which was considered as price. Microstructure models of electronic limit order markets have become quite popular in literature devoted to the problem of optimal liquidation of a portfolio. This particular case differed from the consumption–investment framework due to the terminal condition—predefined volume of the portfolio to be liquidated. The most notable results in this field are from Almgren and Chriss (1999) and Obizhaeva and Wang (2012). The framework has become quite popular in practice due to the simple models and intuitive results. Both approached considered discrete strategies and defined optimality functional not through utility function, but as a weighted sum of expected value and standard deviation of portfolio value.

The work of Obizhaeva and Wang first appeared as a draft in 2005 and considered a flat static structure of the limit book. Their approach has been adopted by many authors, evolving into several directions. The most realistic models were presented by Predoiu et al. (2011) and Fruth et al. (2011). Predoui et al. consider a general form of order distribution inside a book and non-adaptive strategies of liquidation. Fruth et al. postulate a flat but dynamic form of order distribution while allowing for both discrete and continuous trading in the same framework, linear permanence and general temporary price impact; the described model does not allow several kinds of arbitrage and non-adaptive strategies, which proved to be optimal in the framework. Analytical solutions have been obtained for discrete cases and for continuous trading.

4 Comparison of Portfolio Management Strategies

Despite the great potential of the developed models, most of them have not been applied to real data. To prove the usefulness of portfolio management models for practitioners, we apply some of the contemporary results in this field to real MICEX trading data and give recommendations for their usage. Our database consists of the complete tick-by-tick limit order book for MICEX shares from January 2006 through June 2007. We consider only liquid shares, such as LKOH, RTKM and GAZP, because only during sufficiently intensive trading does it become possible to calibrate models for the real market.

We consider the problem of optimal purchase of a single-asset portfolio over a given period and compare the performance of the following strategies:

1. Immediate strategy—portfolio is obtained via a single trade at the moment of decision-making. This strategy must lead to the largest costs but eliminates market risk completely. It is recommended for high-volatility markets or in case of information about unfavourable future price movements.
2. Fruth et al.'s (2011) strategy—this has the same goal as uniform strategy, i.e. minimization of expected transaction costs but not market risk. The main advantage of the model is its flexibility and consideration of several main microstructure effects, such as time-varying immediate price impact, dynamic model of the order book and time-varying resilience rate. Authors define price impact for buy and sell sides (E_t and D_t) as the difference between best price in the book and unaffected price. Permanent impact is proportional to volume of the order and constant over time while immediate response function $K(t,v)=K_t v$ changes over time. Temporary impact decays exponentially with a fixed time-dependent, deterministic recovery rate ρ_t, so that temporary impact of trade v, occurred at time s, at time t equals $K_s e^{-\int_s^t \rho_u du} v$. General framework considers both continuous and discrete time market models. It generalizes Obizhaeva and Wang's approach and postulates the following strategy: when price impact is low and the agent still has much to buy, she buys until the ratio of impact to remaining position is high enough, otherwise she waits for the impact to lower. After that, the agent can make another deal or wait, etc. So, for each moment of time, the agent has a barrier dividing her "Buy" and "Wait" regions.
3. Andreev et al. (2011) approach—a generalization of the Almgren and Chriss framework. Optimality is considered as minimization of both transaction costs and risk. This model has been obtained specifically for the MICEX market and incorporates a parametric dynamic model of cost function, which provides more accurate results: market model uses fundamental price instead of best bid-ask

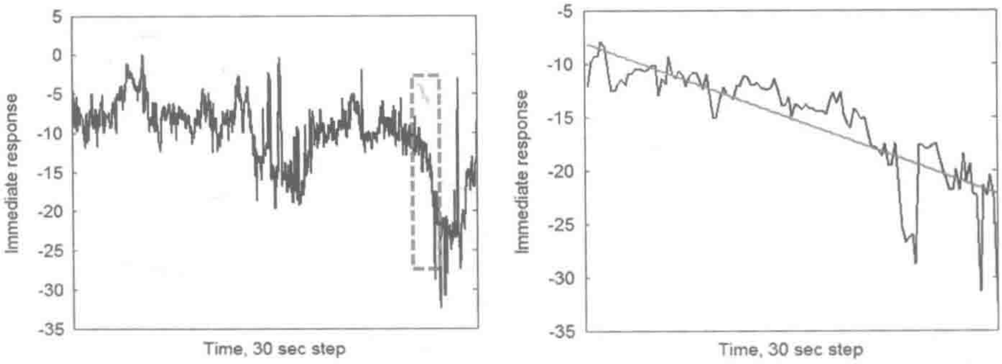

Fig. 4 Immediate response coefficient K_t for the whole trading day (7 February 2006) and dynamics during decline period (LKOH shares)

prices, which follows arithmetic Brownian motion. Transaction costs function has polynomial form (third degree polynom) with stochastic coefficients, which follow simple AR(1) model. No price impact is assumed. The strategy, unlike the previous three, considered agent risk aversion, which is characterized by the weighted sum of two criteria of optimality in minimization of functionality. Thus, problem formulates as minimization of $-E(W_T)+\lambda Var(W_T)$, where W_T is terminal wealth and λ is *a priori* risk aversion parameter.

For example, consider a 100,000 LKOH-share portfolio, liquidated via six consequent trades with 60-s wait periods. Consider also linear immediate response function with coefficient K_t. Rough estimate of K_t is obtained via least-squares method: $K_t = \arg\min \sum_{i=1}^{M} \left(\frac{\partial}{\partial v} C\,(t, v_i) - K_t v_i\right)^2$, where $C(t, v)$ is cost of trade with volume v, reconstructed from order book shape, and $0 < v_1 < \cdots < v_M = \overline{V}$ is *a priori* volume grid, for $\overline{V}$ we take half of available trading volume at the moment. Figure 4 shows dynamics of immediate response coefficient K_t. Liquidation begins when decline in response has been observed for some time (selected region in Fig. 4).

Strategies 2 and 3 are presented in Fig. 5 and have quite different behaviours. The form of the first strategy is obvious from the description. For Strategy 3, we use the simplest calibration assumptions, considering resilience rate a constant and immediate response as linear in time and volume. Assumptions are appropriate for medium periods of time.

We ascertain that the performance of Fruth et al.'s approach is the best of the three, while immediate buy is the worst. This result was expected because Strategy 2 is better adjusted to a specific form of response and can often show better performance if the form was guessed right. The strategy of Almgren and Chriss shows inferior performance and higher aggressiveness (see Fig. 6) due to

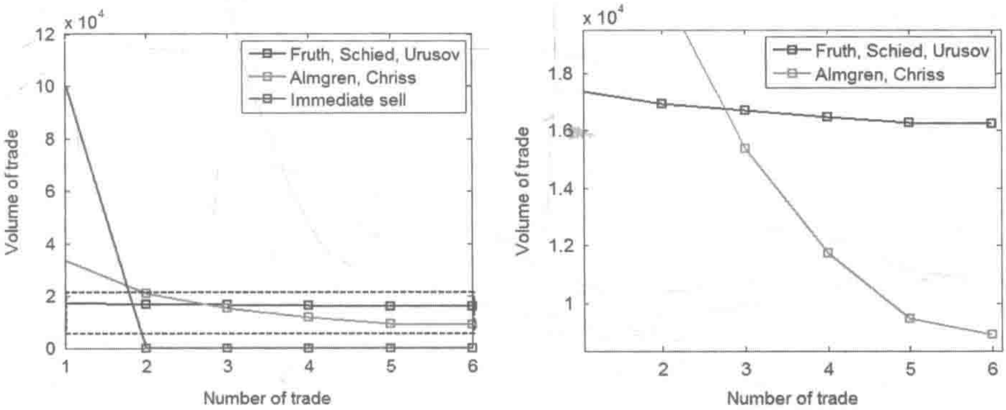

Fig. 5 Trading strategies for immediate strategy, approach by Fruth et al. (2011) and approach by Andreev et al. (2011) with $\lambda = 0.01$ for purchase of portfolio of 100,000 Lukoil shares via six trades with 1-min intervals. Date: February 7, 2006

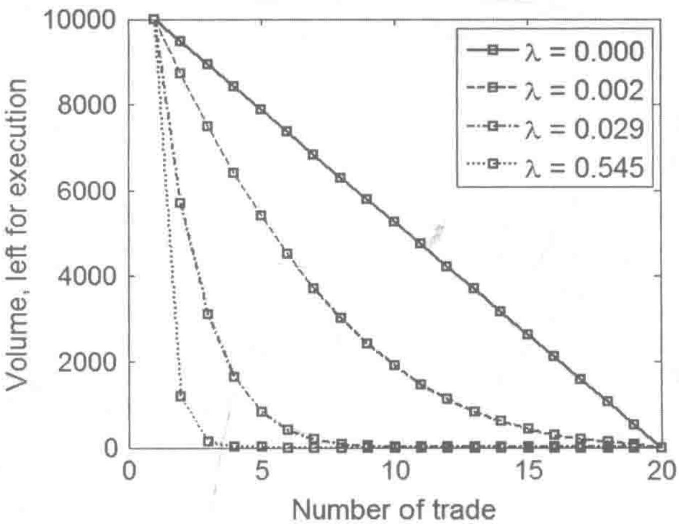

Fig. 6 In Almgren and Chriss framework aggressiveness of the strategy increases with risk-aversion parameter λ. The figure demonstrates how volume left for execution depends on the number of trade for different values of λ. $\lambda = 0$ leads to equal size of trades. Initial volume is 10,000 shares, strategy allows the maximum of 20 trades

minimization of market risk if risk-aversion is sufficiently high.[1] The choice of risk-aversion parameter heavily influences resulting strategy but cannot be chosen automatically. Unfortunately some practitioners interpret this as a misspecification and excessive difficulty of the model and therefore favor simpler strategies. It is also not surprising that the Fruth et al. approach leads to lower costs than immediate

[1]Extreme case of Almgren and Chriss strategy with infinite risk-aversion ($\lambda = \infty$) would be immediate buy.

strategy: the strategies in the model contains immediate buy, and dynamics in the parameters of the market are taken into account. Immediate strategy doesn't consider specifics or the current situation on the market, so it can be frequently outperformed by more elaborate methods.

Conclusion

Due to development of microstructure models and availability of high-frequency historic data, mathematical portfolio selection strategies have been extensively researched since the early 1990s. Nevertheless, very few frameworks were applied by practitioners because underlying models of the market were too unrealistic at the time. The aim of this research is to provide a review of modern accomplishments in the field, including the ongoing work, and demonstrate more realistic market models used in contemporary frameworks. To illustrate the effect of using automatic algorithms of portfolio selection and, in particular, optimal purchase/liquidation, we apply several approaches to real MICEX shares-related trading data and compare the results.

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