

Third Edition

# AERODYNAMICS OF WIND TURBINES



MARTIN O. L. HANSEN

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Third edition

Martin O. L. Hansen

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# Aerodynamics of Wind Turbines

*Aerodynamics of Wind Turbines* is the established essential text for the fundamental solutions to efficient wind turbine design. Now in its third edition, it has been substantially updated with respect to structural dynamics and control. The new control chapter includes details on how to design a classical pitch and torque regulator to control rotational speed and power, while the section on structural dynamics has been extended with a simplified mechanical system explaining the phenomena of forward and backward whirling modes. Readers will also benefit from a new chapter on vertical-axis wind turbines (VAWTs).

Topics covered include increasing mass flow through the turbine, performance at low and high wind speeds, assessment of the extreme conditions under which the turbine will perform, and the theory for calculating the lifetime of the turbine. The classical blade element momentum method is also covered, as are eigenmodes and the dynamic behaviour of a turbine.

The book describes the effects of the dynamics and how this can be modelled in an aeroelastic code, which is widely used in the design and verification of modern wind turbines. Furthermore, it examines how to calculate the vibration of the whole construction, as well as the time varying loads and global case studies.

**Martin O. L. Hansen** is a lecturer at the Technical University of Denmark, where he teaches wind turbine and related technologies.

# Preface

This book has been written based on notes from two courses given at the Technical University of Denmark on the mechanical aspects of wind turbines to give a basic understanding of how a wind turbine converts the kinetic energy of the wind into mechanical shaft power, which can be used to drive a generator to finally produce electrical power. This also includes various ways to control the loads and thus the output of the wind turbine by pitching the blades and by controlling the rotational speed. How to actually apply power electronics to vary the torque on the generator shaft to control the rotational rotor speed is not addressed in detail, and an interested reader is referred to textbooks on electrical motors and power electronics. Chapters describing the most important loads on the wind turbine construction are also included. Chapters 1–8, 10 and 11 could form the basis of an undergraduate course in basic wind turbine technology. From these chapters it is possible for a student knowing the geometry of a wind turbine to write a program based on the blade element momentum (BEM) method that can actually estimate the static loads and thus the mechanical power for various wind speeds, pitch angles and rotational speed. Such a program is a valuable tool in an optimization process, where the load distribution must be calculated many times for different design variables in order to maximize some desired property, such as, e.g., the annual energy production.

Building a wind turbine that will not break down during the design lifetime involves calculating time histories of the internal material loads for various unsteady conditions, the most important being a turbulent inflow, which varies in time, as well as space, over the rotor area. This involves building a so-called aeroelastic method that also takes into account the dynamic response of the wind turbine construction from the time varying aerodynamic, gravitational and other loads. The vibration velocities of the blades directly influence the aerodynamic loads, and there is therefore a strong coupling between the aerodynamics and the structural dynamics. Further, local accelerations must be taken into account when calculating the internal material loads. The outline of such a method is given in Chapters 9 and 12, describing respectively an unsteady blade element momentum method and some basic structural dynamics tools. Together with Chapters 14 and 15 these can be used in a graduate course

concerning wind turbine aeroelasticity. Chapter 14 deals with a method to construct a 3D turbulent inflow and Chapter 15 outlines how the fatigue damage is estimated from the various load time histories. These chapters should be supplemented with some norms describing all the different load cases that a wind turbine must be designed for, both with respect to fatigue and ultimate loads. Finally, the vertical-axis concept is addressed in Chapter 16.

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# 1 General introduction to wind turbines

Before addressing more technical aspects of wind turbine technology, an attempt is made to give a short general introduction to wind energy. This involves a very brief historical part explaining the development of wind power, as well as a part dealing with economy and wind turbine design. It is far from intended to give a full historical review of wind turbines, but merely to mention some major milestones in their development and to show examples of historical exploitation of wind power.

## Short historical review

The force of the wind can be very strong, as seen after the passage of a hurricane or a typhoon. Historically, people have harnessed this force peacefully, its most important use probably being the propulsion of ships using sails before the invention of the steam engine and the internal combustion engine. Wind has also been used in windmills to grind grain or to pump water for irrigation or, as in Holland, to prevent the ocean from flooding low-lying land. At the beginning of the twentieth century electricity came into use and windmills gradually became wind turbines as the rotor was connected to an electric generator.

The first electrical grids consisted of low-voltage DC cables with high losses. Electricity therefore had to be generated close to the site of use. On farms, small wind turbines were ideal for this purpose and in Denmark Poul la Cour, who was among the first to connect a windmill to a generator, gave a course for 'agricultural' electricians. La Cour had great foresight and in his school he installed one of the first wind tunnels in the world in order to investigate rotor aerodynamics. Gradually, however, diesel engines and steam turbines took over the production of electricity and only during the two world wars, when fuel was scarce, did wind power flourish again.

However, even after the Second World War, the development of more efficient wind turbines was still pursued in several countries such as Germany, the USA, France, the UK and Denmark. In Denmark, this work was undertaken by Johannes Juul, who was an employee in the utility company SEAS and a former student of la Cour. In the mid-1950s Juul introduced what was later called

the Danish concept by constructing the famous Gedser turbine, which had an upwind three-bladed, stall-regulated rotor, connected to an AC asynchronous generator running with almost constant speed. With the oil crisis in 1973, wind turbines suddenly became interesting again for countries which wanted to be less dependent on oil imports. Therefore many national research programmes were initiated to investigate the possibilities of utilizing wind energy. Large non-commercial prototypes were built to evaluate the economy of wind-produced electricity and to measure the loads on big wind turbines. Since the oil crisis, commercial wind turbines have gradually become an important multibillion dollar industry with annual growth rates of up to 20 per cent.

### Why use wind power?

As already mentioned, a country or region where energy production is based on imported coal or oil will become more self-reliant by using alternatives such as wind power. Electricity produced from the wind produces no CO<sub>2</sub> emissions and therefore does not contribute to the greenhouse effect. Wind energy is relatively labour intensive and thus creates many jobs. In remote areas or areas with a weak electricity grid, wind energy can be used for charging batteries or can be combined with a diesel engine to save fuel whenever wind is available. Moreover wind turbines can be used for the desalination of water in coastal areas with little fresh water, for instance the Middle East. At windy sites the cost of electricity, measured in \$/kWh, is competitive with the production cost from more conventional methods, for example, from coal-fired power plants.

To reduce the price further and to make wind energy more competitive with other production methods, wind turbine manufacturers are concentrating on bringing down the price of the turbine itself. Other factors, such as interest rates, cost of land and, not least, the amount of wind available at a certain site, also influence the production cost of the electrical energy generated. The production cost is computed as the investment plus the discounted maintenance cost divided by the discounted production measured in kWh over a period of typically 20 years. When the characteristics of a given turbine is known, i.e. the power for a given wind speed, as well as the annual wind distribution, the annual energy production can be estimated at a specific site.

Some of the drawbacks of wind energy can also be mentioned. Wind turbines create a certain amount of noise when they produce electricity. In modern wind turbines, manufacturers have managed to reduce almost all mechanical noise and are now working hard on reducing aerodynamic noise from the rotating blades. Noise is an important competition factor, especially in densely populated areas. Some people think that wind turbines are unsightly in the landscape, but as bigger and bigger machines gradually replace the older smaller machines, the actual number of wind turbines will be reduced and, at the same time, a greater capacity is available. If many turbines are to be erected in a region, it is important to have public acceptance. This can be achieved by allowing those people living close to the turbines to own a part of the project and thus share

the income. Noise and visual impact will in the future be less important as more wind turbines will be sited offshore.

One problem is that wind energy can only be produced when the wind is blowing. This is not a problem for most countries that are connected to big grids and can therefore buy electricity from the grid in the absence of wind. It is, however, an advantage to know in advance what resources will be available in the near future so that conventional power plants can adapt their production. Reliable weather forecasts are desirable since it takes some time for a coal-fired power plant to change its production. Combining wind energy with hydropower would be a perfect match, since it takes almost no time to open or close a valve at the inlet to a water turbine, i.e. one can save water in the reservoirs when the wind is sufficiently strong.

## The wind resource

A wind turbine transforms the kinetic energy in the wind into mechanical energy in a shaft and finally into electrical energy in a generator. The maximum available energy,  $P_{max}$ , is thus obtained if theoretically the wind speed could be reduced to zero:

$$P_{max} = \frac{1}{2} \dot{m} V_o^2 = \frac{1}{2} \rho A V_o^3 \quad (1.1)$$

where  $\dot{m}$  is the massflow through the rotor,  $V_o$  is the wind speed,  $\rho$  the density of the air and  $A$  the area where the wind speed has been reduced. The equation for the maximum available power is very important since it tells us that the power increases with the cube of the wind speed and only linearly with density and area. The available wind speed at a given site is therefore often first measured over a period of time before a project is initiated.

In practice one cannot reduce the wind speed to zero, so a power coefficient  $C_p$  is defined as the ratio between the actual power obtained and the maximum available power as given by Equation 1.1. A theoretical maximum for  $C_p$  exists denoted the Betz limit,  $C_{p, max} = 16/27 = 0.593$ . Modern optimized wind turbines operate close to this limit with  $C_p$  up to 0.5. Statistics have been gathered from many turbines sited in Denmark, and as rule of thumb, they produce approximately 1,000 kWh/m<sup>2</sup>/year. However, the production is very site dependent and the rule of thumb can only be used as a crude estimation and only for a site in Denmark.

Sailors discovered very early that it is more efficient to use the lift force rather than simple drag as the main source of propulsion. Lift and drag are the components of the force perpendicular and parallel to the direction of the relative wind, respectively. It is easy to show theoretically that it is much more efficient to use lift rather than drag when extracting power from the wind. All modern wind turbines therefore consist of a number of rotating blades which look like propeller blades. If the blades are connected to a vertical shaft, the turbine is called a vertical-axis wind turbine, VAWT, and if the shaft is horizontal, the turbine is called a horizontal-axis wind turbine, HAWT. For commercial wind turbines, the majority are HAWTs and the following text therefore mainly focuses on this