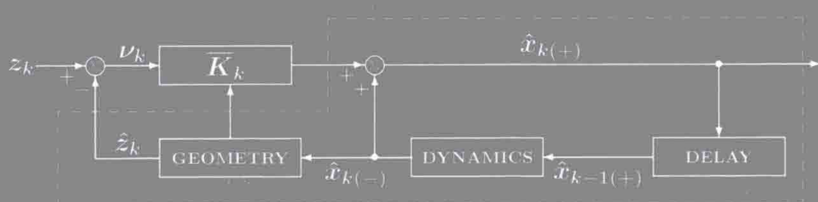
KALMAN FILTERING

Theory and Practice Using MATLAB®



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FOURTH EDITION

WILEY

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Theory and Practice Using MATLAB®

Fourth Edition

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Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data:

Grewal, Mohinder S.
Kalman filtering : theory and practice using MATLAB / Mohinder S. Grewal, Angus P. Andrews. – Fourth edition.
pages cm
Includes index.
ISBN 978-1-118-85121-0 (cloth)
1. Kalman filtering. 2. MATLAB. I. Andrews, Angus P. II. Title.
QA402.3.G695 2015
629.8′312–dc23

2014020208

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

KALMAN FILTERING

PREFACE TO THE FOURTH EDITION

This book is designed to provide our readers a working familiarity with both the theoretical and practical aspects of Kalman filtering by including "real-world" problems in practice as illustrative examples. The material includes the essential technical background for Kalman filtering and the more practical aspects of implementation: how to represent the problem in a mathematical model, analyze the performance of the estimator as a function of system design parameters, implement the mechanization equations in numerically stable algorithms, assess its computational requirements, test the validity of results, and monitor the filter performance in operation. These are important attributes of the subject that are often overlooked in theoretical treatments but are necessary for application of the theory to real-world problems.

In this fourth edition, we have added a new chapter on the attributes of probability distributions of importance in Kalman filtering, added two sections with easier derivations of the Kalman gain, added a section on a new *sigmaRho* filter implementation, updated the treatment of nonlinear approximations to Kalman filtering, expanded coverage of applications in navigation, added many more derivations and implementations for satellite and inertial navigation error models, and included many new examples of sensor integration. For readers who may need more background in matrix mathematics, we have included an Appendix B as a pdf file on the companion Wiley web site at `www.wiley.com/go/kalmanfiltering`.

We have also updated the problem sets and incorporated helpful corrections and suggestions from our readers, reviewers, colleagues, and students for the overall improvement of the textbook.

All software has been provided in MATLAB®, so that users can take advantage of its excellent graphing capabilities and a programming interface that is very close to the mathematical equations used for defining Kalman filtering and its applications. The MATLAB development environment also integrates with the Simulink® simulation environment for code verification on specific applications and code translation to C for the many applications microprocessors with C compilers. Appendix A has descriptions of the MATLAB software included on the companion Wiley web site. The inclusion of the software is practically a matter of necessity, because Kalman filtering would not be very useful without computers to implement it. It is a better learning experience for the student to discover how the Kalman filter works by observing it in action.

The implementation of Kalman filtering on computers also illuminates some of the practical considerations of finite-wordlength arithmetic and the need for alternative algorithms to preserve the accuracy of the results. If the student wishes to apply what she or he learns, then it is essential that she or he experience its workings and failings—and learn to recognize the difference.

The book is organized for use as a text for an introductory course in stochastic processes at the senior level and as a first-year graduate-level course in Kalman filtering theory and application. It could also be used for self-instruction or for purposes of review by practicing engineers and scientists who are not intimately familiar with the subject. Chapter 1 provides an informal introduction to the general subject matter by way of its history of development and application. Chapters 2–4 cover the essential background material on linear systems, probability, stochastic processes, and random process modeling. These chapters could be covered in a senior-level course in electrical, computer, and systems engineering.

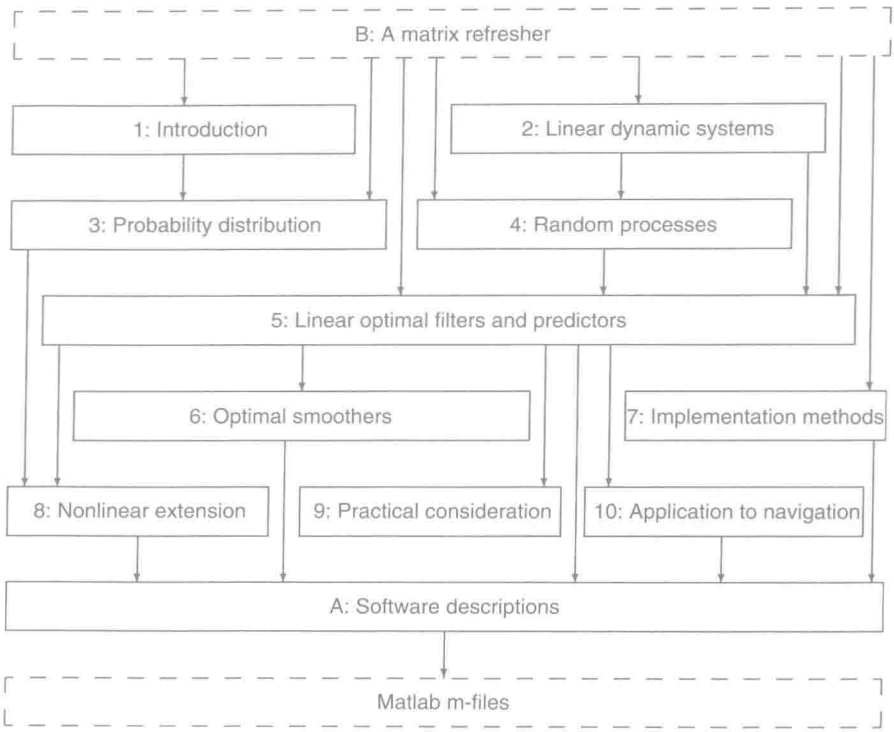
Chapter 5 covers linear optimal filters and predictors, with derivations of the Kalman gain and detailed examples of applications. Chapter 6 is a tutorial-level treatment of optimal smoothing methods based on Kalman filtering models, including more robust implementations. Chapter 7 covers the more recent implementation techniques for maintaining numerical accuracy, with algorithms provided for computer implementation.

Chapter 8 covers approximation methods used for nonlinear applications, including "extended" Kalman filters for "quasilinear" problems and tests for assessing whether extended Kalman filtering is adequate for the proposed application. We also present particle, sigma point, and the "unscented" Kalman filter implementation of Kalman filtering for problems failing the quasilinearity test. Applications of these techniques to the identification of unknown parameters of systems are given as examples. Chapter 9 deals with more practical matters of implementation and use beyond the numerical methods of Chapter 7. These matters include memory and throughput requirements (and methods to reduce them), divergence problems (and effective remedies), and practical approaches to suboptimal filtering and measurement selection.

As a demonstration of how to develop and evaluate applications of Kalman filtering, in Chapter 10, we show how to derive and implement different Kalman filtering configurations for Global Navigation Satellite System (GNSS) receivers and inertial navigation systems (INS) and for integrating GNSS receivers with INS.

Chapters 5–9 cover the essential material for a first-year graduate class in Kalman filtering theory and application or as a basic course in digital estimation theory and application.

The organization of the material is illustrated by the following chapter-level dependency graph, which shows how the subject of each chapter depends upon material in other chapters. The arrows in the figure indicate the recommended order of study. Boxes above another box and connected by arrows indicate that the material represented by the upper boxes is background material for the subject in the lower box. Dashed boxes indicate materials on the Wiley companion web site.



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ACKNOWLEDGEMENTS

The authors express their appreciation to the following individuals for their contributions during the preparation of the core material for this book: E. Richard Cohen, Thomas W. De Vries, Reverend Joseph Gaffney, Thomas L. Gunckel II, Dwayne Heckman, Robert A. Hubbs, Thomas Kailath, Rudolf E. Kalman, Alan J. Laub, Robert F. Nease, John C. Pinson, John M. Richardson, Jorma Rissanen, Gerald E. Runyon, Joseph Smith, and Donald F. Wiberg.

We also thank the following individuals for their review, corrections, and suggestions for improving the second and third editions: Dean Dang, Gordon Inverarity, and Kenneth W. Fertig.

For this fourth edition, we thank Jeffrey Uhlmann and Simon Julier for their assistance on the new material in Chapters 1 and 8, Andrey Podkorytov for his corrections to the Schmidt–Kalman filter, Professor Rudolf E. Kalman for the epigraph to Chapter 1, the late Robert W. Bass (1930–2013) for his corrections to Chapter 1, James Kain for proofreading parts of Chapter 7, John L. Weatherwax for his contributions to the problem set solutions, and Edward H. Martin for providing some early history on GNSS/INS integration.

Most of all, for their dedication, support, and understanding through all editions, we dedicate this book to Sonja Grewal and Jeri Andrews.

—M. S. G., A. P. A

LIST OF ABBREVIATIONS USED

ANSI, American National Standards Institute

arc-sec, second of arc

BMFLS, Biswas–Mahalanabis fixed-lag smoother

bps, bits per second

CEP, circular error probable, the radius of a circle centered at the mean of a probability distribution such that is equally likely that a random sample is inside or outside the circle (also called *circle of equal probability*)

CDMA, code-division multiple access (communications protocol)

dB, decibel

ed., editor or edition

EKF, extended Kalman filter

ENU, east-north-up (coordinates)

f, foot (0.3048 m)

FDMA, frequency-division multiple access (communications protocol)

flops, floating-point operations per second

FLS, fixed-lag smoother

FPS, fixed point smoother

g, 9.80665 m/s^2

gHz, gigahertz

GMLE, Gaussian maximum-likelihood estimator

GNSS, global navigation satellite system

GPS, Global Positioning Service, a GNSS operated by the US Department of Defense

h, hour
Hz, hertz (cycles per second)
IEEE, Institute of Electrical and Electronic Engineers
IEKF, iterated extended Kalman filter
IIR, infinite impulse response
INS, inertial navigation system
ISA, inertial sensor assembly
KF, Kalman filter
km, kilometer
kph, kilometer per hour
LGMLE, linear Gaussian maximum-likelihood estimator
LMSE, least-mean-square estimator
LQ, linear quadratic [estimator]
m, meter
MAP, maximum *a posteriori* probability (estimator).
max, maximum
mHz, megahertz
mi, mile
min, minute of time, or minimum
ML, maximum likelihood
MLE, maximum likelihood estimator
mph, mile per hour
NED, north-east-down (coordinates)
NMi, nautical mile (1852 m)
ppm, part per million
PSD, power spectral density
RMS, root mean squared
RP, random process
RPY, roll–pitch–yaw (vehicle coordinates)
RS, random sequence
RV, random variable
s, second of time
SKF, Schmidt–Kalman filter
SLRD, Schweppe likelihood-ration detection
SPKF, sigma-point Kalman filter
STM, state-transition matrix
SVD, singular value decomposition
UKF, unscented Kalman filter

UT, unscented transform

vs, versus

WSS, wide-sense stationary

μ**,** micrometer (10^{-6} m) or micro (10^{-6} [units])

CONTENTS

1

INTRODUCTION

Once you get the physics right, the rest is mathematics.

—Rudolf E. Kalman

Kailath Lecture, Stanford University, May 11, 2009

1.1 CHAPTER FOCUS

This chapter presents a preview of where we are heading, some history of how others got there before us, an overview showing how all the material fits together, and a common notation and nomenclature to make it more apparent.

1.2 ON KALMAN FILTERING

1.2.1 First of All: What Is a Kalman Filter?

Theoretically, it has been called the *linear least mean squares estimator* (LLSME) because it minimizes the mean-squared estimation error for a linear stochastic system using noisy linear sensors. It has also been called the *linear quadratic estimator* (LQE) because it minimizes a quadratic function of estimation error for a linear dynamic system with white measurement and disturbance noise. Even today, more than half a century after its discovery, it remains a unique accomplishment in

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the history of estimation theory. It is the only practical finite-dimensional solution to the real-time optimal estimation problem for stochastic systems, and it makes very few assumptions about the underlying probability distributions except that they have finite means and second central moments (covariances). Its mathematical model has been found to represent a phenomenal range of important applications involving noisy measurements for estimating the current conditions of dynamic systems with less-than-predictable disturbances. Although many approximation methods have been developed to extend its application to less-than-linear problems, and despite decades of dedicated research directed at generalizing it for nonlinear applications, no comparable general solution[1] for nonlinear problems has been found.

Practically, the Kalman filter is one of the great discoveries of *mathematical engineering*, which uses mathematical modeling to solve engineering problems—in the much same way that mathematical physics is used to solve physics problems, or computational mathematics is used for solving efficiency and accuracy problems in computer implementations.

Its early users would come to consider the Kalman filter to be the greatest discovery in practical estimation theory in the twentieth century, and its reputation has continued to grow over time. As an indication of its ubiquity, a *Google*®; web search for "Kalman filter" or "Kalman filtering" produces more than a million hits. One reason for this is that the Kalman filter has enabled human kind to do many things that could not have been done without it, and it has become as indispensable as silicon in the makeup of many electronic systems. Its most immediate applications have been for the monitoring and control of complex dynamic systems such as continuous manufacturing processes, aircraft, ships, or spacecraft. To control a dynamic system, you must first know what it is doing. For these applications, it is not always possible or desirable to measure every variable that you want to control, and the Kalman filter provides the mathematical framework for inferring the unmeasured variables from indirect and noisy measurements. The Kalman filter is also used for predicting the likely future courses of dynamic systems that people are not likely to control, such as the flow of rivers during flood, the trajectories of celestial bodies, or the prices of traded commodities and securities. It has become a universal tool for integrating different sensor and/or data collection systems into an overall optimal solution.

As an added bonus, the Kalman filter model can be used as a tool for assessing the relative accuracy of alternative sensor system designs for likely scenarios of dynamic system trajectories. Without this capability, development of many complex sensor systems (including Global Navigation Satellite Systems) may not have been possible.

From a practical standpoint, the following are the perspectives that this book will present:

1. *It is only a tool*. It does not solve any problem all by itself, although it can make it easier for you to do it. It is not a *physical* tool, but a *mathematical* one. Mathematical tools make mental work more efficient, just as mechanical tools make physical work more efficient. As with any tool, it is important to

[1] However, a somewhat limited finite-dimensional nonlinear solution has been found [1].