

MECHANICAL ENGINEERING AND SOLID MECHANICS SERIES



# Specification Development

*Mechanical Vibration and Shock Analysis*  
*Revised and Updated 3<sup>rd</sup> Edition*  
*Volume 5*

Christian Lalanne

ISTE

WILEY

Mechanical Vibration and Shock Analysis

*Third edition – Volume 5*

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## Foreword to Series

In the course of their lifetime simple items in everyday use such as mobile telephones, wristwatches, electronic components in cars or more specific items such as satellite equipment or flight systems in aircraft, can be subjected to various conditions of temperature and humidity, and more particularly to mechanical shock and vibrations, which form the subject of this work. They must therefore be designed in such a way that they can withstand the effects of the environmental conditions to which they are exposed without being damaged. Their design must be verified using a prototype or by calculations and/or significant laboratory testing.

Sizing, and later, testing are performed on the basis of specifications taken from national or international standards. The initial standards, drawn up in the 1940s, were blanket specifications, often extremely stringent, consisting of a sinusoidal vibration, the frequency of which was set to the resonance of the equipment. They were essentially designed to demonstrate a certain standard resistance of the equipment, with the implicit hypothesis that if the equipment survived the particular environment it would withstand, undamaged, the vibrations to which it would be subjected in service. Sometimes with a delay due to a certain conservatism, the evolution of these standards followed that of the testing facilities: the possibility of producing swept sine tests, the production of narrowband random vibrations swept over a wide range and finally the generation of wideband random vibrations. At the end of the 1970s, it was felt that there was a basic need to reduce the weight and cost of on-board equipment and to produce specifications closer to the real conditions of use. This evolution was taken into account between 1980 and 1985 concerning American standards (MIL-STD 810), French standards (GAM EG 13) or international standards (NATO), which all recommended the *tailoring of tests*. Current preference is to talk of the *tailoring of the product to its environment* in order to assert more clearly that the environment must be taken into account from the very start of the project, rather than to check the behavior of the material *a*

*posteriori*. These concepts, originating with the military, are currently being increasingly echoed in the civil field.

Tailoring is based on an analysis of the life profile of the equipment, on the measurement of the environmental conditions associated with each condition of use and on the synthesis of all the data into a simple specification, which should be of the same severity as the actual environment.

This approach presupposes a proper understanding of the mechanical systems subjected to dynamic loads and knowledge of the most frequent failure modes.

Generally speaking, a good assessment of the stresses in a system subjected to vibration is possible only on the basis of a finite element model and relatively complex calculations. Such calculations can only be undertaken at a relatively advanced stage of the project once the structure has been sufficiently defined for such a model to be established.

Considerable work on the environment must be performed independently of the equipment concerned either at the very beginning of the project, at a time where there are no drawings available, or at the qualification stage, in order to define the test conditions.

In the absence of a precise and validated model of the structure, the simplest possible mechanical system is frequently used consisting of mass, stiffness and damping (a linear system with one degree of freedom), especially for:

- the comparison of the severity of several shocks (shock response spectrum) or of several vibrations (extreme response and fatigue damage spectra);
- the drafting of specifications: determining a vibration which produces the same effects on the model as the real environment, with the underlying hypothesis that the equivalent value will remain valid on the real, more complex structure;
- the calculations for pre-sizing at the start of the project;
- the establishment of rules for analysis of the vibrations (choice of the number of calculation points of a power spectral density) or for the definition of the tests (choice of the sweep rate of a swept sine test).

This explains the importance given to this simple model in this work of five volumes on “Mechanical Vibration and Shock Analysis”.

Volume 1 of this series is devoted to *sinusoidal vibration*. After several reminders about the main vibratory environments which can affect materials during their working life and also about the methods used to take them into account,

following several fundamental mechanical concepts, the responses (relative and absolute) of a mechanical one-degree-of-freedom system to an arbitrary excitation are considered, and its transfer function in various forms are defined. By placing the properties of sinusoidal vibrations in the contexts of the real environment and of laboratory tests, the transitory and steady-state response of a single-degree-of-freedom system with viscous and then with non-linear damping is evolved. The various sinusoidal modes of sweeping with their properties are described, and then, starting from the response of a one-degree-of-freedom system, the consequences of an unsuitable choice of sweep rate are shown and a rule for choice of this rate is deduced from it.

Volume 2 deals with *mechanical shock*. This volume presents the shock response spectrum (SRS) with its different definitions, its properties and the precautions to be taken in calculating it. The shock shapes most widely used with the usual test facilities are presented with their characteristics, with indications how to establish test specifications of the same severity as the real, measured environment. A demonstration is then given on how these specifications can be made with classic laboratory equipment: shock machines, electrodynamic exciters driven by a time signal or by a response spectrum, indicating the limits, advantages and disadvantages of each solution.

Volume 3 examines the analysis of *random vibration* which encompasses the vast majority of the vibrations encountered in the real environment. This volume describes the properties of the process, enabling simplification of the analysis, before presenting the analysis of the signal in the frequency domain. The definition of the power spectral density is reviewed, as well as the precautions to be taken in calculating it, together with the processes used to improve results (windowing, overlapping). A complementary third approach consists of analyzing the statistical properties of the time signal. In particular, this study makes it possible to determine the distribution law of the maxima of a random Gaussian signal and to simplify the calculations of fatigue damage by avoiding direct counting of the peaks (Volumes 4 and 5). The relationships that provide the response of a one-degree-of-freedom linear system to a random vibration are established.

Volume 4 is devoted to the calculation of *damage fatigue*. It presents the hypotheses adopted to describe the behavior of a material subjected to fatigue, the laws of damage accumulation and the methods for counting the peaks of the response (used to establish a histogram when it is impossible to use the probability density of the peaks obtained with a Gaussian signal). The expressions of mean damage and its standard deviation are established. A few cases are then examined using other hypotheses (mean not equal to zero, taking account of the fatigue limit, non-linear accumulation law, etc.). The main laws governing low cycle fatigue and fracture mechanics are also presented.

Volume 5 is dedicated to presenting the method of *specification development* according to the principle of tailoring. The extreme response and fatigue damage spectra are defined for each type of stress (sinusoidal vibrations, swept sine, shocks, random vibrations, etc.). The process for establishing a specification as from the lifecycle profile of the equipment is then detailed taking into account the uncertainty factor (uncertainties related to the dispersion of the real environment and of the mechanical strength) and the test factor (function of the number of tests performed to demonstrate the resistance of the equipment).

First and foremost, this work is intended for engineers and technicians working in design teams responsible for sizing equipment, for project teams given the task of writing the various sizing and testing specifications (validation, qualification, certification, etc.) and for laboratories in charge of defining the tests and their performance following the choice of the most suitable simulation means.

## Introduction

For many years mechanical environmental specifications have been taken directly from written standards, and this is often still the case today. The values proposed in such documents were determined years ago on the basis of measurements performed on vehicles which are now obsolete. They were transformed into test standards with very wide margins, and were adapted to the constraints of the testing facilities available at the time. A considerable number of tests taking the form of a swept sine vibration can therefore be found. These standards were designed more to verify resistance to the greatest stresses than to demonstrate resistance to fatigue. Generally speaking, the values proposed were extremely severe, resulting in the over-sizing of equipment.

Since the early 1980s, some of those standards (MIL-STD 810, GAM T13) have been upgraded, providing for drafting specifications on the basis of measurements taken under conditions in which the equipment is used. This approach presupposes an analysis of the lifecycle profile of the equipment, by stipulating the various conditions of use (storage, handling, transport facilities, interfaces, durations, etc), and then relating characteristic measurements of the environment to each of the situations identified.

In this volume of the series, a method for the synopsis of the collated data into specifications is presented. The equivalence criteria adopted are a reproduction of greatest stress and fatigue damage. This equivalence is obtained not from the responses of a real structure subjected to vibration, given that such a structure is unknown at the time of drafting specifications, but from the study of a single degree-of-freedom linear reference system. These criteria result in two types of spectra: extreme response spectra (ERS), similar to the older shock response spectra; and fatigue damage spectra (FDS).



Calculation of ERS is presented in Chapter 1 for sinusoidal vibrations (sine and swept sine) and in Chapter 2 for a random vibration.

Chapters 3 to 5 are devoted respectively to calculating the FDS of sinusoidal and random vibrations and of shocks. Chapter 6 shows that ERSs and FDSs are insensitive to the choice of parameters necessary for their calculation.

Specifications may vary considerably, depending on the objectives sought. In Chapter 7 the main types of tests performed are recalled and, after a brief historical recapitulation, the current trend which recommends tailoring and taking into account the environment from the very beginning of the project is outlined.

Results of environmental measurements generally show a scattered pattern, due to the random nature of the phenomena. Moreover, it is well known that the resistance of parts obeys a statistical law and can therefore be described only by a mean value and a standard deviation. The stress–strength comparison can therefore only be drawn by the combination of two statistical laws, which results, when they are known, in a probability of failure, solely dependent, all other things being equal, on the ratio of the means of the two laws. For shocks and vibrations measured during an accident (environmental conditions which are not normal), ratio ( $k$ ) is called the *uncertainty factor* or *safety factor* (Chapter 8).

In practice, the environment with its laws of distribution can be known, but the resistance of the equipment remains as yet unknown. Specifications give the values of the environment to be met, with a maximum tolerated probability of failure. The purpose of the test will therefore be to demonstrate the observance of that probability, namely that the mean resistance is at least equal to  $k$  times the mean environment. For understandable reasons of cost, only a very limited number of tests are performed, frequently only one. This small number simply makes it possible to demonstrate that the mean of the strength is in an interval centered on the level of the test, with a width dependent upon the level of confidence adopted and on the number of tests. To be sure that the mean, irrespective of its real position in the interval, is indeed higher than the required value, the tests must be performed to a greater degree of severity, something which is achieved by the application of a coefficient called the *test factor* (Chapter 10).

Certain items of equipment are used only after a long period in storage, during which their mechanical characteristics may have weakened through aging. For the probability of proper operability to be that which is required after such aging, much more must therefore be required of the equipment when new, at the time of its qualification, resulting in the application of another coefficient, called the *aging factor* (Chapter 9).

These spectra and factors form the basis of the method for drafting *tailored specifications* in four steps, as described in Chapter 11: establishment of the lifecycle profile of the equipment, description of the environment (vibrations, shocks, etc.) for each situation (transport, handling, etc.), synopsis of the data thus collated, and establishment of the testing program.

The sensitivity of specifications developed with the method of equivalence of damages based on the different calculation parameters is studied in Chapter 12.

Chapter 13 provides a few other possible applications of the ERS and FDS, such as the comparison of the different types of vibrations (sinusoidal, random stationary or not, sine on random, shocks, etc.), the comparison between different standards or between standards and real environmental measures, the transformation of a large number of shocks into a specification of random vibration with similar severity, etc.

The Appendices show that the development method of specifications using ERSs and FDSs adds no additional hypothesis in relation to the PSD envelope method, which can lead to specifications which are too high in relation to the real environment if it is used without precaution. Contrary to this last method, the equivalence method of damages makes it possible to easily process more difficult cases, such as, for example, non-stationary vibrations, the establishment of a specification covering different types of vibrations with different application durations.

At the end of the book, a list of formulae combines the major relations established in the five volumes.

## List of Symbols

The list below gives the most frequent definition of the main symbols used in this work. Some of the symbols can have another meaning locally, this will be defined in the text to avoid any confusion.

a	Threshold value of $z(t)$	$F_m$	Maximum value of $F(t)$
aerf	Inverse error function	$f(t)$	Frequency sweeping law
b	Parameter b of Basquin relationship $N \sigma^b = C$	$F(t)$	External force applied to a system
c	Viscous damping constant	g	Acceleration due to gravity
C	Basquin relationship constant ( $N \sigma^b = C$ )	$G(\ )$	Power spectral density for $0 \leq f \leq \infty$
CoV <sub>m</sub>	Coefficient of variation of mean distribution	h	Interval ( $f/f_0$ )
D	Fatigue damage	H	Drop height
e	Error	$H(\ )$	Transfer function
E	Exaggeration factor	i	$\sqrt{-1}$
$\bar{E}$	Mean of the environment	$I_0(\ )$	Zero order Bessel function
$E_E$	Expected environment	J	Damping constant
$E_S$	Selected environment	k	Stiffness or uncertainty factor
erf	Error function	$k_v$	Aging coefficient
ERS	Extreme response spectrum	K	Constant of proportionality between stress and deformation
$E(\ )$	Expectation of ...	$\ell_{rms}$	Rms value of $\ell(t)$
f	Frequency of excitation		
$f_M$	Mean frequency		
$f_0$	Natural frequency		
FDS	Fatigue response spectrum		

$\ell_m$	Maximum value of $\ell(t)$		during time T
$\ell(t)$	Generalized excitation (displacement)	$P(\ )$	Distribution function
$\dot{\ell}(\ )$	First derivative of $\ell(t)$	$q$	$\sqrt{1-r^2}$
$\ddot{\ell}(t)$	Second derivative of $\ell(t)$	$q(u)$	Probability density of maxima
m	Mass	Q	Q factor (quality factor)
MRS	Maximum response spectrum	$Q(u)$	Probability that a maximum is higher than a given threshold
$M_n$	Moment of order $n$	r	Irregularity factor
n	Number of cycles undergone by test-bar or material or Number of measurements or Number of tests	rms	Root mean square (value)
$n_a^+$	Mean number of up-crossings of threshold a with positive slope per second	R	Extreme response spectrum
$n_0^+$	Mean number of zero-crossings with positive slope per second (mean frequency)	$\bar{R}$	Mean of strength
$n_p^+$	Mean number of maxima per second	$R_e$	Yield stress
N	Number of cycles to failure or Mean number of envelope maxima per second or Number of peaks higher than a given threshold	$R_m$	Ultimate tensile strength
$N_a^+$	Mean number of positive maxima higher than a given threshold for a given duration	$R_U$	Response spectrum with given up-crossing risk
$P_v$	Probability of correct operation related to aging	s	Standard deviation
PSD	Power spectrum density	$s_E$	Standard deviation of environment
$p(\ )$	Probability density	$s_R$	Standard deviation of resistance
$p(T)$	Probability density of first passage of a threshold	SRS	Shock Response Spectrum
		t	Time or Random variable of Student distribution law
		$t_s$	Sweeping duration
		T	Duration of vibration
		$T_F$	Test factor
		$T_l$	Time-constant of logarithmic frequency sweep
		TS	Test severity
		u	Ratio of threshold a to rms value $z_{rms}$ of $z(t)$ or value of $u(t)$
		$u_{rms}$	rms value of $u(t)$

$u_m$	Maximum value of $u(t)$	$z_{\text{sup}}$	The largest value of $z(t)$
$u_0$	Threshold value of $u(t)$	$\dot{z}_{\text{rms}}$	rms value of $\dot{z}(t)$
URS	Up-crossing risk Response Spectrum	$\ddot{z}_{\text{rms}}$	rms value of $\ddot{z}(t)$
$u(t)$	Generalized response	$z(t)$	Relative response displacement of the mass of a single degree-of- freedom system with respect to its base
$v_i$	Impact velocity	$\dot{z}(t)$	Relative response velocity
$V_E$	Variation coefficient of real environment	$\ddot{z}(t)$	Relative response acceleration
$V_R$	Variation coefficient of strength of the material	$\alpha$	Risk of up-crossing
$x_m$	Maximum value of $x(t)$	$\delta$	Non-centrality parameter of the non-central t-distribution
$x(t)$	Absolute displacement of the base of a single degree- of-freedom system	$\Delta f$	Frequency interval between half-power points
$\dot{x}(t)$	Absolute velocity of base of a single degree-of- freedom system	$\Delta N$	Number of cycles carried out between half-power points
$\ddot{x}(t)$	Absolute acceleration of the base of a single degree- of-freedom system	$\Delta t$	Time spent between half- power points
$\ddot{x}_{\text{rms}}$	rms value of $\ddot{x}(t)$	$\Delta V$	Velocity change
$\ddot{x}_m$	Maximum value of $\ddot{x}(t)$	$\varepsilon$	Euler's constant (0.577 215 662 ...)
$\ddot{y}_{\text{rms}}$	rms value of $\ddot{y}(t)$	$\gamma(t)$	Incomplete gamma function
$\ddot{y}(t)$	Absolute response acceleration of the mass of a one-degree-of-freedom system	$\Gamma( )$	Gamma function
$z_{\text{rms}}$	rms value of $z(t)$	$\eta$	Dissipation (or loss) coefficient
$z_{a\text{rms}}$	rms value of the response to a random vibration	$\varphi$	Phase
$z_m$	Maximum value of $z(t)$	$\pi$	3.141 592 65 ...
$z_p$	Peak value of $z(t)$	$\pi_0$	Confidence level
$z_s$	Relative response displacement to a sinusoidal vibration	$\sigma$	Stress
$z_{s\text{rms}}$	rms value of the response to a sinusoidal vibration	$\sigma_a$	Alternating stress
		$\sigma_D$	Fatigue limit stress
		$\sigma_m$	Mean stress
		$\sigma_{\text{rms}}$	Rms stress value
		$\sigma_{\text{max}}$	Maximum stress
		$\omega_0$	Natural pulsation ( $2 \pi f_0$ )

$\Omega$	Pulsation of excitation ( $2 \pi f$ )		$\xi$	Damping factor
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