Solid Mechanics and Its Applications

Johan Blaauwendraad
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# Structural Shell Analysis

## Understanding and Application

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ISSN 0925-0042
ISBN 978-94-007-6700-3 ISBN 978-94-007-6701-0 (eBook)
DOI 10.1007/978-94-007-6701-0
Springer Dordrecht Heidelberg New York London

Library of Congress Control Number: 2013938211

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Printed on acid-free paper

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# Solid Mechanics and Its Applications

## Volume 200

*Series Editor*

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## Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why*?, *How*?, and *How much*? The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies: vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

# Preface

The structural and architectural potential of shell structures is used in various fields of civil, architectural, mechanical, aeronautical, and marine engineering. The strength of a (doubly) curved structure is efficiently and economically used, for example, to cover large areas without supporting columns. In addition to the mechanical advantages, the use of shell structures leads to esthetic architectural appearance. Examples of shells used in civil and architectural engineering include shell roofs, liquid storage tanks, silos, cooling towers, containment shells of nuclear power plants, and arch dams. Piping systems, curved panels, pressure vessels, bottles, buckets, and parts of cars, are examples of shells used in mechanical engineering. In aeronautical and marine engineering, shells are used in aircraft, spacecraft, missiles, ships, and submarines.

Similar to plate structures, one dimension of shell structures is small compared to the others. However, because of their spatial shape, the behavior of shells is different from that of plates. In flat plates, external loads are carried either by membrane response or bending response. In shells, the loads are carried by both. Similarly, both extensions and changes of curvature occur. As a result a mathematical description of the properties of a shell is much more elaborate than for beam and plate structures. Therefore many engineers and architects are unacquainted with aspects of shell behavior and design.

It took tens of years in the twentieth century to achieve sufficiently reliable shell theories for the different shell types that occur. Some of the most famous names in this respect are Love, Reissner, Wlassow, Morley, Flügge, Novoshilov, Koiter, Donnell, and Niordson. Well-known textbooks on the subject have also been published by Plüger, Rüdiger, Timoshenko, and Wolmir. Rather than contributing to theory development, this book is a university textbook, with a focus on architectural and civil engineering schools. Of course, practising professionals will profit from it as well. In writing this book we had three aims: (i) providing insight into the behavior of shell structures, (ii) explaining applied shell theories, and (iii) applying numerical programs for design purposes.

The book deals only with thin elastic shells, in particular with cylindrical, conical and spherical shells, and elliptic and hyperbolic paraboloids. The focus is on roofs, chimneys, pressure vessels, and storage tanks. The reader is supposed to be acquainted with the theory of flat plates loaded in-plane (shear walls, etc.) and loaded laterally (slabs, etc.). Material nonlinearity is not considered, and the deformation due to transverse shear is not taken into account. Geometric nonlinearity is considered only in an introductory chapter on buckling of thin shells. A substantial part of the book is derived from research efforts in the middle of the twentieth century at the Civil Engineering Department of Delft University of Technology by Bouma, Loof, and Van Koten. As such, we offer an addition to the archive of literature dealing with developments in shell research that are of continuing importance. Newer parts of the book come from doctoral thesis work of Hoefakker under supervision of Blaauwendraad [18].

The triple aim of the book is realized in the following way. We explain the theory of shells for a number of shell types. We show structural designers how to perform a manual calculation of the main force flow in a shell structure. We teach them how to estimate the stresses and the deformations. Special attention is paid to the characterization of edge bending effects. This is of prominent importance for mesh design in edge zones in case the structural designer performs a Finite Element Analysis.

# Acknowledgments

Donnell's theory of shallow shells was the basis of extensive shell research in The Netherlands in the fifties and sixties of the 20th century. A team of people in the Civil Engineering Faculty of Delft University and the TNO research organisation for Building Research elaborated the theory, including work of Von Kármán and Jenkins. We acknowledge Professor A.L. Bouma, the late Associate Professor H.W. Loof and Mr. H. van Koten for their intensive academic effort to make the theory accessible for design purposes. A large part of the book has been adapted from their lecture notes on shell analysis in past decades, particularly the chapters on roof structures. We are indebted to Professor L.J. Sluys, editor-in-chief of HERON, joint publication of TNO, Delft University of Technology and Eindhoven University of Technology in The Netherlands. He permitted copying of tables and figures from bygone volumes. The extension on chimneys and tanks, on the basis of the Morley-Koiter theory, is more recent work of the authors, reflecting lectures on shell theory and doctoral thesis work. The persevering support of software virtuoso and structural engineer M.J. de Rijke in performing complex FEanalyses is highly rewarded. We warmly thank Springer ladies Nathalie Jacobs and Cynthia Feenstra for dedicated promotion and assistance respectively.

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**Part II Roof Structures**

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# Chapter 1
# Introduction to Shells

This book is concerned with thin elastic shells. A *thin shell* has a very small thickness-to-minimal-radius ratio, often smaller than 1/50. As with plates, an applied load that acts out-of-plane leads to larger displacements than those generated by a load acting in-plane with the same intensity. Due to its initial curvature, a shell is able to transfer an applied load by in-plane as well as out-of-plane actions. A thin shell subjected to an applied load therefore produces mainly in-plane actions, which are called *membrane forces*. These membrane forces are actually resultants of normal stresses and in-plane shear stresses that are uniformly distributed across the thickness.

## 1.1 Shell Theories

A shell is a generalization of an isotropic homogeneous plate. Plates are flat structures of which the dimensions in two in-plane directions are large compared to the third direction perpendicular to the plate. The span in two directions is much larger than the thickness. Plates are defined by their *middle plane*, *thickness* and *material properties*. The *displacements* of the middle plane play the role of degrees of freedom in structural modelling. In-plane loads of plates generate in-plane membrane forces, and out-of-plane loads generate moments and transverse shear forces.

Shells are also defined by their *middle plane*, *thickness* and *material properties*. The difference with plates is that the middle plane of plates is flat, and that it is curved in shells. As a consequence, shells can carry out-of-plane loads by in-plane membrane forces, which is not possible for plates. In fact, this is the major reason why shells are such strong and economic structures.

The theory of this membrane behaviour is called *membrane theory*. However, membrane theory does not satisfy all equilibrium and/or displacement requirements of all cases. For example (Fig. 1.1):

J. Blaauwendraad and J. H. Hoefakker, *Structural Shell Analysis*,
Solid Mechanics and Its Applications 200, DOI: 10.1007/978-94-007-6701-0_1,

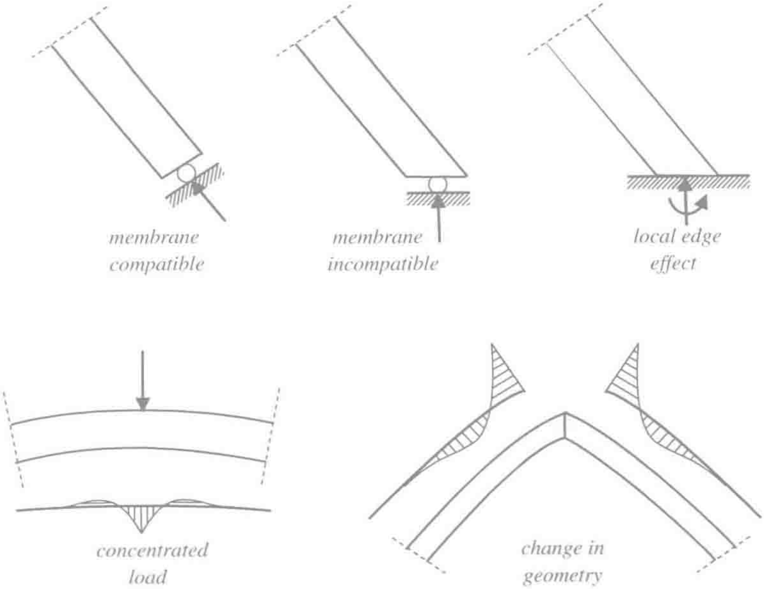

**Fig. 1.1** Membrane and bending conditions

- Boundary conditions and deformation constraints that are incompatible with the requirements of a pure membrane field;
- Concentrated loads;
- Changes in geometry.

In the regions where the membrane theory will not hold, some (or all) of the bending field components are produced to compensate the shortcomings of the membrane field in the disturbed zone. These disturbances have to be described by a more complete analysis, which will lead to a *bending theory* of thin elastic shells.

If the bending components occur, they often have a local range of influence. Theoretical calculations and experiments show that the required bending components attenuate and often bending is confined to boundaries where a pure membrane solution does not exist. Therefore, in many cases the bending behaviour is restricted to an *edge disturbance*. The undisturbed and major part of the shell behaves like a true membrane. This unique property of shells is a result of the curvature of the spatial structure. Efficient structural performance is responsible for the widespread appearance of shells in nature. The continuous progress of numerical methods for computational mechanics, combined with an efficient structural performance and a pleasing shape, makes the application of shell structures more and more possible and favourable. However, for the use of numerical programs, some basic knowledge of the underlying theories and the mechanical behaviour of the structure is needed.