

Solid Mechanics and Its Applications

Johan Blaauwendraad
Jeroen H. Hoefakker

Structural Shell Analysis

Understanding and Application

 Springer

Johan Blaauwendraad · Jeroen H. Hoefakker

Structural Shell Analysis

Understanding and Application

 Springer

Johan Blaauwendraad
Emeritus Professor
Delft University of Technology
Delft
The Netherlands

Jeroen H. Hoefakker
Former Lecturer
Delft University of Technology
Delft
The Netherlands

ISSN 0925-0042

ISBN 978-94-007-6700-3

ISBN 978-94-007-6701-0 (eBook)

DOI 10.1007/978-94-007-6701-0

Springer Dordrecht Heidelberg New York London

Library of Congress Control Number: 2013938211

© Springer Science+Business Media Dordrecht 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law. The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Solid Mechanics and Its Applications

Volume 200

Series Editor

G. M. L. Gladwell

Department of Civil Engineering, University of Waterloo, Waterloo, Canada

For further volumes:

<http://www.springer.com/series/6557>

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?*, *How?*, and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

Preface

The structural and architectural potential of shell structures is used in various fields of civil, architectural, mechanical, aeronautical, and marine engineering. The strength of a (doubly) curved structure is efficiently and economically used, for example, to cover large areas without supporting columns. In addition to the mechanical advantages, the use of shell structures leads to esthetic architectural appearance. Examples of shells used in civil and architectural engineering include shell roofs, liquid storage tanks, silos, cooling towers, containment shells of nuclear power plants, and arch dams. Piping systems, curved panels, pressure vessels, bottles, buckets, and parts of cars, are examples of shells used in mechanical engineering. In aeronautical and marine engineering, shells are used in aircraft, spacecraft, missiles, ships, and submarines.

Similar to plate structures, one dimension of shell structures is small compared to the others. However, because of their spatial shape, the behavior of shells is different from that of plates. In flat plates, external loads are carried either by membrane response or bending response. In shells, the loads are carried by both. Similarly, both extensions and changes of curvature occur. As a result a mathematical description of the properties of a shell is much more elaborate than for beam and plate structures. Therefore many engineers and architects are unacquainted with aspects of shell behavior and design.

It took tens of years in the twentieth century to achieve sufficiently reliable shell theories for the different shell types that occur. Some of the most famous names in this respect are Love, Reissner, Wlassow, Morley, Flügge, Novoshilov, Koiter, Donnell, and Niordson. Well-known textbooks on the subject have also been published by Plüger, Rüdiger, Timoshenko, and Wolmir. Rather than contributing to theory development, this book is a university textbook, with a focus on architectural and civil engineering schools. Of course, practising professionals will profit from it as well. In writing this book we had three aims: (i) providing insight into the behavior of shell structures, (ii) explaining applied shell theories, and (iii) applying numerical programs for design purposes.

The book deals only with thin elastic shells, in particular with cylindrical, conical and spherical shells, and elliptic and hyperbolic paraboloids. The focus is on roofs, chimneys, pressure vessels, and storage tanks. The reader is supposed to be acquainted with the theory of flat plates loaded in-plane (shear walls, etc.) and

loaded laterally (slabs, etc.). Material nonlinearity is not considered, and the deformation due to transverse shear is not taken into account. Geometric nonlinearity is considered only in an introductory chapter on buckling of thin shells. A substantial part of the book is derived from research efforts in the middle of the twentieth century at the Civil Engineering Department of Delft University of Technology by Bouma, Loof, and Van Koten. As such, we offer an addition to the archive of literature dealing with developments in shell research that are of continuing importance. Newer parts of the book come from doctoral thesis work of Hoefakker under supervision of Blaauwendraad [18].

The triple aim of the book is realized in the following way. We explain the theory of shells for a number of shell types. We show structural designers how to perform a manual calculation of the main force flow in a shell structure. We teach them how to estimate the stresses and the deformations. Special attention is paid to the characterization of edge bending effects. This is of prominent importance for mesh design in edge zones in case the structural designer performs a Finite Element Analysis.

Acknowledgments

Donnell's theory of shallow shells was the basis of extensive shell research in The Netherlands in the fifties and sixties of the 20th century. A team of people in the Civil Engineering Faculty of Delft University and the TNO research organisation for Building Research elaborated the theory, including work of Von Kármán and Jenkins. We acknowledge Professor A.L. Bouma, the late Associate Professor H.W. Loof and Mr. H. van Koten for their intensive academic effort to make the theory accessible for design purposes. A large part of the book has been adapted from their lecture notes on shell analysis in past decades, particularly the chapters on roof structures. We are indebted to Professor L.J. Sluys, editor-in-chief of HERON, joint publication of TNO, Delft University of Technology and Eindhoven University of Technology in The Netherlands. He permitted copying of tables and figures from bygone volumes. The extension on chimneys and tanks, on the basis of the Morley-Koiter theory, is more recent work of the authors, reflecting lectures on shell theory and doctoral thesis work. The persevering support of software virtuoso and structural engineer M.J. de Rijke in performing complex FEanalyses is highly rewarded. We warmly thank Springer ladies Nathalie Jacobs and Cynthia Feenstra for dedicated promotion and assistance respectively.

Contents

1	Introduction to Shells	1
1.1	Shell Theories	1
1.2	Classification of Shells	3
1.2.1	Gaussian Curvature of a Surface.	3
1.2.2	Developed and Undeveloped Surfaces.	4
1.2.3	Generated Surfaces	5
1.2.4	Combined Surfaces	6
1.2.5	Folded Plates	6
1.3	Analytical Description of the Shell Surface	7
1.3.1	Circular Plan	7
1.3.2	Rectangular Plan.	9
Part I Membrane Theory and Edge Disturbances		
2	Membrane Theory for Shells with Principal Curvatures	13
2.1	Kinematic Relation	15
2.2	Constitutive Relation.	16
2.3	Equilibrium Relation.	17
3	Membrane Theory for Thin Shells of Arbitrary Curvatures.	21
3.1	Kinematic Relations	21
3.2	Constitutive Equation	25
3.3	Equilibrium Equation	25
3.3.1	Effect of Curvatures	25
3.3.2	Effect of Twist.	27
4	Application of Membrane Theory to Circular Cylindrical Shells	29
4.1	Description of the Circular Cylindrical Surface	29
4.2	Kinematic Relation	30
4.3	Constitutive Relation.	31
4.4	Equilibrium Relation.	31

4.5	General Solution.	31
4.6	Applications of Circular Cylindrical Shells as Beam.	32
	4.6.1 Simply Supported Tube Beam with Diaphragms.	35
	4.6.2 Circular Shell as a Cantilever Beam	39
4.7	Circular Beam Under a Transverse Load	40
	4.7.1 Solution in Beam Theory.	40
	4.7.2 Comparison of Beam Solution with Shell Membrane Solution.	43
4.8	In-Extensional Deformation of a Circular Storage Tank	46
4.9	Circular Shell Under an Axisymmetric Load	48
	4.9.1 Application to Water Tank.	49
4.10	Concluding Remarks.	50
5	Edge Disturbance in Circular Cylindrical Shells Under Axisymmetric Load	51
5.1	Problem Assignment	51
5.2	Derivation of a Differential Equation	52
5.3	Application to a Water Tank	55
5.4	Solution for a Long Shell Subject to Edge Loads	59
	5.4.1 Edge Force	60
	5.4.2 Edge Torque	61
	5.4.3 Edge Force and Torque	62
5.5	Reconsidering the Water Tank by Force Method	63
5.6	Four Elementary Cases	64
	5.6.1 Elementary Case A.	64
	5.6.2 Elementary Case B.	66
	5.6.3 Elementary Case C.	69
	5.6.4 Elementary Case D.	69
5.7	Concluding Remarks.	69
	Reference	70

Part II Roof Structures

6	Donnell Bending Theory for Shallow Shells.	73
6.1	Introduction	73
6.2	Kinematic Relation	75
6.3	Constitutive Relation.	76
6.4	Equilibrium Relation.	77
6.5	Differential Equation for One Displacement.	77
	6.5.1 In-Plane State.	78
	6.5.2 Out-of-Plane State	79
	6.5.3 Coupled States	80
6.6	Boundary Conditions.	81
	Reference	82

7	Circular Cylindrical Roof	83
7.1	Introduction	83
7.2	Differential Equation for Circular Cylinder	84
7.3	Boundary Conditions at a Straight Edge	85
7.4	Expressions for Shell Forces and Displacements.	86
7.5	Homogeneous Solution for a Straight Edge	87
7.5.1	Exact Solution	87
7.5.2	Approximate Solution	90
7.6	Displacements and Shell Forces of the Homogeneous Solution.	91
7.7	Application to a Shell Roof Under its Own Weight	93
7.7.1	Uniform Load	96
7.7.2	Vertical Load.	98
7.7.3	Comparison of Solutions for a Concrete Roof	101
7.8	Circular Shell Roof Compared with Beam Theory	103
	References	106
8	Hyperbolic- and Elliptic-Paraboloid Roofs	107
8.1	Geometry of the Hyppar Surface with Straight Edges	107
8.2	Set of Relations for Hyppar with Straight Edges	109
8.2.1	Kinematic Relation	109
8.2.2	Constitutive Relation.	110
8.2.3	Equilibrium Relation.	110
8.3	Membrane Solution for a Uniform Load on Hyppar with Straight Edges.	111
8.3.1	Concluding Remarks About the Membrane Solution.	112
8.4	Bending of Hyppar with Straight Edges.	113
8.4.1	Differential Equation.	113
8.4.2	Approximate Bending Solution for Hyppar with Straight Edges.	114
8.4.3	Edge Disturbances for Uniform Load	116
8.5	Hyppar Roof Examples	118
8.5.1	Single Hyppar on Two Supports.	118
8.5.2	Composed Hyppar Roofs.	121
8.6	Elpars and Hyppars with Curved Edges.	125
8.6.1	Doubly Curved Shells Supported on Two Opposite Edges.	126
8.6.2	Doubly Curved Shells Supported Along All Edges.	128
	References	128

Part III Chimneys and Storage Tanks

9	Morley Bending Theory for Circular Cylindrical Shells	131
9.1	Introduction	131
9.1.1	Leading Term in Differential Equations.	132
9.1.2	Geometrical Considerations	132
9.1.3	Load Considerations	133
9.1.4	Three Load-Deformation Behaviours.	133
9.2	Sets of Equations	135
9.2.1	Kinematical Relation.	135
9.2.2	Constitutive Relation.	136
9.2.3	Equilibrium Relation.	137
9.2.4	Boundary Conditions.	137
9.3	Differential Equations for Load p_z	138
9.3.1	Differential Equations for Displacements.	138
9.3.2	Single Differential Equation.	140
9.4	Homogeneous Solution of the Differential Equation for a Curved Edge	141
9.4.1	Exact Solution	141
9.4.2	Approximate Solution	143
9.5	Influence Length.	144
9.5.1	Axisymmetric Mode	144
9.5.2	Beam Mode	145
9.5.3	Self-Balancing Modes	145
9.6	Displacements and Shell Forces of the Homogeneous Solution for Self-Balancing Modes	146
9.6.1	Comparison with Donnell Solution	149
9.7	Inhomogeneous Solution for Self-Balancing Modes	149
9.8	Complete Solution for Self-Balancing Modes.	158
9.9	Complete Solution for the Axisymmetric Load and Beam Load	158
9.9.1	Axisymmetric Mode	158
9.9.2	Beam Mode	159
	References	160
10	Semi-Membrane Concept Theory for Circular Cylindrical Shells	161
10.1	Introduction	161
10.2	Sets of Equations	162
10.3	Differential Equations for Load p_z	164
10.3.1	The Differential Equations for Displacements	164
10.3.2	The Single Differential Equation	165
10.4	Homogeneous Solution of the Differential Equation for a Curved Edge	165

10.5	Influence Length.	167
10.6	Displacements and Shell Forces of the Homogeneous Solution for the Self-Balancing Modes	167
10.7	Inhomogeneous Solution	170
10.8	Complete Solution	170
10.9	Remark Considering Accuracy	171
	Reference	172
11	Analysis by Circular Cylindrical Super Elements	173
11.1	Introduction of Super Element Analysis	173
11.2	Outline of Super Element Analysis.	174
	11.2.1 Consideration of Super Element Level	174
	11.2.2 Load on Circular Node	178
	11.2.3 Assembling and Solving Procedure.	178
11.3	Calculation Scheme.	179
11.4	Features of the Program CShell	179
	11.4.1 Structure, Supports and Loading.	179
	11.4.2 Shell Theory to be Chosen.	181
	11.4.3 Ring Elements	181
	11.4.4 Verification	182
	11.4.5 Output.	182
11.5	Overview of the Analysed Structures	182
12	Chimneys.	183
12.1	Wind Load.	183
12.2	Fixed Base: Free End	184
	12.2.1 Closed-Form Solution	184
	12.2.2 Applicability Range of Formulas	193
12.3	SMC Approximation.	194
12.4	Effect of Ring Stiffeners	196
	12.4.1 Closed-Form Solution (SMC).	196
	12.4.2 Applicability Range of Formulas	199
12.5	Effect of Elastic Supports	200
	12.5.1 Derivation of Formulas	201
	12.5.2 Applicability Range of Formulas	205
12.6	Summary of Chimney Design Formulas	207
	12.6.1 Design Formula for Chimneys with Rigid Base	208
	12.6.2 Design Formula for Chimney with Stiffening Rings	208
	12.6.3 Design Formula for Chimneys with Elastic Supports	209

13 Storage Tanks	211
13.1 Problem Statement	211
13.2 Load-Deformation Conditions and Analysed Cases	212
13.3 Stresses Due to Content.	213
13.3.1 Concrete Tank	213
13.3.2 Steel Tank	213
13.4 Stresses Due to Wind Load	215
13.4.1 Concrete Tank	215
13.4.2 Steel Tank	217
13.5 Settlement Induced Stresses	219

Part IV Cones and Spheres

14 Membrane Behaviour of Shells of Revolution Under Axisymmetric Loading	225
14.1 Description of the Surface	225
14.2 Kinematic Relation	227
14.3 Constitutive Relation.	228
14.4 Equilibrium Relation.	229
14.5 Membrane Forces and Displacements	231
14.5.1 Membrane Forces	231
14.5.2 Displacements	233
14.6 Geometry of Conventional Shells of Revolution.	234
14.7 Application to a Spherical Shell Under its Own Weight	235
14.7.1 Comparison with Famous Domes	238
14.7.2 Approximation as Short Beam	238
14.8 Application to a Conical Shell Subject to its Own Weight.	239
14.8.1 Alternate Derivation of Membrane Forces	242
15 Edge Disturbance in Shell of Revolution Due to Axisymmetric Loading	243
15.1 Problem Statement	243
15.2 Recall of Solution for Circular Cylinder	244
15.3 Extension to Cones.	246
15.3.1 Computational Verification	251
15.4 Application to Clamped Sphere Cap	252
15.4.1 Membrane Solution.	253
15.4.2 Bending Solution	254
15.5 Application to a Pressured Hemispherical Boiler Cap	257
15.6 Application to a Pressured Shallow Spherical Boiler Cap	264
References	267

Part V Capita Selecta

16 Introduction to Buckling.	271
16.1 Problem Statement	271
16.2 Beam-Column Buckling	271
16.3 Shell Buckling Study on the Basis of Donnell Equation	273
16.4 Buckling Check for Beam-Column	274
16.5 Check for Flat Plate	274
16.6 Arch Buckling	275
16.7 Buckling of a Curved Plate and Cylinder Under Lateral Pressure.	276
16.8 Buckling of Axially-Pressed Cylinder	278
16.9 Buckling of Spheres, Hyppars and Elpars Subject to Lateral Pressure	279
16.10 Reducing Effect of Imperfections	280
Reference	282
17 FEA for Shells of Irregular Shape.	283
17.1 Finite Element Analysis.	283
17.1.1 Method 1	284
17.1.2 Method 2	286
17.2 Example of Irregular Shell.	286
17.2.1 Geometry and Mesh	286
17.2.2 Computational Results.	288
17.3 Check of FEA-Results by Theory of Cones	291
17.3.1 Membrane Solution.	293
17.3.2 Bending Moment Due to Edge Disturbance	293
References	295
Index	297

Chapter 1

Introduction to Shells

This book is concerned with thin elastic shells. A *thin shell* has a very small thickness-to-minimal-radius ratio, often smaller than $1/50$. As with plates, an applied load that acts out-of-plane leads to larger displacements than those generated by a load acting in-plane with the same intensity. Due to its initial curvature, a shell is able to transfer an applied load by in-plane as well as out-of-plane actions. A thin shell subjected to an applied load therefore produces mainly in-plane actions, which are called *membrane forces*. These membrane forces are actually resultants of normal stresses and in-plane shear stresses that are uniformly distributed across the thickness.

1.1 Shell Theories

A shell is a generalization of an isotropic homogeneous plate. Plates are flat structures of which the dimensions in two in-plane directions are large compared to the third direction perpendicular to the plate. The span in two directions is much larger than the thickness. Plates are defined by their *middle plane*, *thickness* and *material properties*. The *displacements* of the middle plane play the role of degrees of freedom in structural modelling. In-plane loads of plates generate in-plane membrane forces, and out-of-plane loads generate moments and transverse shear forces.

Shells are also defined by their *middle plane*, *thickness* and *material properties*. The difference with plates is that the middle plane of plates is flat, and that it is curved in shells. As a consequence, shells can carry out-of-plane loads by in-plane membrane forces, which is not possible for plates. In fact, this is the major reason why shells are such strong and economic structures.

The theory of this membrane behaviour is called *membrane theory*. However, membrane theory does not satisfy all equilibrium and/or displacement requirements of all cases. For example (Fig. 1.1):

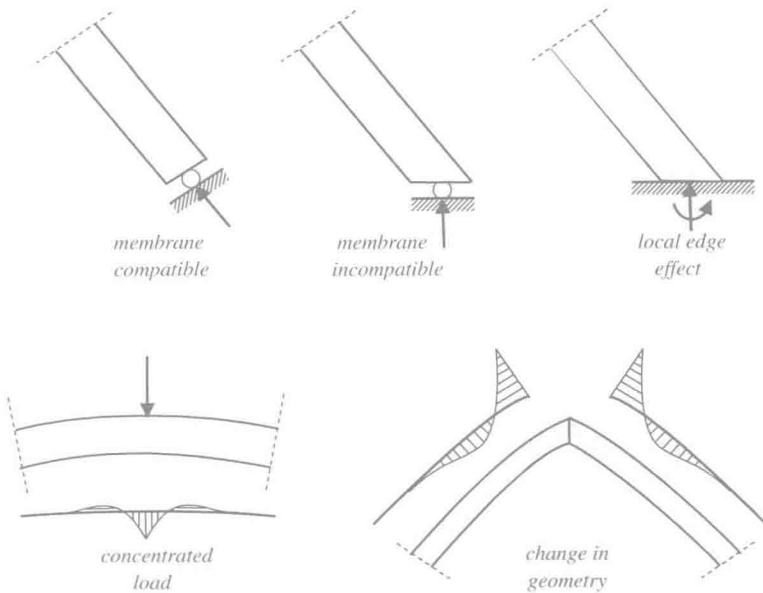


Fig. 1.1 Membrane and bending conditions

- Boundary conditions and deformation constraints that are incompatible with the requirements of a pure membrane field;
- Concentrated loads;
- Changes in geometry.

In the regions where the membrane theory will not hold, some (or all) of the bending field components are produced to compensate the shortcomings of the membrane field in the disturbed zone. These disturbances have to be described by a more complete analysis, which will lead to a *bending theory* of thin elastic shells.

If the bending components occur, they often have a local range of influence. Theoretical calculations and experiments show that the required bending components attenuate and often bending is confined to boundaries where a pure membrane solution does not exist. Therefore, in many cases the bending behaviour is restricted to an *edge disturbance*. The undisturbed and major part of the shell behaves like a true membrane. This unique property of shells is a result of the curvature of the spatial structure. Efficient structural performance is responsible for the widespread appearance of shells in nature. The continuous progress of numerical methods for computational mechanics, combined with an efficient structural performance and a pleasing shape, makes the application of shell structures more and more possible and favourable. However, for the use of numerical programs, some basic knowledge of the underlying theories and the mechanical behaviour of the structure is needed.