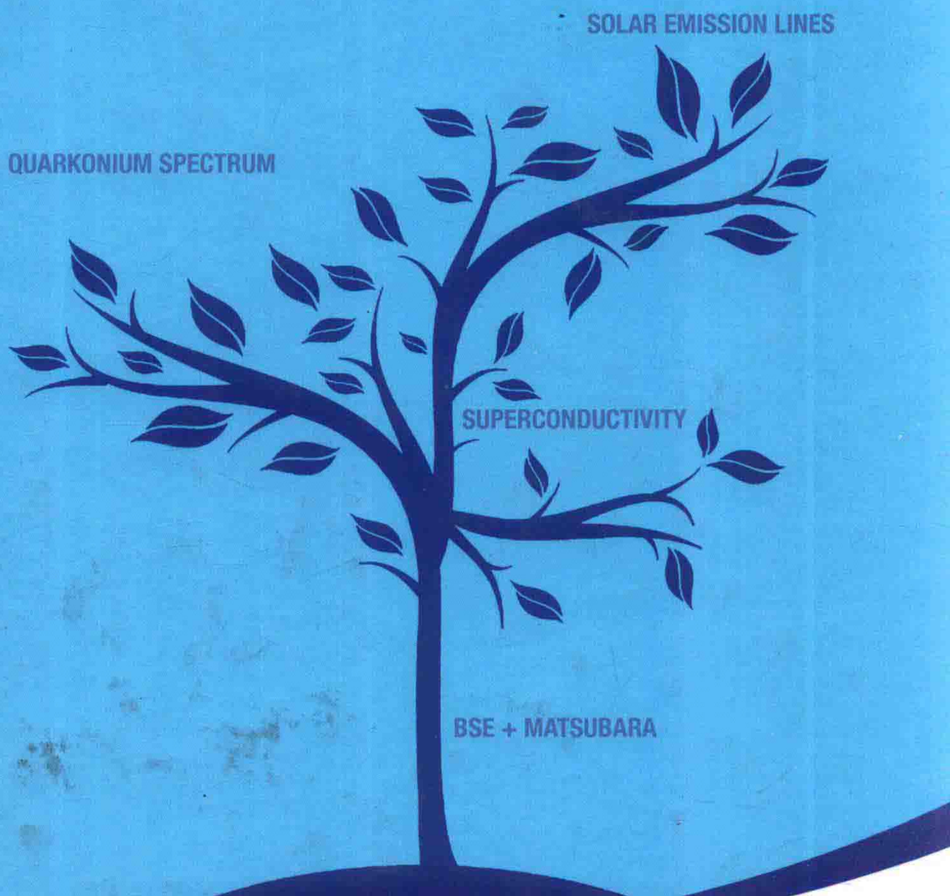
Series on Directions in Condensed Matter Physics – Volume 21

SUPERCONDUCTIVITY

A New Approach Based on the Bethe–Salpeter Equation in the Mean-Field Approximation

G P Malik



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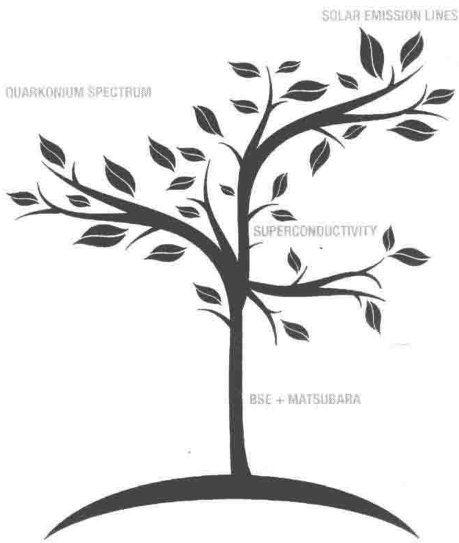

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The author wishes to dedicate this monograph to the memory of his father

Mr. R.K. Malik (March 22, 1914–August 8, 1996)

Preface

BCS theory (1957) emerged 46 years after the experimental discovery of superconductivity. This is a long period considering the large number of scientists — including some of the top names of the twentieth century — who toiled in this field. As is invariably the case for a new theory, it took time for BCS theory to be generally accepted — a status that it had by and large acquired prior to 1987. This however was the case when the highest T_c of any known superconductor (SC) was about 23 K. A radical change in the status quo occurred after Bednorz and Müller discovered an SC that had $T_c \approx 38$ K. An avalanche of activity followed thereafter — witnessed perhaps only once before when X-rays were discovered — leading to the discovery of several SCs such as YBCO and the Tl- and the Bi-based SCs that have T_cs greater than even the liquefaction temperature of nitrogen. Since BCS is a weak-coupling theory, it was found wanting to deal with such high-T_c SCs (HTSCs). It was also felt to be inadequate in accounting for the properties of a class of SCs, e.g., heavy-fermion SCs (HFSCs), for which therefore the term *exotic* or *unconventional* was coined.

We show in this monograph that a generalization of BCS equations (GBCSEs) enables one to address the superconducting features of non-elemental SCs in the manner elemental SCs are dealt with in the original theory. To elaborate, given Debye temperature of an elemental SC and its T_c, BCS theory enables one to predict the value of its gap Δ_0 at $T = 0$, or vice versa. Likewise, given the Debye temperature of a non-elemental

SC and any two parameters of the set $\{T_c, \Delta_{10}, \Delta_{20} > \Delta_{10}\}$, GBCSEs enable one to predict the remaining parameter. The desired generalization is achieved by adopting the "language" of Bethe–Salpeter equation (BSE). The importance of the choice of a language for description of physical phenomena cannot be over-emphasized. To state that dealing with the hydrogen atom by employing Cartesian coordinates would be *clumsy* is to state the obvious. A more pertinent example is provided by tensors which afford the most economical and elegant language for the general theory of relativity. To suggest that BSE provides a similar vehicle for superconductivity would seem to be bizarre, if not ridiculous, because the relativistic domain for which BSE was invented and the domain of superconductivity are as far apart as the north and the south poles.

To obtain results valid in the non-relativistic domain from a relativistic equation, however, is not really a problem since, to give a well-known example, one can obtain Schrodinger equation from Dirac equation by making appropriate approximations. To address superconductivity, we shall likewise resort to the non-relativistic approximation. This raises the legitimate question: Why then bother with the all the complexities of a 4-dimensional BSE? Salient advantages of adopting the BSE-based approach are noted below.

(a) It enables one at the outset — via the Matsubara technique — to temperature-generalize the pairing equation. The equation so obtained leads to the expression for the binding energy in the celebrated Cooper problem in the T = 0 limit; besides, with an appropriate change of limits, it leads also to
(b) The BCS equation for T_c of an elemental SC and to an alternative equation for Δ(T). More importantly,
(c) It leads to a manifest generalization of BCS equations which enable one to address HTSCs in the manner elemental SCs are dealt with in the original theory. This is achieved by employing for the kernel of the T-generalized BSE a superpropagator — in lieu of the usual one-phonon propagator employed for elemental SCs. As will be seen, we are then enabled to quantitatively address the observed high-T_cs and multiple gaps of HTSCs.

(d) It leads to new dynamics-based equations for the critical magnetic field and critical current density of an elemental and a non-elemental SC.

The presentation in this monograph has been inspired by two celebrated books, one authored by Sakurai[1] and the other by Dirac.[2] The former deals with an *enormous* task (major advances in the fundamentals of quantum physics from 1927 to 1967) and claims to do so "in a manner that cannot be made any simpler" — a claim that is amply justified. The latter book presents "the indispensible material in a direct and concise form" on general theory of relativity so as to enable students to pass through "the main obstacles" "with the minimum expenditure of time."

At the root of this monograph are quantum field theory (QFT) and its finite-temperature version — finite-temperature field theory (FTFT). Since the former *per se* is a formidable subject, and latter perhaps even more so, it seems imperative that we specify the level of preparation on the part of the reader to be able to easily follow the contents herein. The good news is that QFT is needed here to the extent of obtaining BSE Eq. (2.1) — no more! Since intuitive considerations that lead to Eq. (2.1) have been dealt with in Chapter 1 in a step-by-step manner, these can easily be followed by even those who have only a cursory familiarity with Feynman diagrams. Similarly, FTFT is needed here to the extent that it provides the recipe given in Eqs. (2.12) — no more. It will be seen that learning the use of these additional tools is a small price to pay for the rich dividend it yields: Unified treatment of various features of the superconductivity of both elemental and non-elemental SCs. The literature on QFT and FTFT is huge and, for the uninitiated, can be rather intimidating. In order not to overwhelm the reader for reasons already stated, references given herein are sparse. While due deference has been paid to classics in the field, referencing to other sources has been kept at a minimum and, as always, reflects the bias (or ignorance) of the author.

The author regards the rather unusual choice of topics covered in this monograph an important feature of it. While this reflects his own interests, he believes that it will be seen that there is a logical continuity to the topics that have been dealt with. He should also like to add that he has been privileged

[1] J.J. Sakurai, *Advanced Quantum Mechanics* (Addison Wesley; Boston, 1967).

[2] P.A.M. Dirac, *General Theory of Relativity* (John Wiley, NY, 1975).

to know serious workers who have devoted a lifetime to the field, but have been reluctant to go beyond certain confined areas of it. He hopes that this monograph will promote a widening of interest among those interested in this fascinating and challenging field.

The work reported here is a labour of love: Most of it was done post-retirement — without financial support from any agency. While he owes a debt of gratitude to many people for reasons acknowledged elsewhere, he would specially like to thank Lalit Pande, his friend and colleague for more than three decades, for inspiring him to *dig deep*, Manuel de Llano, for innumerable e-exchnges and for being an inspiration, and Arun Attri, who was a student when the author joined JNU and subsequently his colleague, for the gift of a Math software *after* which the author learnt to supplement his analytic work with numerical work, without which this monograph would not have been possible.

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More Frequently Used Abbreviations in the Text

BCS:	Bardeen, Cooper, and Schrieffer
BEC:	Bose–Einstein condensation
Bi-2212:	$Bi_2Sr_2CaCu_2O_8$
BSE:	Bethe–Salpeter equation
CP:	Cooper pair
FTFT:	Finite-temperature field theory
GBCSEs:	Generalized BCS equations
HFSC:	Heavy-fermion superconductor
HTSC:	High-T_c superconductor
IA:	Instantaneous approximation
LCO:	La_2CuO_4
MDTs:	Multiple Debye temperatures
MFA:	Mean-field approximation
OPEM:	One-phonon exchange mechanism
QFT:	Quantum field theory
SC:	Superconductor
Tl-2212:	$Tl_2Ba_2CaCu_2O_8$
TPEM:	Two-phonon exchange mechanism
YBCO:	$YBa_2Cu_3O_8$

Symbols Used in More than One Chapter

β: $1/k_B T$

$\Delta(T)$: BCS energy gap at temperature T

Δ_0: BCS energy gap at $T = 0$

Δ_{10}: Smaller of the two gaps of an SC at $T = 0$

Δ_{20}: Larger of the two gaps of an SC at $T = 0$

Δ_F: Feynman propagator for a scalar field

E_F: Fermi energy

k_B: Boltzmann constant

$H_c(T)$: Critical magnetic field at temperature T

H_0: Critical magnetic field at $T = 0$

κ: Thermal conductivity

κ_{es}: Electronic part of thermal conductivity in the superconducting state

κ_{gs}: Lattice part of thermal conductivity in the superconducting state

j_c: Critical current density

j_0: Critical current density at $T = 0$

λ_A: Dimensionless BCS interaction parameter ([N(0)V]) of an elemental SC A

λ_m: Dimensionless BCS interaction parameter for an elemental SC in an external magnetic field

λ_A^c: Dimensionless BCS interaction parameter of a non-elemental SC of which element A is a constituent

μ: Chemical potential

μ_0: Chemical potential at $T = 0$

μ_1: Chemical potential at $T = T_c$

$N(0)$: Density of states at the Fermi surface for one spin, in units of $\mathrm{eV}^{-1}\mathrm{cm}^{-3}$

p_μ: Relative 4-momentum of two particles bound together

P_μ: 4-momemtum of the centre-of mass of two particles bound together

T: Temperature

T_c: Critical temperature

θ_D: Debye temperature

θ_D^A: Debye temperature of A ions in their free state

θ_D^{Ac}: Debye temperature of A ions when they occur as constituents of a composite material

$-V$: BCS parameter for net attraction between a pair of electrons because of attraction due to the ion-lattice and Coulomb repulsion, in units of eVcm^3

$|W(T)|$: (half) the binding energy of a Cooper pair at temperature T

$|W_0|$: (half) the binding energy of a Cooper pair at $T = 0$; equals Δ_0 in the limit of infinesimal λ

$|W_{10}|$: (half) the binding energy of a Cooper pair at $T = 0$ in the OPEM scenario; to be identified with Δ_{10}

$|W_{20}|$: (half) the binding energy of a Cooper pair at $T = 0$ in the TPEM scenario; to be identified with Δ_{20}

ω_c: Debye cut off frequency $(k_B\theta_D/\hbar)$

Units: cgs and Natural

1. Values of some physical and numerical constants

$$c = 2.99792458 \times 10^{10} \text{ cm sec}^{-1},$$
$$1 \text{ eV} = 1.6021892 \times 10^{-12} \text{ gm cm}^2 \text{ sec}^{-1} \text{ (erg)},$$
$$\hbar = 6.582173 \times 10^{-16} \text{ eV sec},$$
$$\hbar c = 1.973285 \times 10^{-5} \text{ eV cm}-,$$
$$e^2/\hbar c = 1/137.03604,$$
$$k_{\text{Boltzmann}} = 8.61735 \times 10^{-5} \text{ K}^{-1} \text{ eV},$$
$$m_{\text{electron}} = 0.5110034 \text{ MeV/c}^2.$$

2. With a_i $(i = 1, 2, 3)$ as given below

$$a_1 = 5.60958616 \times 10^{32}, \quad a_2 = 5.06772886 \times 10^4,$$
$$a_3 = 1.51926689 \times 10^{15},$$

the following identities can easily be verified by substituting for eV, c and $\hbar$ the numerical values given above:

$$1 \text{ gm} = a_1 \text{ eV}(\mathbf{c}^{-2})$$
$$1 \text{ cm} = a_2 \text{ eV}^{-1}(\hbar\mathbf{c})$$
$$1 \text{ sec} = a_3 \text{ eV}^{-1}(\hbar).$$

It is seen from these relations that all physical properties derivable from them, e.g., momentum, force, pressure, charge, etc., can be expressed in terms of eV, c, and $\hbar$. As an example, the cgs unit of magnetic field (gauss) can be written as:

$$1\,\text{gauss} = \text{gm}^{1/2}\text{cm}^{-1/2}\,\text{sec}^{-1} = (\text{a}_1^{1/2}\text{a}_2^{-1/2}\text{a}_3^{-1})\ \text{eV}^2(\hbar\mathbf{c})^{-3/2}.$$

3. It follows from paragraph 2 that if one were to adopt (eV, c, $\hbar$) as the basic units in lieu of (gm, cm, sec) *and* choose $c = \hbar = 1$, then every physical property can be expressed as some power of eV *only*. In the resulting *natural system of units* (*NSU*), the dimensions of any term in an equation can easily be checked without carrying along the factors of c and $\hbar$ in any calculation.
4. We have employed NSU in this monograph. However, since appropriate conversion factors have been incorporated into the equations, one actually needs to use the familiar BCS units for the input variables, which therefore lead to the output too in the same units. To elaborate, input/output for V (as in [N(0)V]) is in terms of eV cm^3, and gauss for the magnetic field H, and so on.

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