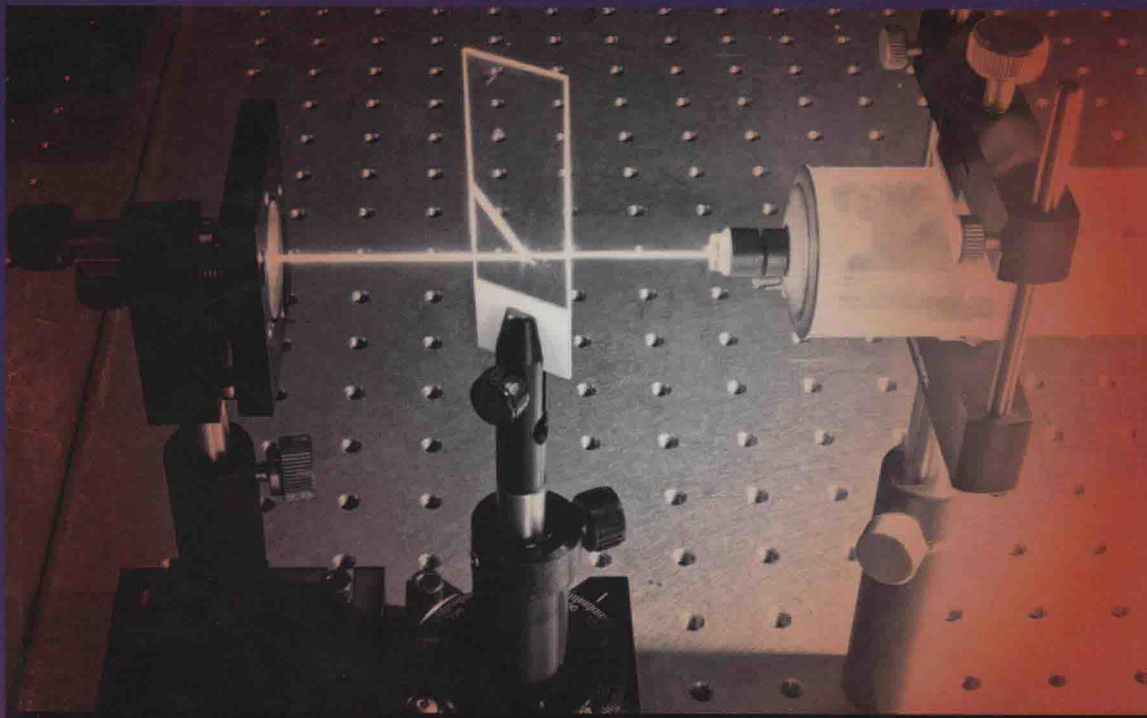




# LASER MODELING

A Numerical Approach with Algebra and Calculus



**Mark Steven Csele**



CRC Press  
Taylor & Francis Group



# **L A S E R M O D E L I N G**

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# **L A S E R M O D E L I N G**

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# Preface

The approach taken in this book is simple: present laser theory in an understandable way and one that can be applied immediately, and numerically, to real laser systems. With that in mind, the approach in this text is to present each theory along with a real, solved example—in most cases, based on commercial lasers. As a professor of laser science, I am fortunate to have a lab equipped with many different types of lasers; many of those lasers are included here in examples.

In making the theory “accessible,” both a calculus-based and an algebraic approach are shown in tandem; a prime example of this is the presentation of both the calculus-based Rigrod model and an algebra-based model for the prediction of various laser parameters in Chapters 3 and 4. Readers drawn to numerically grounded solutions to problems (dare we say “engineers”?) will find the algebraic approach a refreshing demonstration of how concepts actually work and are applied, while those with more mathematical thought processes will appreciate the complementary calculus-based models. Either way, the results are similar (and, as I tell my students, it doesn’t matter how you learn it). As an educator, I appreciate the fact that we all learn in different ways.

The actual use of algebra-based solutions originated with our four-year bachelor program at Niagara College. Although at the inception of the program we intended to use calculus-based theory exclusively, it became apparent that students were spending more time on the math than on the concepts (i.e., they were often “buried in the math,” with the mathematical rigor of solutions getting in the way of understanding the concepts). In an attempt to reverse that situation, an algebra-based solution was presented in conjunction with the traditional approach. It was very well received by students and has been a staple of my laser courses since.

An example of the value of the algebraic approach is the Q-switch model presented in Chapter 6. This model predicts how inversion builds in time, and it was found that students spent more time developing and solving derivatives than understanding the concepts involved; by using an algebraic model, one could easily see how inversion builds, then decays, in a laser medium. This nicely demonstrates the concept, after which implementing a more complex calculus-based model is considerably easier for many students.

Although some knowledge of basic physics (such as the nature of light, emission of radiation, and some basic atomic physics) is assumed, the text begins with Chapter 1 providing a review of basic concepts and definitions of terms. A wide variety of approaches can be found in various texts and research papers, so it is worthwhile to provide a frame of reference. Most material in this chapter is a basis for concepts explored later in the text.

Since all models start with, or require, an accurate gain threshold equation, Chapter 2 is devoted entirely to this topic. While superficially simple, the formulation of this equation is more complex to accurately reflect the laser system and all components—one size does NOT fit all here, and the basic threshold equation familiar to

many readers does not apply to all lasers. For ultimate accuracy, a unique threshold equation is required for each laser configuration. Most importantly, the methodology of developing this equation is outlined and many examples are given such that readers can formulate an equation specific for their application. Once the equation is developed, several simple applications of this equation are outlined, including the determination of small-signal gain (one of the key parameters of all lasing media).

The second most important concept, gain saturation, is covered in Chapter 3 along with one of the most powerful models, the pass-by-pass model. Although this model could be implemented in any computer programming language, it is shown here completed on a spreadsheet, since this is essentially a universal tool on all computers. From a learning perspective, this model is an excellent vehicle to demonstrate a host of concepts. This algebraic model may be used with almost any configuration of laser and can predict a host of performance parameters. It allows rapid implementation of “what if?” scenarios. Spreadsheet-based models used in this text are available from the publisher’s website as well so that readers may modify them to suit their application.

Armed with the basic concepts from Chapters 2 and 3, the calculus-based Rigrod approach is outlined in Chapter 4. This approach, the “gold standard” for laser work, is developed in a simplified manner and applied to several lasers. Limitations of the approach are outlined and comparison made to the algebraic approach of Chapter 3. Used in tandem, both approaches (algebraic and Rigrod) allow maximum understanding of laser processes (appreciating, again, that we all learn in different ways).

With the increase in the number of solid-state lasers available, Chapter 5 outlines thermal effects on these (as well as other) lasers. Specifically, thermal population of the lower-lasing levels of many new quasi-three-level materials, as well as the effect of temperature on diode and DPSS lasers, is examined in detail. This topic is one that seems to be lacking in many texts (perhaps for no other reason than the relative increase in importance of solid-state lasers in the past few years and the ongoing search for newer and more efficient materials—what was a “lab curiosity” only a few years ago is “commercially viable” today).

Chapter 5 also presents an outline of how the convolution technique can be applied to predict the effect of temperature drift on a DPSS system. The effects of temperature on a pump diode, coupled with the characteristics of solid-state amplifier materials, lead to systems that are often unexpectedly sensitive to temperature change. Convolution, long used in the world of digital signal processing (DSP) for filtering signals, is implemented here (again using a spreadsheet for simplicity) to predict such effects.

Q-switching, an important technique integral to many lasers, is presented in Chapter 6. Having outlined the technology involved (i.e., how real Q-switches actually work), a simple model is presented to predict the output power of a Q-switched laser. Numerical examples are given outlining application to both a “normal” Q-switched laser as well as a considerably more unusual double-pulsed laser system (which serves as an excellent example of application of the theory).

Chapter 7 serves as a bit of a “crash course” in non-linear optics, which are so prevalent in modern laser systems. The theory of non-linear radiation generation is presented with practical examples given on determination of irradiance for a Q-switched laser to determine the suitability of a particular non-linear material. A simple model

for predicting the optimal crystal length is developed (“simple,” since it makes a number of assumptions, but effective).

Finally, Chapter 8 presents an overview of many common laser systems, including a summary of parameters (many of which are used in examples throughout the text). For the reader curious about the physical implementation of many lasers (e.g., “why does an argon-ion laser have a ceramic plasma tube?” or “why is the flashlamp so big on a ruby?”), this section attempts to answer a few basic questions on the design of these lasers.

I wish to thank Niagara College, Canada, for their cooperation in producing this text. With a dedicated photonics program, the labs at Niagara College have afforded me the ability to test numerous models on real lasers. Most of the photos in this text were shot at the college labs which feature an enormous range of lasers. I would also like to thank Ashley Gasque, acquiring editor, and Amber Donley, production coordinator, at Taylor & Francis, for their assistance.

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# The Author

**Mark Csele** is a full-time professor at Niagara College, Canada, in the heart of the beautiful Niagara Region. A physicist and a professional engineer, he has taught for over twenty years in programs ranging from a two-year technician level to a four-year undergraduate level. Currently, he teaches in the college's photonics programs, which feature an array of dedicated laboratories hosting a variety of laser systems. He has authored a previous book on fundamental laser concepts as well as several articles in magazines and trade encyclopedias.



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# 1 Basic Laser Processes

In the course of developing models to predict laser performance, which is the focus of this text, we often need to start at the beginning. The purpose of this chapter, then, is to provide a basic overview of processes and parameters relevant to the laser. Some readers may already be familiar with many of these concepts but may not have seen them applied specifically to laser systems.

Key concepts outlined in this chapter include an atomic view of the laser processes of emission and absorption (including the important process of stimulated emission), a basic overview of rate equations involved in various laser levels (which will be useful when developing models later), methods of pumping a laser amplifier, and the nature of gain and loss in a laser amplifier. Only the basics are presented here and many of these concepts will be expanded upon in later chapters as required.

## 1.1 THE LASER AND LASER LIGHT

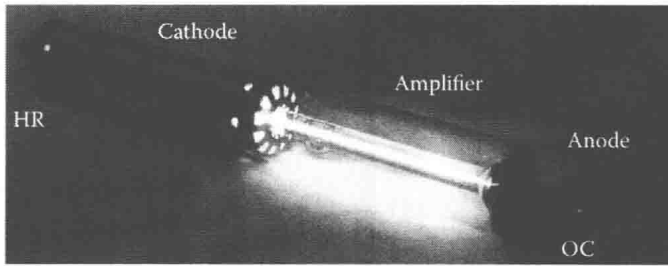
At first glance, a laser seems simple enough: an amplifier (often a glass tube filled with gas, a semiconductor diode, or a rod of glass-like material) pumped by some means, and surrounded by two mirrors—one of which is partially transmitting—from which an output beam emerges. As we shall see in this text, the arrangement is anything but haphazard.

Consider first the common helium-neon (HeNe) laser, which will be one of several lasers used extensively in examples in this text. Once the most popular visible laser, it has been replaced in many applications by smaller and cheaper semiconductor diode lasers. Still, the high-quality beam lends itself well to a host of laboratory applications (and, from the standpoint of an example laser, it represents a design with very “standard” elements. The structure of a typical HeNe laser is shown in the annotated photograph of Figure 1.1.

The HeNe laser is typical of many lasers: an amplifying medium surrounded by two cavity mirrors. The actual amplifying medium, in this case, is a mixture of helium and neon gases at low pressures and excited by a high-voltage discharge. In Figure 1.1, the actual discharge is seen to be confined to a narrow tube inside the larger laser tube and occurs between the anode (on which the output coupler, or OC, from which the output beam emerges is mounted) and the cylindrical cathode. (Note that not all tubes have the OC mounted on the anode; this varies by tube and may also be on the cathode end.)

Lasers are classed by the amplifying medium employed and may use gas, liquid (dye), solid-state (i.e. glass-like crystals), or semiconductor materials. Each has a





**FIGURE 1.1** A basic HeNe gas laser.

special set of characteristics and a different method of pumping energy into the system. (Most solid-state lasers, for example, are optically pumped by an intense lamp source or another laser, whereas gas lasers are usually pumped by an electrical discharge through the gas medium.)

In a practical laser, a single photon of radiation passes through the amplifier many times, with the flux of photons becoming more powerful on each pass and eventually building to power where a usable beam results. The cavity mirrors surrounding the amplifier are required to produce lasers of a manageable length. If length were not a concern one could produce a HeNe laser tube over 80m in length and forgo mirrors altogether—but this is hardly practical and the use of mirrors allows a “folding” arrangement where photons are made to pass through an amplifier many times.

The required reflectivity of the cavity mirrors depends on the gain of the amplifier employed. Low-gain amplifiers require high-reflectivity mirrors—an example being the HeNe gas laser, which commonly has a high reflector (HR) of almost 100% reflectivity and an OC of about 99% reflectivity (with 1% of the intra-cavity power exiting through this optic to become the output beam). On the other hand, higher-gain lasers optimally use lower reflectivity optics and feature a higher transmission of the output coupler. Chapters 3 and 4 address the issue of optimal coupling.

The output of a laser is quite unique, as evident from simple observation. The most important property of laser light is coherence: every photon in the beam is in phase with each other. This is a consequence of the mechanism of the laser itself, stimulated emission, which ensures that all photons produced are essentially clones of an original photon—they must therefore all be of the exact same wavelength and relative phase.

Another important property of laser light is directionality and the extraordinarily low divergence of many laser beams. This property is primarily a consequence of the arrangement of optical elements in the laser. Most lasers are standing-wave lasers which have the general form of an amplifier surrounded by two cavity mirrors aligned very parallel to each other (as we shall see later, though, this is not the only possible arrangement). Photons emitted from the front of the laser will have made many passes through the amplifier (hundreds, or perhaps thousands) and so, in order to be amplified at all they must be on a specific trajectory perfectly aligned to the optical axis of the laser. Photons that are divergent, even slightly, will often strike the side of the amplifier tube and hence never be amplified to any extent.