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An Introduction to Complex Analysis and Geometry

复分析与几何引论

John P. D'Angelo

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图字：01-2016-2501 号

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Fufenxi yu Jihe Yinlun

图书在版编目 (CIP) 数据

复分析与几何引论 = An Introduction to Complex Analysis and Geometry : 英文 / (美) 约翰 · P · 迪 · 安格罗 (John P. D'Angelo) 著 . —影印本 . —北京 : 高等教育出版社 , 2017.4
ISBN 978-7-04-046998-1
Ⅰ. ①复… Ⅱ. ①约… Ⅲ. ①复分析—英文②几何学—英文Ⅳ. ①O174.5 ②O181
中国版本图书馆 CIP 数据核字 (2016) 第 327004 号

策划编辑 李 鹏　　责任编辑 李 鹏
封面设计 张申申　　责任印制 赵义民

出版发行 高等教育出版社
社址 北京市西城区德外大街4号
邮政编码 100120
购书热线 010-58581118
咨询电话 400-810-0598
网址 http://www.hep.edu.cn
http://www.hep.com.cn
网上订购 http://www.hepmall.com.cn
http://www.hepmall.com
http://www.hepmall.cn
印刷 北京中科印刷有限公司

开本 787mm × 1092 mm 1/16
印张 11.5
字数 280 千字
版次 2017 年 4 月第 1 版
印次 2017 年 4 月第 1 次印刷
定价 99.00 元

[物 料 号 46998-00]

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出版者的话

近年来，我国的科学技术取得了长足进步，特别是在数学等自然科学基础领域不断涌现出一流的研究成果。与此同时，国内的科研队伍与国外的交流合作也越来越密切，越来越多的科研工作者可以熟练地阅读英文文献，并在国际顶级期刊发表英文学术文章，在国外出版社出版英文学术著作。

然而，在国内阅读海外原版英文图书仍不是非常便捷。一方面，这些原版图书主要集中在科技、教育比较发达的大中城市的大型综合图书馆以及科研院所的资料室中，普通读者借阅不甚容易；另一方面，原版书价格昂贵，动辄上百美元，购买也很不方便。这极大地限制了科技工作者对于国外先进科学技术知识的获取，间接阻碍了我国科技的发展。

高等教育出版社本着植根教育、弘扬学术的宗旨服务我国广大科技和教育工作者，同美国数学会（American Mathematical Society）合作，在征求海内外众多专家学者意见的基础上，精选该学会近年出版的数十种专业著作，组织出版了“美国数学会经典影印系列”丛书。美国数学会创建于 1888 年，是国际上极具影响力的专业学术组织，目前拥有近 30000 会员和 580 余个机构成员，出版图书 3500 多种，冯·诺依曼、莱夫谢茨、陶哲轩等世界级数学大家都是其作者。本影印系列涵盖了代数、几何、分析、方程、拓扑、概率、动力系统等所有主要数学分支以及新近发展的数学主题。

我们希望这套书的出版，能够对国内的科研工作者、教育工作者以及青年学生起到重要的学术引领作用，也希望今后能有更多的海外优秀英文著作被介绍到中国。

高等教育出版社

2016 年 12 月

Preface

This book developed from a course given in the Campus Honors Program at the University of Illinois Urbana-Champaign in the fall semester of 2008. The aims of the course were to introduce bright students, most of whom were freshmen, to complex numbers in a friendly, elegant fashion and to develop reasoning skills belonging to the realm of elementary complex geometry. In the spring semester of 2010 I taught another version of the course, in which a draft of this book was available online. I therefore wish to acknowledge the Campus Honors Program at UIUC for allowing me to teach these courses and to thank the 27 students who participated in them.

Many elementary mathematics and physics problems seem to simplify magically when viewed from the perspective of complex analysis. My own research interests in functions of several complex variables and CR geometry have allowed me to witness this magic daily. I continue the preface by mentioning some of the specific topics discussed in the book and by indicating how they fit into this theme.

Every discussion of complex analysis must spend considerable time with power series expansions. We include enough basic analysis to study power series rigorously and to solidify the backgrounds of the typical students in the course. In some sense two specific power series dominate the subject: the geometric and exponential series.

The geometric series appears all throughout mathematics and physics and even in basic economics. The Cauchy integral formula provides a way of deriving from the geometric series the power series expansion of an arbitrary complex analytic function. Applications of the geometric series appear throughout the book.

The exponential series is of course also crucial. We define the exponential function via its power series, and we define the trigonometric functions by way of the exponential function. This approach reveals the striking connections between the functional equation $e^{z+w} = e^z e^w$ and the profusion of trigonometric identities.

Using the complex exponential function to simplify trigonometry is a compelling aspect of elementary complex analysis and geometry. Students in my courses seemed to appreciate this material to a great extent.

One of the most appealing combinations of the geometric series and the exponential series appears in Chapter 4. We combine them to derive a formula for the sums

$$\sum_{j=1}^{n} j^p,$$

in terms of Bernoulli numbers.

We briefly discuss ordinary and exponential generating functions, and we find the ordinary generating function for the Fibonacci numbers. We then derive Binet's formula for the n-th Fibonacci number and show that the ratio of successive Fibonacci numbers tends to the golden ratio $\frac{1+\sqrt{5}}{2}$.

Fairly early in the book (Chapter 3) we discuss hyperbolas, ellipses, and parabolas. Most students have seen this material in calculus or even earlier. In order to make the material more engaging, we describe these objects by way of Hermitian symmetric quadratic polynomials. This approach epitomizes our focus on complex numbers rather than on pairs of real numbers.

The geometry of the unit circle also allows us to determine the Pythagorean triples. We identify the Pythagorean triple (a, b, c) with the complex number $\frac{a}{c}+i\frac{b}{c}$; we then realize that a Pythagorean triple corresponds to a rational point (in the first quadrant) on the unit circle. After determining the usual rational parametrization of the unit circle, one can easily find all these triples. But one gains much more; for example, one discovers the so-called $\tan(\frac{\theta}{2})$ substitution from calculus. During the course several students followed up this idea and tracked down how the indefinite integral of the secant function arose in navigation.

This book is more formal than was the course itself. The list of approximately two hundred eighty exercises in the book is also considerably longer than the list of assigned exercises. These exercises (as well as the figures) are numbered by chapter, whereas items such as theorems, propositions, lemmas, examples, and definitions are numbered by section. Unless specified otherwise, a reference to a section, theorem, proposition, lemma, example, or definition is to the current chapter. The overall development in the book closely parallels that of the courses, although each time I omitted many of the harder topics. I feel cautiously optimistic that this book can be used for similar courses. Instructors will need to make their own decisions about which subjects can be omitted. I hope however that the book has a wider audience including anyone who has ever been curious about complex numbers and the striking role they play in modern mathematics and science.

Chapter 1 starts by considering various number systems and continues by describing, slowly and carefully, what it means to say that the real numbers are a complete ordered field. We give an interesting proof that there is no *rational* square root of 2, and we prove carefully (based on the completeness axiom) that positive real numbers have square roots. The chapter ends by giving several possible definitions of the field of complex numbers.

Chapter 2 develops the basic properties of complex numbers, with a special emphasis on the role of complex conjugation. The author's own research in complex analysis and geometry has often used *polarization*; this technique makes precise the sense in which we may treat z and $\overline{z}$ as independent variables. We will view complex analytic functions as those independent of $\overline{z}$. In this chapter we also include precise definitions about convergence of series and related elementary analysis. Some instructors will need to treat this material carefully, while others will wish to review it quickly. Section 5 treats uniform convergence and some readers will wish to postpone this material. The subsequent sections however return to the basics of complex geometry. We define the exponential function by its power series and the cosine and sine functions by way of the exponential function. We can and therefore do discuss logarithms and trigonometry in this chapter as well.

Chapter 3 focuses on geometric aspects of complex numbers. We analyze the zero-sets of quadratic equations from the point of view of complex rather than real variables. For us hyperbolas, parabolas, and ellipses are zero-sets of quadratic Hermitian symmetric polynomials. We also study linear fractional transformations and the Riemann sphere.

Chapter 4 considers power series in general; students and instructors will find that this material illuminates the treatment of series from calculus courses. The chapter includes a short discussion of generating functions, Binet's formula for the Fibonacci numbers, and the formula for sums of p-th powers mentioned above. We close Chapter 4 by giving a test for when a power series defines a rational function.

Chapter 5 begins by posing three possible definitions of complex analytic function. These definitions involve locally convergent power series, the Cauchy-Riemann equations, and the limit quotient version of complex differentiability. We postpone the proof that these three definitions determine the same class of functions until Chapter 6 after we have introduced integration. Chapter 5 focuses on the relationship between real and complex derivatives. We define the Cauchy-Riemann equations using the $\frac{\partial}{\partial \overline{z}}$ operator. Thus complex analytic functions are those functions *independent* of $\overline{z}$. This perspective has profoundly influenced research in complex analysis, especially in higher dimensions, for at least fifty years. We briefly consider harmonic functions and differential forms in Chapter 5; for some audiences there might be too little discussion about these topics. It would be nice to develop potential theory in detail and also to say more about closed and exact differential forms, but then perhaps too many readers would drown in deep water.

Chapter 6 treats the Cauchy theory of complex analytic functions in a simplified fashion. The main point there is to show that the three possible definitions of analytic function introduced in Chapter 5 all lead to the same class of functions. This material forms the basis for both the theory and application of complex analysis. In short, Chapter 5 considers derivatives and Chapter 6 considers integrals.

Chapter 7 offers many applications of the Cauchy theory to ordinary integrals. In order to show students how to apply complex analysis to things they have seen before, we evaluate many interesting real integrals using residues and contour integration. We also include sections on the Fourier transform on the Gamma function.

Chapter 8 introduces additional appealing topics such as the fundamental theorem of algebra (for which we give three proofs), winding numbers, Rouche's theorem, Pythagorean triples, conformal mappings, the quaternions, and (a brief mention of) complex analysis in higher dimensions. The section on conformal mappings includes a brief discussion of non-Euclidean geometry. The section on quaternions includes the observation that there are many quaternionic square roots of -1, and hence it illuminates the earliest material used in defining $\mathbf{C}$. The final result proved concerns polarization; it justifies treating z and $\overline{z}$ as independent variables, and hence it also unifies much of the material in the book.

Our bibliography includes many excellent books on complex analysis in one variable. One naturally asks how this book differs from those. The primary difference is that this book begins at a more elementary level. We start at the logical beginning, by discussing the natural numbers, the rational numbers, and the real numbers. We include detailed discussion of some truly basic things, such as the existence of square roots of positive real numbers, the irrationality of $\sqrt{2}$, and several different definitions of $\mathbf{C}$ itself. Hence most of the book can be read by a smart freshman who has had some calculus, but not necessarily any real analysis. A second difference arises from the desire to engage an audience of bright freshmen. I therefore include discussion, examples, and exercises on many topics known to this audience via real variables, but which become more transparent using complex variables. My ninth grade mathematics class (more than forty years ago) was tested on being able to write word-for-word the definitions of hyperbola, ellipse, and parabola. Most current college freshmen know only vaguely what these objects are, and I found myself reciting those definitions when I taught the course. During class I also paused to carefully prove that .999... really equals 1. Hence the book contains various basic topics, and as a result it enables spiral learning. Several concepts are revisited with high multiplicity throughout the book. A third difference from the other books arises from the inclusion of several unusual topics, as described throughout this preface.

I hope, with some confidence, that the text conveys my deep appreciation for complex analysis and geometry. I hope, but with more caution, that I have purged all errors from it. Most of all I hope that many will enjoy reading it and solving the exercises in it.

I began expanding the sketchy notes from the course into this book during the spring 2009 semester, during which I was partially supported by the Kenneth D. Schmidt Professorial Scholar award. I therefore wish to thank Dr. Kenneth Schmidt and also the College of Arts and Sciences at UIUC for awarding me this prize. I have received considerable research support from the NSF for my work in complex analysis; in particular I acknowledge support from NSF grant DMS-0753978. The students in the first version of the course survived without a text; their enthusiasm and interest merit praise. Over the years many other students have inspired me to think carefully how to present complex analysis and geometry with elegance. Another positive influence on the evolution from sketchy notes to this book was working through some of the material with Bill Heiles, Professor of Piano at UIUC and one who appreciates the art of mathematics. Jing Zou, computer science student at UIUC, prepared the figures in the book. Tom Forgacs, who invited me to speak at California State University, Fresno on my experiences teaching this course, also made useful comments. My colleague Jeremy Tyson made many valuable suggestions on both the mathematics and the exposition. I asked several friends to look at the N-th draft for various large N. Bob Vanderbei, Rock Rodini, and Mike Bolt all made many useful comments which I have incorporated. I thank Sergei Gelfand and Ed Dunne of the American Mathematical Society for encouragement; Ed Dunne provided me marked-up versions of two drafts and shared with me, in a lengthy phone conversation, his insights on how to improve and complete the project. Cristin Zannella and Arlene O'Sean of the AMS oversaw the final editing and other finishing touches. Finally I thank my wife Annette and our four children for their love.

Preface for the student

I hope that this book reveals the beauty and usefulness of complex numbers to you. I want you to enjoy both reading it and solving the problems in it. Perhaps you will spot something in your own area of interest and benefit from applying complex numbers to it. Students in my classes have found applications of ideas from this book to physics, music, engineering, and linguistics. Several students have become interested in historical and philosophical aspects of complex numbers. I have not yet seen anyone get excited about the hysterical aspects of complex numbers.

At the very least you should see many places where complex numbers shed a new light on things you have learned before. One of my favorite examples is trig identities. I found them rather boring in high school and later I delighted in proving them more easily using the complex exponential function. I hope you have the same experience. A second example concerns certain definite integrals. The techniques of complex analysis allow for stunningly easy evaluations of many calculus integrals and seem to lie within the realm of science fiction.

This book is meant to be readable, but at the same time it is precise and rigorous. Sometimes mathematicians include details that others feel are unnecessary or obvious, but do not be alarmed. If you do many of the exercises and work through the examples, then you should learn plenty and enjoy doing it. I cannot stress enough two things I have learned from years of teaching mathematics. First, students make too few sketches. You should strive to merge geometric and algebraic reasoning. Second, definitions are your friends. If a theorem says something about a concept, then you should develop both an intuitive sense of the concept and the discipline to learn the precise definition. When asked to verify something on an exam, start by writing down the definition of that something. Often the definition suggests exactly what you should do!

Some sections and paragraphs introduce more sophisticated terminology than is necessary at the time, in order to prepare for later parts of the book and even for subsequent courses. I have tried to indicate all such places and to revisit the crucial ideas. In case you are struggling with any material in this book, remain calm. The magician will reveal his secrets in due time.

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Contents

Chapter 1

From the Real Numbers to the Complex Numbers

1. Introduction

Many problems throughout mathematics and physics illustrate an amazing principle: ideas expressed within the realm of real numbers find their most elegant expression through the unexpected intervention of complex numbers. Many of these delightful interventions arise in elementary, recreational mathematics. On the other hand most college students either never see complex numbers in action or they wait until the junior or senior year in college, at which time the sophisticated courses have little time for the elementary applications. Hence too few students witness the beauty and elegance of complex numbers. This book aims to present a variety of elegant applications of complex analysis and geometry in an accessible but precise fashion. We begin at the beginning, by recalling various number systems such as the integers **Z**, the rational numbers **Q**, and the real numbers **R**, before even defining the complex numbers **C**. We then provide three possible equivalent definitions. Throughout we strive for as much geometric reasoning as possible.

2. Number systems

The ancients were well aware of the so-called *natural numbers*, written $1, 2, 3, \ldots$. Mathematicians write **N** for the collection of natural numbers together with the usual operations of addition and multiplication. Partly because subtraction is not always possible, but also because negative numbers arise in many settings such as financial debts, it is natural to expand the natural number system to the larger system **Z** of integers. We assume that the reader has some understanding of the integers; the set **Z** is equipped with two distinguished members, written 1 and 0, and two operations, called addition $(+)$ and multiplication $(*)$, satisfying familiar laws. These operations make **Z** into what mathematicians call a *commutative ring* with unit 1. The integer 0 is special. We note that each n in **Z** has an additive inverse $-n$ such that

$$n + (-n) = (-n) + n = 0. \tag{1}$$

Of course 0 is the only number whose additive inverse is itself.

Let a, b be given integers. As usual we write $a-b$ for the sum $a+(-b)$. Consider the equation $a + x = b$ for an unknown x. We learn to solve this equation at a young age; the idea is that subtraction is the inverse operation to addition. To solve $a + x = b$ for x, we first add $-a$ to both sides and use (1). We can then substitute b for $a + x$ to obtain the solution

$$x = 0 + x = (-a) + a + x = (-a) + b = b + (-a) = b - a.$$

This simple principle becomes a little more difficult when we work with multiplication. It is not always possible, for example, to divide a collection of n objects into two groups of equal size. In other words, the equation $2 * a = b$ does not have a solution in $\mathbf{Z}$ unless b is an even number. Within $\mathbf{Z}$, most integers (± 1 are the only exceptions) do not have multiplicative inverses.

To allow for division, we enlarge $\mathbf{Z}$ into the larger system $\mathbf{Q}$ of rational numbers. We think of elements of $\mathbf{Q}$ as fractions, but the definition of $\mathbf{Q}$ is a bit subtle. One reason for the subtlety is that we want $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{50}{100}$ all to represent the same rational number, yet the expressions as fractions differ. Several approaches enable us to make this point precise. One way is to introduce the notion of equivalence class and then to define a rational number to be an equivalence class of pairs of integers. See **[4]** or **[8]** for this approach. A second way is to think of the rational number system as known to us; we then write elements of $\mathbf{Q}$ as letters, x, y, u, v, and so on, without worrying that each rational number can be written as a fraction in infinitely many ways. We will proceed in this second fashion. A third way appears in Exercise 1.2 below. Finally we emphasize that we cannot divide by 0. Surely the reader has seen alleged proofs that, for example, $1 = 2$, where the only error is a cleverly disguised division by 0.

▶ **Exercise 1.1.** Find an invalid argument that $1 = 2$ in which the only invalid step is a division by 0. Try to obscure the division by 0.

▶ **Exercise 1.2.** Show that there is a one-to-one correspondence between the set $\mathbf{Q}$ of rational numbers and the following set L of lines. The set L consists of all lines through the origin, except the vertical line $x = 0$, that pass through a nonzero point (a, b) where a and b are integers. (This problem sounds sophisticated, but one word gives the solution!)

The rational number system forms a *field.* A field consists of objects which can be added and multiplied; these operations satisfy the laws we expect. We begin our development by giving the precise definition of a field.

Definition 2.1. A field $\mathbf{F}$ is a mathematical system consisting of a collection of objects and two operations, addition and multiplication, subject to the following axioms.

1) For all x, y in $\mathbf{F}$, we have $x + y = y + x$ and $x * y = y * x$ (the commutative laws for addition and multiplication).

2) For all x, y, t in $\mathbf{F}$, we have $(x+y)+t = x+(y+t)$ and $(x*y)*t = x*(y*t)$ (the associative laws for addition and multiplication).

3) There are distinct distinguished elements 0 and 1 in $\mathbf{F}$ such that, for all x in $\mathbf{F}$, we have $0+x = x+0 = x$ and $1*x = x*1 = x$ (the existence of additive and multiplicative identities).

4) For each x in $\mathbf{F}$ and each y in $\mathbf{F}$ such that $y \neq 0$, there are $-x$ and $\frac{1}{y}$ in $\mathbf{F}$ such that $x+(-x) = 0$ and $y * \frac{1}{y} = 1$ (the existence of additive and multiplicative inverses).

5) For all x, y, t in $\mathbf{F}$ we have $t*(x+y) = (t*x)+(t*y) = t*x+t*y$ (the distributive law).

For clarity and emphasis we repeat some of the main points. The rational numbers provide a familiar example of a field. In any field we can add, subtract, multiply, and divide as we expect, although we cannot divide by 0. The ability to divide by a nonzero number distinguishes the rational numbers from the integers. In more general settings the ability to divide by a nonzero number distinguishes a field from a commutative ring. Thus every field is a commutative ring but a commutative ring need not be a field.

There are many elementary consequences of the field axioms. It is easy to prove that each element has a unique additive inverse and that each nonzero element has a unique multiplicative inverse, or reciprocal. The proof, left to the reader, mimics our early argument showing that subtraction is possible in $\mathbf{Z}$.

Henceforth we will stop writing $*$ for multiplication; the standard notation of xy for $x*y$ works adequately in most contexts. We also write x^2 instead of xx as usual. Let t be an element in a field. We say that x is a square root of t if $t = x^2$. In a field, taking square roots is not always possible. For example, we shall soon prove that there is no rational square root of 2 and that there is no real square root of -1.

At the risk of boring the reader we prove a few basic facts from the field axioms; the reader who wishes to get more quickly to geometric reasoning could omit the proofs, although writing them out gives one some satisfaction.

Proposition 2.1. *In a field the following laws hold:*

1) $0+0=0$.

2) *For all x, we have $x0 = 0x = 0$.*

3) $(-1)^2 = (-1)(-1) = 1$.

4) $(-1)x = -x$ *for all* x.

5) *If $xy = 0$ in $\mathbf{F}$, then either $x = 0$ or $y = 0$.*

Proof. Statement 1) follows from setting $x = 0$ in the axiom $0+x = x$. Statement 2) uses statement 1) and the distributive law to write $0x = (0+0)x = 0x+0x$. By property 4) of Definition 2.1, the object $0x$ has an additive inverse; we add this inverse to both sides of the equation. Using the meaning of additive inverse and then the associative law gives $0 = 0x$. Hence $x0 = 0x = 0$ and 2) holds. Statement 3) is a bit more interesting. We have $0 = 1+(-1)$ by axiom 4) from Definition 2.1.