

MECHANICAL ENGINEERING PRINCIPLES

John Bird & Carl Ross
THIRD EDITION



Mechanical Engineering Principles

Third Edition

John Bird BSc(Hons), CEng, CMath, CSci. FIMA. FIET, FCollT

Carl Ross BSc(Hons), PhD, DSc, CEng, FRINA, MSNAME

Third edition published 2015
by Routledge
2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN

and by Routledge
711 Third Avenue, New York, NY 10017

Routledge is an imprint of the Taylor & Francis Group, an informa business

© 2015 John O. Bird and Carl T. F. Ross

The right of John O. Bird and Carl T. F. Ross to be identified as authors of this work has been asserted by them in accordance with sections 77 and 78 of the Copyright, Designs and Patents Act 1988.

All rights reserved. No part of this book may be reprinted or reproduced or utilised in any form or by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying and recording, or in any information storage or retrieval system, without permission in writing from the publishers.

This publication presents material of a broad scope and applicability. Despite stringent efforts by all concerned in the publishing process, some typographical or editorial errors may occur, and readers are encouraged to bring these to our attention where they represent errors of substance. The publisher and author disclaim any liability, in whole or in part, arising from information contained in this publication. The reader is urged to consult with an appropriate licensed professional prior to taking any action or making any interpretation that is within the realm of a licensed professional practice.

First edition published by Elsevier in 2002
Second edition published by Routledge in 2012

Trademark notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloguing-in-Publication Data
Bird, J. O.
Mechanical engineering principles / John Bird and Carl Ross. -- 3rd edition.
pages cm

ISBN 978-1-138-78157-3 (pbk. : alk. paper) -- ISBN 978-1-315-76980-6 (ebook)
1. Mechanical engineering--Textbooks. 2. Mechanical engineering--Problems, exercises, etc. I. Ross, C. T. F., 1935- II. Title.
TJ159.B49 2015
621--dc23
2014024745

ISBN: 9781138781573 (pbk)
ISBN: 9781315769806 (ebk)

Typeset in Times by
Servis Filmsetting Ltd, Stockport, Cheshire

Printed by Bell and Bain Ltd, Glasgow



Mechanical Engineering Principles

Third Edition

Why are competent engineers so vital?

Engineering is among the most important of all professions. It is the authors' opinions that engineers save more lives than medical doctors (physicians). For example, poor water, or the lack of it, is the second largest cause of human death in the world, and if engineers are given the 'tools', they can solve this problem. The largest cause of human death is caused by the malarial mosquito, and even death due to malaria can be decreased by engineers – by providing helicopters for spraying areas infected by the mosquito and making and designing medical syringes and pills to protect people against catching all sorts of diseases. Most medicines are produced by engineers! How does the engineer put 1 mg of 'medicine' precisely and individually into millions of pills, at an affordable price?

Moreover, one of the biggest contributions by humankind was the design of the agricultural tractor, which was designed and built by engineers to increase food production many-fold, for a human population which more-or-less quadruples every century! It is also interesting to note that the richest countries in the world are very heavily industrialized. Engineers create wealth! Most other professions don't!

Even in blue sky projects, engineers play a major role. For example, most rocket scientists are chartered engineers or their equivalents and Americans call their chartered engineers (and their equivalents), scientists. Astronomers are space scientists and not rocket scientists; they could not design a rocket to conquer outer space. Even modern theoretical physicists are mainly interested in astronomy and cosmology and also nuclear science. In general a theoretical physicist cannot, without special training, design a submarine structure to dive to the bottom of the Mariana Trench, which is 11.52 km or 7.16 miles deep, or design a very long bridge, a tall city skyscraper or a rocket to conquer outer space. It may be shown that the load on a submarine pressure hull of diameter 10 m and length 100 m is equivalent to carrying the total weight of about 7 million London double-decker buses!

This book presents a solid foundation for the reader in mechanical engineering principles, on which s/he can safely build tall buildings and long bridges that may last for a thousand years or more. It is the authors' experience that it is most unwise to attempt to build such structures on shaky foundations; they may come tumbling down – with disastrous consequences.

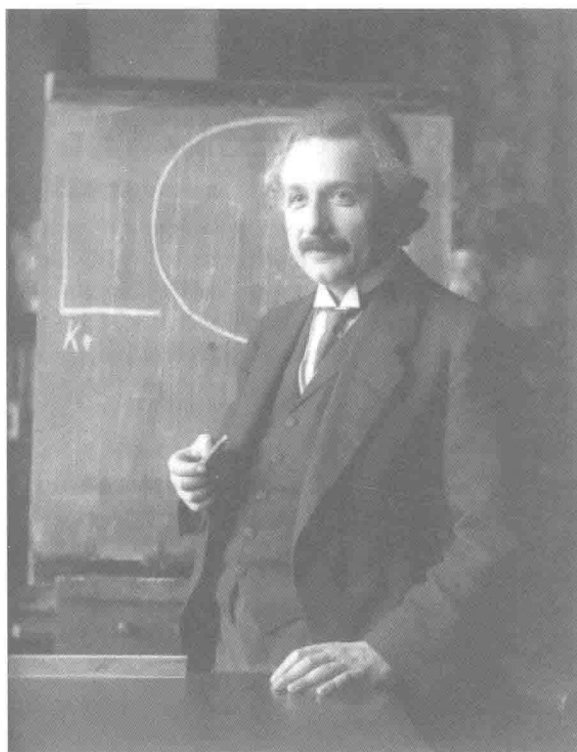
John Bird is the former Head of Applied Electronics in the Faculty of Technology at Highbury College, Portsmouth, U.K. More recently, he has combined freelance lecturing at the University of Portsmouth, with Examiner responsibilities for Advanced Mathematics with City and Guilds, and examining for the International Baccalaureate Organisation. He is the author of over 125 textbooks on engineering and mathematical subjects with worldwide sales of one million copies. He is currently a Senior Training Provider at the Defence School of Marine Engineering in the Defence College of Technical Training at H.M.S. Sultan, Gosport, Hampshire, U.K.

Carl Ross gained his first degree in Naval Architecture, from King's College, Durham University; his PhD in Structural Engineering from the Victoria University of Manchester; and was awarded his DSc in Ocean Engineering from the CNA, London. His research in the field of engineering led to advances in the design of submarine pressure hulls. His publications and guest lectures to date exceed some 290 papers and books, etc., and he is Professor of Structural Dynamics at the University of Portsmouth, UK.

See Carl Ross's website below, which has an enormous content on science, technology and education.

<http://tiny.cc/6kvqhx>

Some quotes from Albert Einstein (14 March 1879–18 April 1955)



‘Scientists investigate that which already is; Engineers create that which has never been’

‘Imagination is more important than knowledge. For knowledge is limited to all we now know and understand, while imagination embraces the entire world, and all there ever will be to know and understand’

‘Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid’

‘To stimulate creativity, one must develop the childlike inclination for play’

Preface

Mechanical Engineering Principles 3rd Edition aims to broaden the reader's knowledge of the basic principles that are fundamental to mechanical engineering design and the operation of mechanical systems.

Modern engineering systems and products still rely upon static and dynamic principles to make them work. Even systems that appear to be entirely electronic have a physical presence governed by the principles of statics.

In this third edition of *Mechanical Engineering Principles*, a further chapter has been added on revisionary mathematics; it is not possible to progress in engineering studies without a reasonable knowledge of mathematics, a fact that soon becomes obvious to both students and teachers alike. It is therefore hoped that this further chapter on mathematics revision will be helpful and make engineering studies more comprehensible. Minor modifications, some further worked problems, a glossary of terms and famous engineers' biographies have all been added to the text.

More has been added to the website for this new edition – such as full solutions being made available to both students and staff, and much more besides – see page x.

For clarity, the text is divided into **four sections**, these being:

- Part 1 Revision of Mathematics**
- Part 2 Statics and Strength of Materials**
- Part 3 Dynamics**
- Part 4 Heat Transfer and Fluid Mechanics**

Mechanical Engineering Principles 3rd Edition is suitable for the following:

- (i) National Certificate/Diploma courses in Mechanical Engineering
- (ii) Undergraduate courses in Mechanical, Civil, Structural, Aeronautical & Marine Engineering, together with Naval Architecture
- (iii) Any introductory/access/foundation course involving Mechanical Engineering Principles at University, and Colleges of Further and Higher education.

Although pre-requisites for the modules covered in this book include Foundation Certificate/diploma, or similar, in Mathematics and Science, **each topic considered in the text is presented in a way that assumes that the reader has little previous knowledge of that topic.**

Mechanical Engineering Principles 3rd Edition contains over **400 worked problems**, followed by over **700 further problems** (all **with answers**). The further problems are contained within some **150 Exercises**; each Exercise follows on directly from the relevant section of work, every few pages. In addition, the text contains **298 multiple-choice questions** (all **with answers**), and **260 short answer questions**, the answers for which can be determined from the preceding material in that particular chapter. Where at all possible, the problems mirror practical situations found in mechanical engineering. **387 line diagrams** enhance the understanding of the theory.

At regular intervals throughout the text are some **9 Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in Chapter 1, Test 2 covers the material in Chapter 2, Test 3 covers the material in Chapters 3 to 6, and so on. No answers are given for the questions in the Revision Tests, but an **Instructor's guide** has been produced giving full solutions and suggested marking scheme. The guide is offered online free to lecturers/instructors – see below.

At the end of the text, a list of relevant **formulae** is included for easy reference, together with a **glossary of terms**.

'**Learning by Example**' is at the heart of *Mechanical Engineering Principles, 3rd Edition*.

JOHN BIRD
Defence College of Technical Training,
HMS Sultan, formerly
University of Portsmouth and
Highbury College, Portsmouth
CARL ROSS Professor, University of Portsmouth

Free Web downloads

The following support material is available from <http://www.routledge.com/cw/bird>

For Students:

1. Full worked solutions to all 700 further questions contained in the 150 Practice Exercises
2. A list of Essential Formulae
3. A full glossary of terms
4. Multiple-choice questions
5. Information on 20 Famous Engineers mentioned in the text

6. Video links to practical demonstrations by Professor Carl Ross <http://tiny.cc/6kvqhxx>

For Lecturers/Instructors:

- 1–6. As per students 1–6 above.
7. Full solutions and marking scheme for each of the 9 Revision Tests; also, each test may be downloaded for distribution to students.
8. All 387 illustrations used in the text may be downloaded for use in PowerPoint presentations.

Contents

Preface	ix
---------	----

Part One Revision of Mathematics 1

1 Revisionary mathematics	3
1.1 Introduction	3
1.2 Radians and degrees	4
1.3 Measurement of angles	4
1.4 Triangle calculations	5
1.5 Brackets	8
1.6 Fractions	8
1.7 Percentages	10
1.8 Laws of indices	12
1.9 Simultaneous equations	14

Revision Test 1 Revisionary mathematics 18

2 Further revisionary mathematics	20
2.1 Units, prefixes and engineering notation	21
2.2 Metric – US/Imperial conversions	24
2.3 Straight line graphs	28
2.4 Gradients, intercepts and equation of a graph	30
2.5 Practical straight line graphs	32
2.6 Introduction to calculus	34
2.7 Basic differentiation revision	34
2.8 Revision of integration	36
2.9 Definite integrals	38
2.10 Simple vector analysis	39

Revision Test 2 Further revisionary mathematics 43

Part Two Statics and Strength of Materials 45

3 The effects of forces on materials	47
3.1 Introduction	48
3.2 Tensile force	48
3.3 Compressive force	48
3.4 Shear force	48
3.5 Stress	49
3.6 Strain	50
3.7 Elasticity, limit of proportionality and elastic limit	52
3.8 Hooke's law	53

3.9 Ductility, brittleness and malleability	57
3.10 Modulus of rigidity	57
3.11 Thermal strain	57
3.12 Compound bars	58

4 Tensile testing	64
4.1 The tensile test	64
4.2 Worked problems on tensile testing	66
4.3 Further worked problems on tensile testing	67
4.4 Proof stress	69

5 Forces acting at a point	71
5.1 Scalar and vector quantities	71
5.2 Centre of gravity and equilibrium	72
5.3 Forces	72
5.4 The resultant of two coplanar forces	73
5.5 Triangle of forces method	74
5.6 The parallelogram of forces method	75
5.7 Resultant of coplanar forces by calculation	76
5.8 Resultant of more than two coplanar forces	76
5.9 Coplanar forces in equilibrium	78
5.10 Resolution of forces	80
5.11 Summary	83

6 Simply supported beams	86
6.1 The moment of a force	86
6.2 Equilibrium and the principle of moments	87
6.3 Simply supported beams having point loads	89
6.4 Simply supported beams with couples	93

Revision Test 3 Forces, tensile testing and beams 97

7 Forces in structures	98
7.1 Introduction	98
7.2 Worked problems on mechanisms and pin-jointed trusses	99
7.3 Graphical method	100
7.4 Method of joints (a mathematical method)	104
7.5 The method of sections (a mathematical method)	109
8 Bending moment and shear force diagrams	112
8.1 Bending moment (M)	112

8.2	Shearing force (F)	113	13.4	Further equations of motion	174
8.3	Worked problems on bending moment and shearing force diagrams	113	13.5	Relative velocity	176
8.4	Uniformly distributed loads	122	14	Linear momentum and impulse	180
9	First and second moments of area	127	14.1	Linear momentum	180
9.1	Centroids	127	14.2	Impulse and impulsive forces	183
9.2	The first moment of area	128	15	Force, mass and acceleration	188
9.3	Centroid of area between a curve and the x -axis	128	15.1	Introduction	188
9.4	Centroid of area between a curve and the y -axis	128	15.2	Newton's laws of motion	189
9.5	Worked problems on centroids of simple shapes	129	15.3	Centripetal acceleration	192
9.6	Further worked problems on centroids of simple shapes	130	15.4	Rotation of a rigid body about a fixed axis	193
9.7	Second moments of area of regular sections	131	15.5	Moment of inertia (I)	194
9.8	Second moment of area for 'built-up' sections	138	16	Work, energy and power	197
Revision Test 4 Forces in structures, bending moment and shear force diagrams, and second moments of area		144	16.1	Work	197
10	Bending of beams	145	16.2	Energy	201
10.1	Introduction	145	16.3	Power	202
10.2	To prove that $\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$	146	16.4	Potential and kinetic energy	205
10.3	Worked problems on the bending of beams	147	16.5	Kinetic energy of rotation	208
11	Torque	151	Revision Test 6 Linear and angular motion, momentum and impulse, force, mass and acceleration, work, energy and power		211
11.1	Couple and torque	151	17	Friction	212
11.2	Work done and power transmitted by a constant torque	152	17.1	Introduction to friction	212
11.3	Kinetic energy and moment of inertia	154	17.2	Coefficient of friction	213
11.4	Power transmission and efficiency	157	17.3	Applications of friction	214
12	Twisting of shafts	161	17.4	Friction on an inclined plane	215
12.1	To prove that $\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$	161	17.5	Motion up a plane with the pulling force P parallel to the plane	215
12.2	Worked problems on the twisting of shafts	163	17.6	Motion down a plane with the pulling force P parallel to the plane	216
Revision Test 5 Bending of beams, torque and twisting of shafts		167	17.7	Motion up a plane due to a horizontal force P	216
Part Three Dynamics		169	17.8	The efficiency of a screw jack	219
13	Linear and angular motion	171	18	Motion in a circle	223
13.1	The radian	171	18.1	Introduction	223
13.2	Linear and angular velocity	171	18.2	Motion on a curved banked track	225
13.3	Linear and angular acceleration	173	18.3	Conical pendulum	226
			18.4	Motion in a vertical circle	228
			18.5	Centrifugal clutch	230
			19	Simple harmonic motion	232
			19.1	Introduction to simple harmonic motion (SHM)	232
			19.2	The spring-mass system	233
			19.3	The simple pendulum	235
			19.4	The compound pendulum	236
			19.5	Torsional vibrations	237
			20	Simple machines	239
			20.1	Machines	239

20.2	Force ratio, movement ratio and efficiency	239	24	Fluid flow	290
20.3	Pulleys	241	24.1	Differential pressure flowmeters	290
20.4	The screw-jack	243	24.2	Orifice plate	291
20.5	Gear trains	243	24.3	Venturi tube	292
20.6	Levers	245	24.4	Flow nozzle	292
Revision Test 7 Friction, motion in a circle, simple harmonic motion and simple machines			24.5	Pitot-static tube	292
			24.6	Mechanical flowmeters	293
			24.7	Deflecting vane flowmeter	293
			24.8	Turbine type meters	294
			24.9	Float and tapered-tube meter	294
			24.10	Electromagnetic flowmeter	295
			24.11	Hot-wire anemometer	296
			24.12	Choice of flowmeter	296
			24.13	Equation of continuity	296
			24.14	Bernoulli's equation	297
			24.15	Impact of a jet on a stationary plate	298
Part Four Heat Transfer and Fluid Mechanics			25	Ideal gas laws	301
			25.1	Boyle's law	301
21	Heat energy and transfer	253	25.2	Charles' law	303
21.1	Introduction	253	25.3	The pressure or Gay-Lussac's law	304
21.2	The measurement of temperature	254	25.4	Dalton's law of partial pressure	305
21.3	Specific heat capacity	255	25.5	Characteristic gas equation	306
21.4	Change of state	256	25.6	Worked problems on the characteristic gas equation	306
21.5	Latent heats of fusion and vaporisation	257	25.7	Further worked problems on the characteristic gas equation	308
21.6	A simple refrigerator	259	26	The measurement of temperature	312
21.7	Conduction, convection and radiation	259	26.1	Liquid-in-glass thermometer	312
21.8	Vacuum flask	260	26.2	Thermocouples	314
21.9	Use of insulation in conserving fuel	260	26.3	Resistance thermometers	315
22	Thermal expansion	263	26.4	Thermistors	317
22.1	Introduction	263	26.5	Pyrometers	317
22.2	Practical applications of thermal expansion	264	26.6	Temperature indicating paints and crayons	319
22.3	Expansion and contraction of water	264	26.7	Bimetallic thermometers	319
22.4	Coefficient of linear expansion	264	26.8	Mercury-in-steel thermometer	319
22.5	Coefficient of superficial expansion	266	26.9	Gas thermometers	319
22.6	Coefficient of cubic expansion	267	26.10	Choice of measuring devices	320
Revision Test 8 Heat energy and transfer, and thermal expansion			Revision Test 9 Hydrostatics, fluid flow, gas laws and temperature measurement		322
23	Hydrostatics	272	<i>A list of formulae for mechanical engineering principles</i> 323		
23.1	Pressure	272	<i>Metric to Imperial conversions and vice versa</i> 328		
23.2	Fluid pressure	274	<i>Greek alphabet</i> 329		
23.3	Atmospheric pressure	275	<i>Glossary of terms</i> 330		
23.4	Archimedes' principle	276	<i>Answers to multiple-choice questions</i> 335		
23.5	Measurement of pressure	278	Index 337		
23.6	Barometers	278			
23.7	Absolute and gauge pressure	280			
23.8	The manometer	280			
23.9	The Bourdon pressure gauge	281			
23.10	Vacuum gauges	282			
23.11	Hydrostatic pressure on submerged surfaces	282			
23.12	Hydrostatic thrust on curved surfaces	284			
23.13	Buoyancy	284			
23.14	The stability of floating bodies	284			

Part One

Revision of Mathematics

Chapter 1

Revisionary mathematics

Why it is important to understand: Revisionary mathematics

Mathematics is a vital tool for professional and chartered engineers. It is used in mechanical & manufacturing engineering, in electrical & electronic engineering, in civil & structural engineering, in naval architecture & marine engineering and in aeronautical & rocket engineering. In these various branches of engineering, it is very often much cheaper and safer to design your artefact with the aid of mathematics – rather than through guesswork. ‘Guesswork’ may be reasonably satisfactory if you are designing an artefact similar to one that has already proven satisfactory; however, the classification societies will usually require you to provide the calculations proving that the artefact is safe and sound. Moreover, these calculations may not be readily available to you and you may have to provide fresh calculations, to prove that your artefact is ‘roadworthy’. For example, if you design a tall building or a long bridge by ‘guesswork’, and the building or bridge do not prove to be structurally reliable, it could cost you a fortune to rectify the deficiencies. This cost may dwarf the initial estimate you made to construct these artefacts, and cause you to go bankrupt. Thus, without mathematics, the prospective professional or chartered engineer is very severely handicapped.

At the end of this chapter you should be able to:

- convert radians to degrees
- convert degrees to radians
- calculate sine, cosine and tangent for large and small angles
- calculate the sides of a right-angled triangle
- use Pythagoras’ theorem
- use the sine and cosine rules for acute-angled triangles
- expand equations containing brackets
- be familiar with summing vulgar fractions
- understand and perform calculations with percentages
- understand and use the laws of indices
- solve simple simultaneous equations

1.1 Introduction

As highlighted above, it is not possible to understand aspects of mechanical engineering without a good

knowledge of mathematics. This chapter highlights some areas of mathematics which will make the understanding of the engineering in the following chapters a little easier.

1.2 Radians and degrees

There are 2π radians or 360° in a complete circle, thus:

$$\pi \text{ radians} = 180^\circ \quad \text{from which,}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi} \quad \text{or} \quad 1^\circ = \frac{\pi}{180} \text{ rad}$$

where $\pi = 3.14159265358979323846 \dots$ to 20 decimal places!

Problem 1. Convert the following angles to degrees correct to 3 decimal places:

- (a) 0.1 rad (b) 0.2 rad (c) 0.3 rad

- (a) $0.1 \text{ rad} = 0.1 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \mathbf{5.730^\circ}$
 (b) $0.2 \text{ rad} = 0.2 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \mathbf{11.459^\circ}$
 (c) $0.3 \text{ rad} = 0.3 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \mathbf{17.189^\circ}$

Problem 2. Convert the following angles to radians correct to 4 decimal places:

- (a) 5° (b) 10° (c) 30°

- (a) $5^\circ = 5^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{\pi}{36} \text{ rad} = \mathbf{0.0873 \text{ rad}}$
 (b) $10^\circ = 10^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{\pi}{18} \text{ rad} = \mathbf{0.1745 \text{ rad}}$
 (c) $30^\circ = 30^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{\pi}{6} \text{ rad} = \mathbf{0.5236 \text{ rad}}$

Now try the following Practice Exercise

Practice Exercise 1 Radians and degrees

1. Convert the following angles to degrees correct to 3 decimal places (where necessary):

- (a) 0.6 rad (b) 0.8 rad
 (c) 2 rad (d) 3.14159 rad
 [(a) 34.377° (b) 45.837°]
 [(c) 114.592° (d) 180°]

2. Convert the following angles to radians correct to 4 decimal places:

- (a) 45° (b) 90°
 (c) 120° (d) 180°

- [(a) $\frac{\pi}{4} \text{ rad}$ or 0.7854 rad
 (b) $\frac{\pi}{2} \text{ rad}$ or 1.5708 rad
 (c) $\frac{2\pi}{3} \text{ rad}$ or 2.0944 rad
 (d) $\pi \text{ rad}$ or 3.1416 rad]

1.3 Measurement of angles

Angles are measured starting from the horizontal 'x' axis, in an **anticlockwise direction**, as shown by θ_1 to θ_4 in Figure 1.1. An angle can also be measured in a **clockwise direction**, as shown by θ_5 in Figure 1.1, but in this case the angle has a negative sign before it. If, for example, $\theta_4 = 300^\circ$ then $\theta_5 = -60^\circ$.

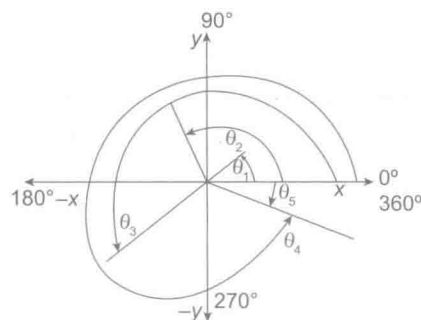


Figure 1.1

Problem 3. Use a calculator to determine the cosine, sine and tangent of the following angles, each measured anticlockwise from the horizontal 'x' axis, each correct to 4 decimal places:

- (a) 30° (b) 120° (c) 250°
 (d) 320° (e) 390° (f) 480°

- (a) $\cos 30^\circ = \mathbf{0.8660}$ $\sin 30^\circ = \mathbf{0.5000}$
 $\tan 30^\circ = \mathbf{0.5774}$
 (b) $\cos 120^\circ = \mathbf{-0.5000}$ $\sin 120^\circ = \mathbf{0.8660}$
 $\tan 120^\circ = \mathbf{-1.7321}$
 (c) $\cos 250^\circ = \mathbf{-0.3420}$ $\sin 250^\circ = \mathbf{-0.9397}$
 $\tan 250^\circ = \mathbf{2.7475}$
 (d) $\cos 320^\circ = \mathbf{0.7660}$ $\sin 320^\circ = \mathbf{-0.6428}$
 $\tan 320^\circ = \mathbf{-0.8391}$

$$(e) \cos 390^\circ = 0.8660 \quad \sin 390^\circ = 0.5000$$

$$\tan 390^\circ = 0.5774$$

$$(f) \cos 480^\circ = -0.5000 \quad \sin 480^\circ = 0.8660$$

$$\tan 480^\circ = -1.7321$$

These angles are now drawn in Figure 1.2. Note that cosine and sine always lie between -1 and $+1$ but that tangent can be >1 and <1

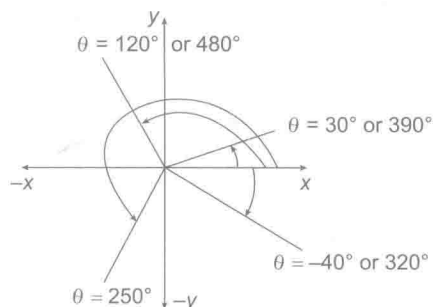


Figure 1.2

Note from Figure 1.2 that $\theta = 30^\circ$ is the same as $\theta = 390^\circ$ and so are their cosines, sines and tangents. Similarly, note that $\theta = 120^\circ$ is the same as $\theta = 480^\circ$ and so are their cosines, sines and tangents. Also, note that $\theta = -40^\circ$ is the same as $\theta = +320^\circ$ and so are their cosines, sines and tangents.

It is noted from above that

- in the **first quadrant**, i.e. where θ varies from 0° to 90° , all (A) values of cosine, sine and tangent are positive
- in the **second quadrant**, i.e. where θ varies from 90° to 180° , only values of sine (S) are positive
- in the **third quadrant**, i.e. where θ varies from 180° to 270° , only values of tangent (T) are positive
- in the **fourth quadrant**, i.e. where θ varies from 270° to 360° , only values of cosine (C) are positive

These positive signs, A , S , T and C are shown in Figure 1.3.

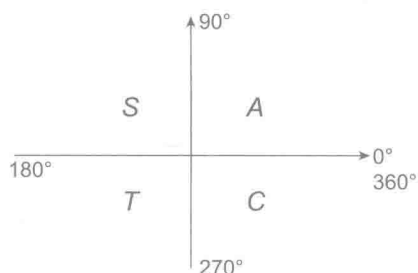


Figure 1.3

Now try the following Practice Exercise

Practice Exercise 2 Measurement of angles

- Find the cosine, sine and tangent of the following angles, where appropriate each correct to 4 decimal places:

- | | | |
|-----------------|-----------------|-----------------|
| (a) 60° | (b) 90° | (c) 150° |
| (d) 180° | (e) 210° | (f) 270° |
| (g) 330° | (h) -30° | (i) 420° |
| (j) 450° | (k) 510° | |

- [(a) 0.5, 0.8660, 1.7321
 (b) 0, 1, ∞
 (c) -0.8660 , 0.5, -0.5774
 (d) -1 , 0, 0
 (e) -0.8660 , -0.5 , 0.5774
 (f) 0, -1 , $-\infty$
 (g) 0.8660 , -0.5000 , -0.5774
 (h) 0.8660 , -0.5000 , -0.5774
 (i) 0.5, 0.8660, 1.7321
 (j) 0, 1, ∞
 (k) -0.8660 , 0.5, -0.5774]

1.4 Triangle calculations

(a) Sine, cosine and tangent

From Figure 1.4, $\sin \theta = \frac{bc}{ac}$ $\cos \theta = \frac{ab}{ac}$

$$\tan \theta = \frac{bc}{ab}$$

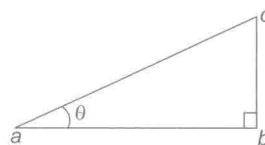


Figure 1.4

Problem 4. In Figure 1.4, if $ab = 2$ and $ac = 3$, determine the angle θ .

It is convenient to use the expression for $\cos \theta$, since 'ab' and 'ac' are given.

Hence, $\cos \theta = \frac{ab}{ac} = \frac{2}{3} = 0.66667$

from which, $\theta = \cos^{-1}(0.66667) = 48.19^\circ$

Problem 5. In Figure 1.4, if $bc = 1.5$ and $ac = 2.2$, determine the angle θ .

It is convenient to use the expression for $\sin \theta$, since 'bc' and 'ac' are given.

Hence, $\sin \theta = \frac{bc}{ac} = \frac{1.5}{2.2} = 0.68182$

from which, $\theta = \sin^{-1}(0.68182) = 42.99^\circ$

Problem 6. In Figure 1.4, if $bc = 8$ and $ab = 1.3$, determine the angle θ .

It is convenient to use the expression for $\tan \theta$, since 'bc' and 'ab' are given.

Hence, $\tan \theta = \frac{bc}{ab} = \frac{8}{1.3} = 6.1538$

from which, $\theta = \tan^{-1}(6.1538) = 80.77^\circ$

(b) Pythagoras' theorem

Pythagoras' theorem* states that:
(hypotenuse)² = (adjacent side)² + (opposite side)²
i.e. in the triangle of Figure 1.5,

$$ac^2 = ab^2 + bc^2$$

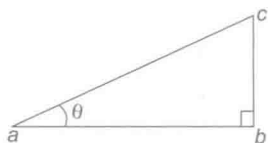


Figure 1.5

Problem 7. In Figure 1.5, if $ab = 5.1$ m and $bc = 6.7$ m, determine the length of the hypotenuse, ac .

From Pythagoras, $ac^2 = ab^2 + bc^2$
 $= 5.1^2 + 6.7^2 = 26.01 + 44.89$
 $= 70.90$

from which, $ac = \sqrt{70.90} = 8.42$ m

Now try the following Practice Exercise

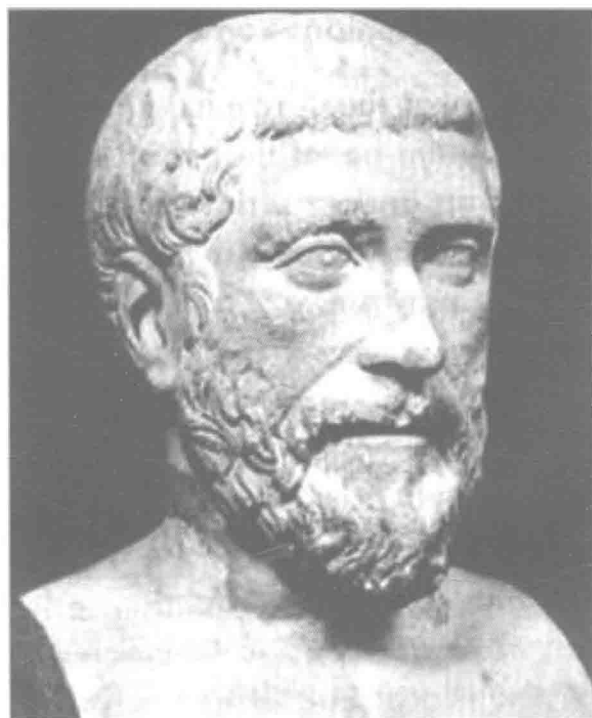
Practice Exercise 3 Sines, cosines and tangents and Pythagoras' theorem

In problems 1 to 5, refer to Figure 1.5.

1. If $ab = 2.1$ m and $bc = 1.5$ m, determine angle θ . [35.54°]
2. If $ab = 2.3$ m and $ac = 5.0$ m, determine angle θ . [62.61°]
3. If $bc = 3.1$ m and $ac = 6.4$ m, determine angle θ . [28.97°]
4. If $ab = 5.7$ cm and $bc = 4.2$ cm, determine the length ac . [7.08 cm]
5. If $ab = 4.1$ m and $ac = 6.2$ m, determine length bc . [4.65 m]

(c) The sine and cosine rules

For the triangle ABC shown in Figure 1.6,



*Pythagoras of Samos (born approximately 570BC and died around 495BC) was an Ionian Greek philosopher and mathematician, best known for the Pythagorean Theorem. To find out more go to www.routledge.com/cw/bird

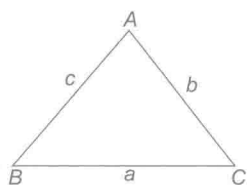


Figure 1.6

the sine rule states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

and the cosine rule states: $a^2 = b^2 + c^2 - 2bc \cos A$

Problem 8. In Figure 1.6, if $a = 3$ m, $A = 20^\circ$ and $B = 120^\circ$, determine lengths b , c and angle C .

Using the sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\text{i.e.} \quad \frac{3}{\sin 20^\circ} = \frac{b}{\sin 120^\circ}$$

$$\begin{aligned} \text{from which, } b &= \frac{3 \sin 120^\circ}{\sin 20^\circ} = \frac{3 \times 0.8660}{0.3420} \\ &= 7.596 \text{ m} \end{aligned}$$

$$\text{Angle, } C = 180^\circ - 20^\circ - 120^\circ = 40^\circ$$

Using the sine rule again gives: $\frac{c}{\sin C} = \frac{a}{\sin A}$

$$\begin{aligned} \text{i.e.} \quad c &= \frac{a \sin C}{\sin A} = \frac{3 \times \sin 40^\circ}{\sin 20^\circ} \\ &= 5.638 \text{ m} \end{aligned}$$

Problem 9. In Figure 1.6, if $b = 8.2$ cm, $c = 5.1$ cm and $A = 70^\circ$, determine the length a and angles B and C .

From the cosine rule,

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= 8.2^2 + 5.1^2 - 2 \times 8.2 \times 5.1 \times \cos 70^\circ \\ &= 67.24 + 26.01 - 2(8.2)(5.1)\cos 70^\circ \\ &= 64.643 \end{aligned}$$

$$\text{Hence, length, } a = \sqrt{64.643} = 8.04 \text{ cm}$$

$$\text{Using the sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\text{i.e.} \quad \frac{8.04}{\sin 70^\circ} = \frac{8.2}{\sin B}$$

$$\text{from which, } 8.04 \sin B = 8.2 \sin 70^\circ$$

$$\text{and} \quad \sin B = \frac{8.2 \sin 70^\circ}{8.04} = 0.95839$$

$$\text{and} \quad B = \sin^{-1}(0.95839) = 73.41^\circ$$

Since $A + B + C = 180^\circ$, then

$$C = 180^\circ - A - B = 180^\circ - 70^\circ - 73.41^\circ = 36.59^\circ$$

Now try the following Practice Exercise

Practice Exercise 4 Sine and cosine rules

In problems 1 to 4, refer to Figure 1.6.

1. If $b = 6$ m, $c = 4$ m and $B = 100^\circ$, determine angles A and C and length a .
[$A = 38.96^\circ$, $C = 41.04^\circ$, $a = 3.83$ m]
2. If $a = 15$ m, $c = 23$ m and $B = 67^\circ$, determine length b and angles A and C .
[$b = 22.01$ m, $A = 38.86^\circ$, $C = 74.14^\circ$]
3. If $a = 4$ m, $b = 8$ m and $c = 6$ m, determine angle A .
[28.96°]
4. If $a = 10.0$ cm, $b = 8.0$ cm and $c = 7.0$ cm, determine angles A , B and C .
[$A = 83.33^\circ$, $B = 52.62^\circ$, $C = 44.05^\circ$]
5. In Figure 1.7, PR represents the inclined jib of a crane and is 10.0 m long. PQ is 4.0 m long. Determine the inclination of the jib to the vertical (i.e. angle P) and the length of tie QR .

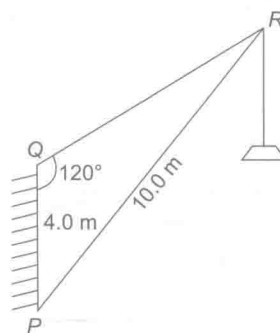


Figure 1.7

$$[P = 39.73^\circ, QR = 7.38 \text{ m}]$$