# NUCLEAR TECHNIQUES IN DIAGNOSTIC MEDICINE

edited by
P.P. van Rijk

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# Preface

This book was conceived by Professor K.H. Ephraim, the former director of the Institute for Nuclear Medicine of the University Hospital of Utrecht. Unfortunately, due to a serious illness, he was not able to finish the work he started. He is, however, very pleased to know that the book is, nevertheless, being published. In principle the volume consists of two parts.

The first is dedicated to basic science and technology in nuclear medicine. It provides the data which are necessary to a clear understanding of the possibilities and limitations of investigations which make use of radioactive materials.

The second part of the book covers those disciplines in medicine in which nuclear medicine can be of help in solving certain clinical problems. Each chapter can be read separately, even without thorough knowledge of the first part of the book.

The contributors to this book come from both Europe and North-America. Each of them has written his chapter out of long-standing personal interest in his particular field of nuclear medicine.

This book will be of value to a wide variety of professionals. It is of interest not only to clinicians of various specialties, but also to diagnostic professionals, i.e. radiologists and nuclear medicine clinicians. Last but not least it will be of use to physicians in training.

It is our hope that by linking technology and clinical problems as we have done, a more precise application of nuclear medicine investigation will result.

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# Contents

# 1. Radioactivity: Measurements and instrumentation *

M. de BRUIN

## 1. THE DECAY OF RADIOACTIVE NUCLEI

### 1.1 *Introduction*

An atomic nucleus can be considered to consist of two kinds of nuclear particles: the positively charged protons and uncharged neutrons. When these nuclear particles or nucleons combine to form an atomic nucleus, the binding energy $E_b$ is released. Since the total energy content of the nucleus is lower than the sum of the energies of the individual nucleons, the mass of the nucleus is lower than the sum of the masses of the composing nucleons. This mass defect $\Delta m$ and the binding energy $E_b$ are related according to Einstein's relation.

$$E_b = \Delta mc^2$$

where: c = the velocity of light in vacuum.

For the nucleus $^{12}_{6}C$ this leads to:

$$6\,E_p + 6\,E_n = E(^{12}_{6}C) + E_b$$

and $$6\,m_p c^2 + 6\,m_n c^2 = m(^{12}_{6}C)\,c^2 + E_b$$

On basis of this relation and the masses of the proton, neutron and the nucleus $^{12}_{6}C$, the total binding energy and the binding energy per nucleon can be calculated. However, the commonly used mass tables list the masses of neutral atoms, including the atomic electrons, so that these atomic electrons have to be included in the mass and energy balance:

$$6\,E_p + 6\,E_n + 6\,E_e = E(^{12}_{6}C) + E_{b,n} + E_{b,e}$$

where: $E_{b,n}$ = binding energy of the nucleons;
$E_{b,e}$ = binding energy of the atomic electrons.

* For Definitions see page 66.

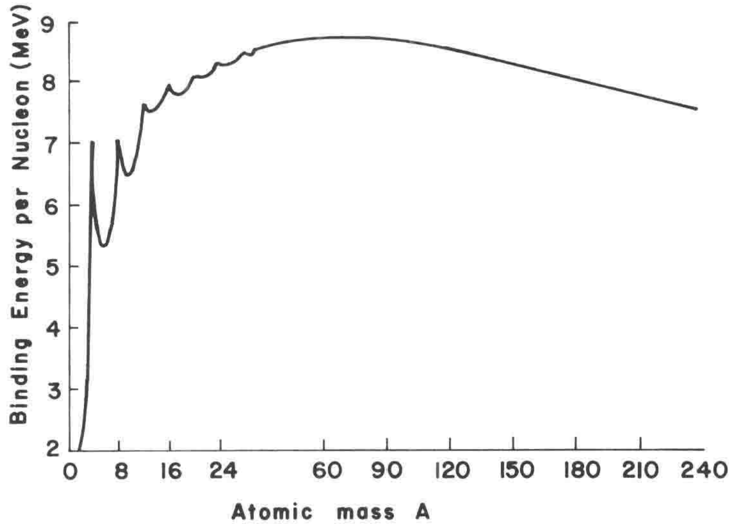


*Figure 1.* Binding energy per nucleon for stable nuclides as function of increasing mass number.

$E_{b,e}$ is approximately $10^5$ times smaller than $E_{b,n}$ and can be neglected in calculations of binding energies from mass defects. Calculation of the binding energy of the nucleons in $^{12}_{6}C$ leads to:

| | | | |
|---|---|---|---|
| mass 6 protons | = | 6.0437 amu | |
| mass 6 neutrons | = | 6.0520 amu | |
| mass 6 electrons | = | 0.0033 amu | |
| total | | 12.0990 amu | |
| mass atom $^{12}_{6}C$ | = | 12.0000 amu | (by definition!) |
| mass defect | | 0.0990 amu | $= 0.0990 \times 931.5$ MeV $= 92.2$ MeV |

For $^{12}_{6}C$ the total binding energy is 92.2 MeV and the mean binding energy per nucleon is 7.7 MeV. Figure 1 shows the dependence of the binding energy per nucleon on the mass number of the atom considered. This figure also indicates that energy can be obtained from fusion of light nuclei as well as from fission of heavy nuclei.

The existence of a maximum of the binding energy suggests that all light and heavy atoms might transmute by fusion or fission to atoms with mass numbers close to 60, but such a preference has played a role only during the early days of the earth, resulting in relatively high concentrations of iron and nickel in the total earth mass (34.6 and 2.4 percent respectively). For most natural nuclei a high potential barrier inhibits spontaneous transmution under terrestrial conditions, Even for the "easily" fusable tritium ($^3_1H$), energy has to be supplied until an apparent temperature of $10^7$ °K is reached, before fusion occurs.

## 1.2 *Decay schemes, chart of the nuclides*

The decay (disintegration) of nuclei of a specific nuclide has three properties characteristic for that specific nuclide:

disintegration probability;
type of transition;
transition energy.

For practical application of radioactive nuclides and for selection of an appropriate radiation detection method, basic information on the indicated properties is required. This information is represented in a comprehensive way in decay schemes and charts of nuclides.

Decay schemes are, for each mass number, a two dimensional representation of the known isobars, with in horizontal direction the atomic number and in vertical direction the possible energy states of the isobaric nuclides (Figure 2).

In a chart of the nuclides, all nuclides are represented in a single two dimensional graph. They are shown in order of increasing number of neutrons and number of protons along the horizontal and vertical axes respectively. For stable nuclides, atomic weight and natural abundance are indicated; for radioactive nuclides the chart gives a limited amount of information on the decay properties (Figure 3).

In the discussion of the individual decay processes, attention will be paid to the representation of these processes in decay schemes and chart of nuclides.

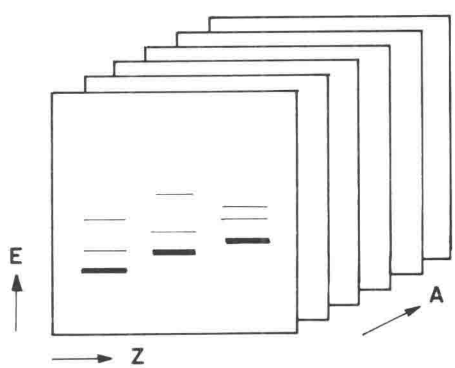


*Figure 2.* Organisation of decay schemes.

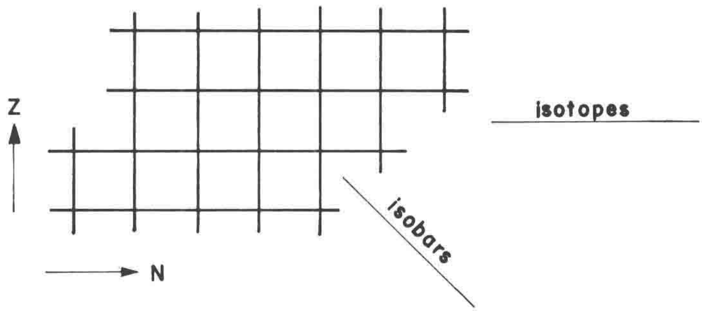


*Figure 3.* Organisation of a chart of nuclides.

## 1.3 *Transition probability, half-life*

### 1.3.a *General*

The decay constant $\lambda$ is a measure for the decay rate of a nuclide. This decay constant is the probability of a transition of a nucleus to occur within one second; the dimension of $\lambda$ is $s^{-1}$. For a radioactive sample, at the moment t consisting of N (t) atoms of a radioactive nuclide, the decay rate or activity A (t) is given by

$$A(t) = \lambda N(t)$$

When $\lambda$ is known, for each other value of t the corresponding activity can be calculated. For these calculations one can distinguish different cases, the most common ones being:

a. simple decay;
b. decay of a nuclide formed in the decay of an other nuclide;
c. decay of a nuclide formed with a constant rate.

### 1.3.b *Simple decay*

The activity of a radioactive source at the point of time t is given by:

$$A(t) = \lambda N(t)$$

As a result of the decay, the number of radioactive nuclei decreases:

$$dN = -\lambda \cdot N \cdot dt$$

Integration of this formula yields:

$$N(t) = N(0)\, e^{-\lambda t}$$

and:

$$A(t) = A(0)\, e^{-\lambda t}$$

where: N (0), A (0) and N (t), A (t) represent the numbers of nuclei present or

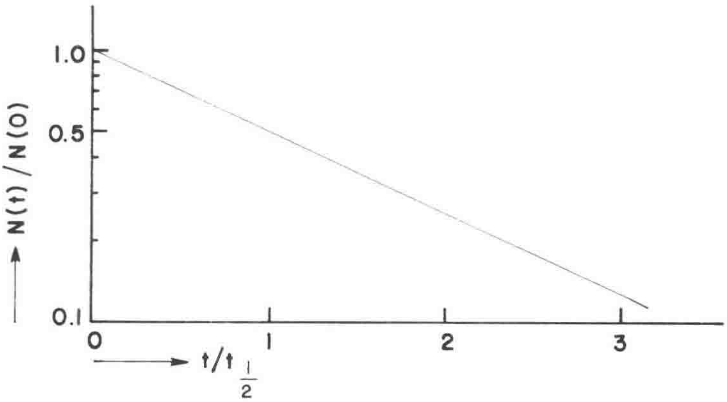


*Figure 4.* Exponential decay.

activities at time t = 0 and t = t respectively; e is the base of the natural logarithm.

Such an exponential relation is graphically represented by a straight line in a graph of log N(t) or log A(t) versus t (Figure 4).

The half-life $t_{\frac{1}{2}}$ of a nuclide is the period required for the number of such nuclei to reduce to one half of the number of nuclei originally present:

$$N(t_{\frac{1}{2}}) = \tfrac{1}{2} N(0) = N(0)\, e^{-\lambda t_{\frac{1}{2}}}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

and:

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}}$$

1.3.c *Decay of a nuclide formed in the decay of an other nuclide*
In this situation a radioactive nuclide Y is formed in the decay of a 'parent' nuclide X. Between X and Y exists a 'parent-daughter' relation

$$X \rightarrow Y \rightarrow$$

The rate of increase and decrease $dN_y$ of $N_y$ is the difference between formation rate $\lambda_x N_x$ of Y from the decay of X, and the decay rate $\lambda_y N_y$ of Y:

$$\frac{dN_y}{dt} = \lambda_x N_x - \lambda_y N_y$$

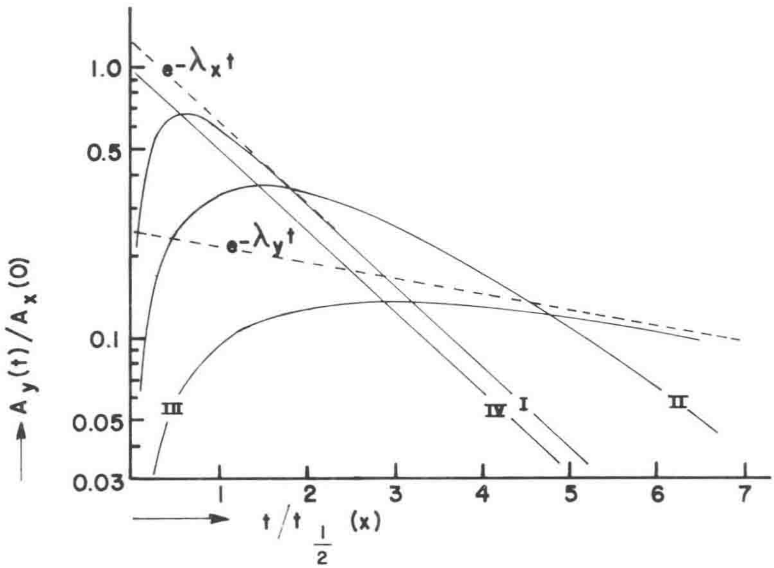


*Figure 5.* Activity of a nuclide Y formed in the decay of a nuclide X for: $\lambda_x = \frac{1}{5}\lambda_y$ (I), $\lambda_x = \lambda_y$ (II) and $\lambda_x = 5\,\lambda_y$ (III). Curve IV represents the activity of X.

Integration of this formula leads to a relation between $A_y(t)$ and t:

$$A_y(t) = A_x(0) \cdot \frac{\lambda_y}{\lambda_y - \lambda_x} \cdot (e^{-\lambda_x t} - e^{-\lambda_y t})$$

For $\lambda_x \gg \lambda_y$ and $\lambda_x t \gg 1$, this relation can be approximated by:

$$A_y(t) = A_x(0) \cdot - \frac{\lambda_y}{\lambda_y - \lambda_x} \cdot e^{-\lambda_y t}$$

Under the conditions indicated, the activity of Y decays according to the decay constant of Y.

For $\lambda_y \gg \lambda_x$ and $\lambda_x t \gg 1$, $A_y(t)$ can be expressed by:

$$A_y(t) = A_x(0) \cdot \frac{\lambda_y}{\lambda_y - \lambda_x} \cdot e^{-\lambda_x t}$$

In this case the activity of Y decreases with the decay constant of the parent nuclide.

In Figure 5 $A_y$ is represented as a function of t for $\lambda_x \gg \lambda_y$ $\lambda_x \simeq \lambda_y$ and $\lambda_y \gg \lambda_x$ respectivily.

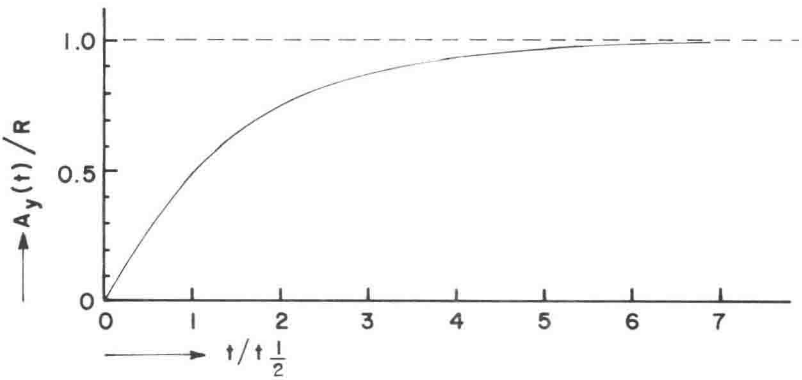


*Figure 6.* Activity of a nuclide formed with constant rate R.

Parent-daughter relations are of importance for nuclide generators, in which the product nuclide is always a daughter of a parent nuclide with longer half-life.

1.3.d *Decay of a nuclide formed with a constant rate*

For a nuclide X, formed with a constant rate of R nuclei per second, the following relation is applicable:

$$\frac{dN_x}{dt} = R - \lambda_x N_x$$

Integration yields:

$$N_x(t) = \frac{R}{\lambda_x}(1 - e^{-\lambda_x t})$$

and $$A_x(t) = R(1 - e^{-\lambda_x t})$$

This relation between $A_x(t)$ and t is represented in Figure 6. From the formulas and the figure it can be observed that for $t > 8\,t_{\frac{1}{2}}$, $A_x(t)$ reaches a constant value, referred to as the saturation activity.

Examples of nuclides formed with constant rates are the daughters of nuclides with extreme long half-lives ($^{226}Ra$ and $^{232}Th$) and the formation of nuclides through nuclear reactions in a fission reactor or particle accelerator.

## 1.4 *Modes of decay*

### 1.4.a *Introduction*

The decay of nuclei of radioactive atoms may occur through different pathways. Although the desintegration of a nucleus in general takes place through more than one process, in this chapter these processes will be discussed separately. The processes are successively:

$\beta^-$-emission
electron capture
$\beta^+$-emission
$\alpha$-emission
$\gamma$-ray emission
internal conversion
spontaneous fission

Moreover, attention will be paid to time relations existing between successive processes and to atomic processes accompanying nuclear transitions.

### 1.4.b $\beta^-$*-emission*

In $\beta^-$-emission a negative electron is emitted by the decaying nucleus, and the charge of the nucleus increases with one charge unit. Together with the electron a neutrino is emitted and the available energy surplus ($Q_{\beta^-}$) is carried by the electron and the neutrino. To compensate for the additional charge of the nucleus an atomic electron is captured. The mass of the emitted and captured electrons cancel in the mass balance:

$$Q_{\beta^-} = \{m({}^{A}_{Z}M) - m({}_{Z+1}^{\;\;A}M)\}\, c^2$$
$$Q_{\beta^-} = E_{\beta^-} + E_{\nu}$$
$$0 \leq E_{\beta^-} \leq Q_{\beta^-}$$
$$0 \leq E_{\nu} \leq Q_{\beta^-}$$

*Table I.* $\beta^-$-emitting nuclides

| Nuclides | $E_{\beta^-}$(max) in keV | $t_{\frac{1}{2}}$ |
|---|---|---|
| $^{3}H$ | 19 | 12.35 Y |
| $^{14}C$ | 156 | 5730 Y |
| $^{32}P$ | 1710 | 14.3 d |
| $^{40}K$ | 1340 | $1.26 \times 10^{9}$ Y |
| $^{45}Ca$ | 255 | 165 d |
| $^{131}I$ | 610 | 8.07 d |