CONCEPTS OF NUCLEAR PHYSICS

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Preface

The structure of nuclei is now about as well-understood as the electronic structure of atoms, but there is a tremendous difference in the extent to which this understanding has been diffused. Atomic structure is taught for the first time in the fourth or fifth grade of elementary school, and more advanced treatments are presented at least twice more before the end of secondary school. Modern-physics courses covering atomic structure are taken by nearly all scientists and engineers on an elementary level and by all physics majors at the senior level. A course on quantum mechanics, required of all physics graduate students, includes more details of atomic structure.

The structure of nuclei, on the other hand, is not taught at all in elementary and secondary schools and is essentially ignored even in the education of physics majors up to the advanced graduate level. Thus while atomic structure is familiar in outline to hundreds of millions of people, nuclear structure is not even familiar to many with Ph.D.s in physics.

Often an elective advanced undergraduate or first-year graduate course in nuclear physics is offered, but it usually devotes a large block of time to experimental aspects of the subject and gives highly phenomenological treatments of decay and reaction processes, heavily influenced by the historical order in which things were discovered. Only near the end does it present a short discussion of nuclear models, in which the impression is given that we are still digging in the dark in our efforts to understand nuclear structure. This would be the equivalent of teaching atomic physics by spending the majority of time on such subjects as the nature of light, optical spectroscopy, and atomic collisions, and only briefly near the end by presenting a brief discussion on models of atomic structure.

Atomic physics is taught in a much more logical way, and this book represents an effort to introduce similar logic into the teaching of an advanced undergraduate or first-year graduate course in nuclear physics. The only absolute prerequisite is an elementary course in modern physics such as the one usually taught as part of the elementary physics sequence. Whenever matters are discussed which would not be understandable to students with that preparation, this is clearly indicated in the text and these discussions can be omitted without loss of continuity. Quantum theory is widely used, but it is reviewed for these students in Chapter 2 with a further extension in Section 10-1. Every physicist has his own way of introducing quantum mechanics, and many may not like my approach. For this I can only apologize and encourage instructors to handle the subject in accordance with their own tastes.

The book originally developed out of courses for students with this minimal preparation given at the University of Pittsburgh during the fall terms of 1967 and 1968, although many sections were omitted or covered only briefly. On the other hand, concepts which can best be understood by the use of more advanced quantum-mechanical techniques are generally treated in that way for the benefit of advanced students. The book was used for a first-year graduate course in the spring of 1969. With all the advanced material included and nearly all the book being covered, there was no indication that the course was too easy.

To my colleagues working in nuclear physics, I would like to offer apologies for weighting the material covered heavily toward areas in which I have had research experience. I find it most difficult to write about subjects I do not thoroughly understand, and most subjects on which I have had no research experience fall into that category. I also want to apologize for so frequently using my own work in examples. This has the advantages that the results are readily available and well-understood, they are usually presented in a manner attuned to my tastes and my style of writing, and the original data are available for replotting or combining in different ways. (A special apology is in order for the use of our old data in Figure 13-4 when so much newer and better data are available, but after hours of searching, I could find nothing that gives coverage to the full mass range.) To avoid giving an unbalanced impression, I have not included authors' names on figures from our data, for which I apologize to my collaborators.

I am greatly indebted to Miss Barbara Ezarik for an outstanding job of typing, to Drs. F. Tabakin, N. Austern, E. Sanderson, R. M. Drisko, D. A. Bromley, and R. A. Sorenson for helpful discussions and suggestions, and to the students who suffered through the developmental stages of this material without a textbook to fall back on.

Bernard L. Cohen

Contents

Chapter 1
Introduction to the Nucleus

An atom consists of a small, massive core called the *nucleus*, surrounded by orbiting electrons. It is the purpose of this book to explain all aspects of the nucleus, its structure, its behavior under various conditions, and its effect on nature and on mankind. In this chapter we introduce some of its most basic characteristics, its mass, size, shape, and other externally observable properties. We also consider some deeper questions such as the force that holds the nucleus together and the mechanical laws that are in effect. We shall introduce more problems than we solve, but our purpose will be to lay out a framework for later discussions.

1-1 Mass, Charge, and Constituents of the Nucleus

Let us begin by reviewing a few fundamental facts that are probably already familiar. The nucleus is made up of neutrons and protons, two particles which are about 1,840 times more massive than electrons. They are spoken of collectively as *nucleons*. The number of protons in a nucleus is just equal to its atomic number Z, and the total number of nucleons A is the integer closest to its atomic weight; hence the number of neutrons is $A - Z$. Thus the nucleus of $_{11}Na^{23}$, a sodium atom which has atomic number 11 and atomic weight very close to 23, contains 11 protons and 12 neutrons. This is a relatively light nucleus; a typical heavy nucleus is $_{79}Au^{197}$, which obviously contains 79 protons and 118 neutrons. The mass of the nucleus is very nearly equal to the mass of the atom; in kilograms it is the atomic weight divided by Avogadro's number, 6.03×10^{26}.

The nucleus was first discovered in 1911 in experiments conducted by Lord Rutherford and his associates on scattering of alpha particles by atoms. He found that the scattering pattern could be explained if atoms consist of a small, massive, positively charged core surrounded by orbiting electrons. While most of his results could be calculated on the basis of an infinitely small nucleus, deviations indicated that the nuclear size is of the order of 10^{-14} m. Since this is 10,000 times smaller than the diameter of atoms, it is small enough to be negligible in practically all atomic problems. For studies of the nucleus itself, however, we must have more accurate size determinations.

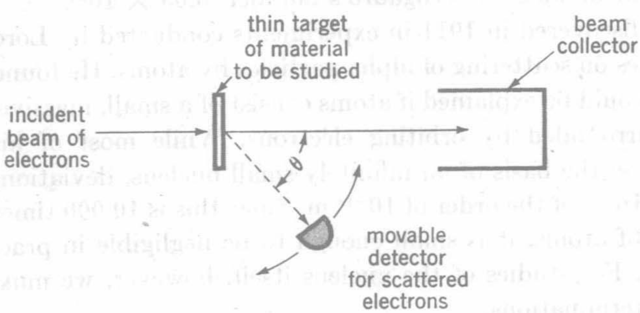
1-2 Nuclear Size and the Distribution of Nucleons

The straightforward approach to studying the size and shape of nuclei is to shoot probing particles at them and measure the effects produced. There is, however one well-known limitation in this endeavor: the wavelength of the probing particles must be of the order of the size of the nuclei being studied or smaller. Since ordinary light, for example, has a wavelength of about 10^{-7} m, which is many orders of magnitude larger than the nuclear size, it is not suitable. Light of very short wavelength, i.e., gamma rays, is also unsuitable because nuclei always occur in nature surrounded by electrons and electromagnetic waves interact more strongly with these electrons than with the nucleus. It is therefore better to employ particles such as electrons, protons, and neutrons as probes, all three of which have been used. Neutrons and protons have the advantage that their wavelength is sufficiently short for energies of about 20 MeV,[1] whereas for electrons over 100 MeV of energy is required, which is much more difficult to obtain. However, electrons have the advantage that their interaction with the nucleus is very well known (it is the familiar electromagnetic interaction), so the most accurate results have been obtained with electrons as probes.

The experiments consist of shooting high-energy electrons at a thin target

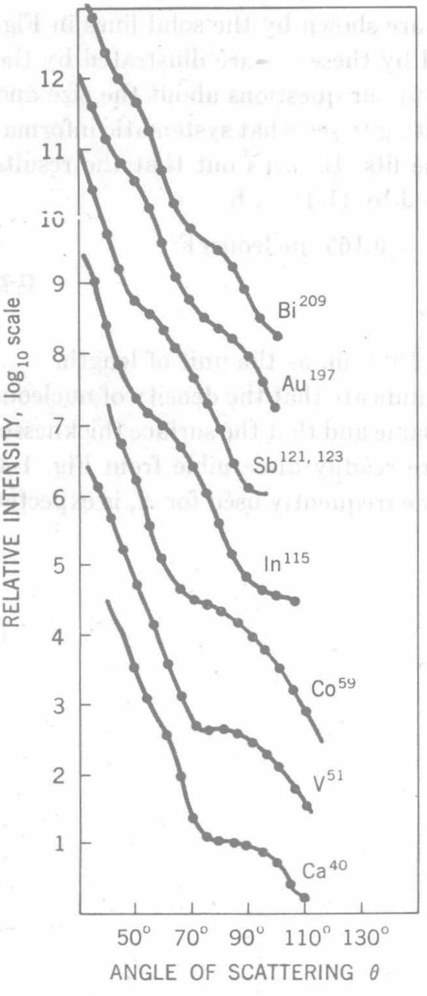
[1] MeV is million electron volts, the unit of energy we shall generally use; 1 MeV $= 10^6$ eV $= 1.6 \times 10^{-13}$ joule (J).

FIGURE 1-1 Experimental arrangement for measuring the angular variation of electron scattering from nuclei. The angle θ is varied by moving the detector, and for each θ measurements are made of the ratio between the number of scattered electrons it detects and the number of electrons in the beam as determined by the collector. (Since very few electrons are deflected by large angles, practically all of the beam reaches the collector.) Typical results of these measurements are shown in Fig. 1-2. The detector is actually a very large and complex group of instruments capable of determining the energies of the electrons.



of the material under study and observing the probability of various angular deflections, as shown in Fig. 1-1. In concept, it is very similar to the Rutherford scattering experiments, in which the nucleus was first discovered. Some typical results of these measurements are shown in Fig. 1-2. If one assumes some density distribution $\rho(r)$ for the nucleons in the nucleus and assumes that the neutrons have the same density distribution as the protons, the probability of various

FIGURE 1-2 Angular distributions of 185-MeV electrons scattered from various nuclei. The curves through the data are theoretical fits. [*From B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev.*, **101**: 1131 (1956).]



angular deflections can be calculated and compared with the experimental results. If they do not fit, other $\rho(r)$ can be tried until a fit is obtained.

The experiments have been performed and analyzed for a great many nuclei and at several incident electron energies. All the results can be approximately explained by a charge distribution given by

$$\rho(r) = \frac{\rho_0}{1 + \exp\,[(r - R)/a]} \tag{1-1}$$

A plot of (1-1) is shown in Fig. 1-3, where the physical significance of the various parameters is illustrated. We see there that ρ_0 is the nucleon density near the center of the nucleus, R is the radius at which the density has decreased by a factor of 2 below its central value, and a is a measure of the surface thickness such that the distance over which the density falls from 90 percent of ρ_0 to 10 percent of ρ_0 is $4.4a$.

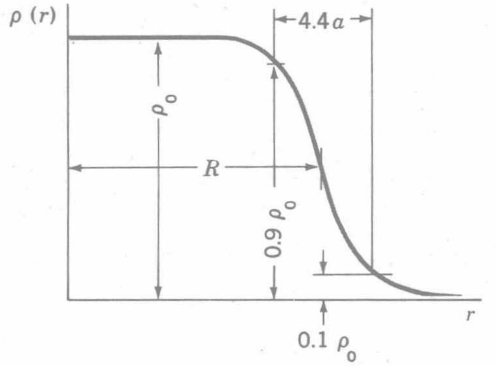
The fits to the data obtained with (1-1) are shown by the solid lines in Fig. 1-2, and the density distributions determined by these fits are illustrated by the curves in Fig. 1-4. These give us the answer to our questions about the size and density distributions in nuclei, but it is interesting to see what systematic information about nuclei can be obtained from these fits. It turns out that the results for all nuclei are reasonably well approximated by (1-1) with

$$\begin{aligned} \rho_0 &\simeq 1.65 \times 10^{44} \text{ nucleons/m}^3 = 0.165 \text{ nucleons/F}^3 \\ R &\simeq 1.07 A^{1/3} \text{ F} \\ a &\simeq 0.55 \text{ F} \end{aligned} \tag{1-2}$$

Note that we use the fermi (abbreviated F), 10^{-15} m, as the unit of length.

These results are extremely simple; they indicate that the density of nucleons in the inner regions of all nuclei is about the same and that the surface thicknesses of all nuclei are very similar. These facts are readily discernible from Fig. 1-4. The $A^{1/3}$ variation of the *nuclear radius*, a name frequently used for R, is expected

FIGURE 1-3 Plot of Eq. (1-1) for $\rho(r)$ vs. r. The meaning of ρ_0, R, and a are illustrated.



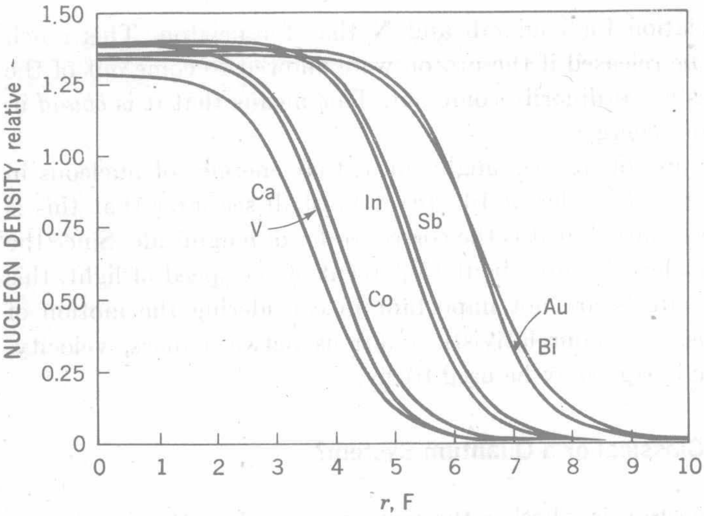


FIGURE 1-4 Nucleon density in various nuclei as obtained from the fits to the data shown in Fig. 1-2. [*From B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev.,* **101:** 1131 (1956).]

from the constancy of ρ_0 since this requires that the volume of the nucleus be proportional to A, and the volume is, of course, proportional to R^3.

1-3 Energies of Nucleons in the Nucleus

We shall eventually be treating the kinetic energies of nucleons in the nucleus in great detail, but at this stage we require an order-of-magnitude estimate. The energies of beta rays and gamma rays emitted from nuclei are typically of the order of 1 MeV, but these processes are transitions of nucleons from one state to another so their energies are differences between nucleon energies in two different states; the actual nucleon energy should be much larger.

One approach to the problem is to calculate the electrostatic energy E_c required to insert a proton into a nucleus. This is approximately

$$E_c = \frac{Ze^2}{4\pi\epsilon_0 R} \tag{1-3}$$

which for a medium-weight nucleus ($Z = 50$, $A = 120$) is

$$E_c = \frac{50(1.6 \times 10^{-19})^2\ \text{C}^2}{4\pi(8.9 \times 10^{-12})\ \dfrac{\text{C}^2}{\text{N-m}^2}\ 1.07 \times 120^{1/3} \times 10^{-15}\ \text{m}} \frac{1\ \text{J}}{1\ \text{N-m}} \frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}}$$

$$= 13 \times 10^6\ \text{eV} = 13\ \text{MeV} \tag{1-3a}$$

where C is the abbreviation for coulomb and N that for newton. This much coulomb energy would be released if the proton were allowed to come out of the nucleus, but still it does not ordinarily come out. This means that it is *bound* in the nucleus by even more energy.

From these simple arguments, we might guess that energies of nucleons in the nucleus are of the general order of 10 MeV. We shall see later that this is something of an underestimate, but it is the correct order of magnitude. Since the velocity of a 10-MeV nucleon is only about 15 percent of the speed of light, this means that relativistic effects are not important in considering the motion of nucleons in the nucleus. The nonrelativistic relations between mass, velocity, momentum, and kinetic energy may be used freely.

1-4 Is the Nucleus a Classical or a Quantum System?

The next interesting question is whether the wave nature of matter is relevant in a nucleus, as it is in atoms, or whether the nucleus is more like systems encountered in our everyday life, where classical mechanics is a sufficiently good approximation. As a general rule, the wave nature of matter is relevant where the wavelength of the particles is of the order of the size of the system, so let us compare them.

The wavelength of a nucleon with an energy of about 10 MeV is

$$\lambda = \frac{h}{Mv} = \frac{h}{\sqrt{2ME}}$$

$$= \frac{6.6 \times 10^{-34}\ \text{J-s} \times \dfrac{1\ \text{kg m}^2/\text{s}^2}{1\ \text{J}}}{\left(2 \times \dfrac{1\ \text{kg}}{6 \times 10^{26}} \times 10 \times 10^6\ \text{eV} \times 1.6 \times 10^{-19}\ \dfrac{\text{J}}{\text{eV}} \times \dfrac{1\ \text{kg m}^2/\text{s}^2}{1\ \text{J}}\right)^{1/2}}$$

$$= 9.3 \times 10^{-15}\ \text{m} = 9.3\ \text{F}$$

This is clearly of the order of the size of a nucleus as given by (1-2), so the wave nature of matter is indeed relevant. The motions of nucleons in the nucleus are governed by the laws of quantum physics; classical pictures in which nucleons are considered as little balls moving around—applied so successfully in describing gases or liquids—are of limited usefulness. We shall therefore have to use and expand our knowledge of the wave nature of matter. A review of this subject is presented in Chap. 2.

1-5 What Holds the Nucleus Together?

The next question we have to face is perhaps the most difficult if we have only our previous experience to go on: What holds the nucleons together in a nucleus?

Systems are held together by forces, and the only forces we have encountered in classical physics or in atomic physics have been the gravitational and electromagnetic forces. Can these do the job? The electromagnetic force most certainly cannot. Neutrons have no electric charge, so they do not experience the electromagnetic force at all,[1] and the principal electromagnetic force between protons is a strong coulomb repulsion, which tends to tear the nucleus apart. The gravitational force is an attractive one between every pair of nucleons, but it is smaller by a factor of about 10^{39} than the electrical force between two protons. Its effects are completely negligible in all nuclear and atomic phenomena.

Thus, the only two forces we have previously encountered cannot account for the existence of nuclei. The only explanation is to recognize that there is a third force in nature, known as the *nuclear force*. We see immediately that this force must be very strong at distances of the order of the nuclear size, since it must more than compensate the coulomb repulsion between protons. On the other hand, molecular structure can be accurately accounted for by the electromagnetic force alone, so we may conclude that at distances of the order of the spacing between nuclei in molecules ($\sim 10^{-10}$ m) the nuclear force must be negligible. It is therefore a *short-range* force, falling off more rapidly with distance than $1/r^2$.

Before we can proceed very far in studying the structure of the nucleus, we must learn more about the nuclear force. This will form the subject matter of Chap. 3.

1-6 Some Other Properties of Nuclei

We learned in elementary physics that if there are no external torques acting on a system, its angular momentum is conserved. Since an isolated nucleus is such a system, its angular momentum is one of its constant properties. Methods of measuring angular momenta of nuclei by use of atomic beams in Stern-Gerlach experiments and by studying the hyperfine structure of atomic spectral lines with and without applied magnetic fields are generally discussed in modern physics courses. Several other methods will be developed later in this book.

In quantum physics, conserved quantities are represented by quantum numbers. The quantum number for the total angular momentum of a nucleus is I; the two are related by

$$\text{Total angular momentum} = \sqrt{I(I+1)}\,\hbar \tag{1-4}$$

[1] Actually since, as we shall see in the next section, the neutron possesses a magnetic moment, it experiences a force in a nonuniform magnetic field, but this is too small to matter here.

where $\hbar$ is Planck's constant (6.25×10^{-34} J-s) divided by 2π. Values of I will be given and explained in many connections throughout this book. A compilation of directly measured values is given in Table A-2 of the Appendix.

In courses on electromagnetism it is shown that a current loop enclosing an area $\mathcal{a}$ and carrying a current i has a magnetic dipole moment μ given by

$$\mu = ir \tag{1-5}$$

For a circular orbit of radius r, traversed f times per second by a charge e moving with velocity v

$$i = ef = \frac{ev}{2\pi r}$$

$$\mathcal{a} \doteq \pi r^2$$

whence, from (1-5),

$$\mu = \frac{e}{2} vr = \frac{e}{2M} L$$

where L is the angular momentum, Mvr. More generally

$$\mu = \frac{e}{2M} Lg \tag{1-6}$$

where g is a factor called the gyromagnetic ratio. In accordance with the above derivation, $g = 1$ when the charge and mass distributions coincide, as when a particle traverses an orbit. In quantum theory, L is a quantum number times $\hbar$.†
For orbital motion with quantum number l, $g_l = 1$, whence, from (1-6),

$$\mu = \frac{e\hbar}{2M} l \tag{1-7}$$

The magnetic moment due to spin is a more complex problem which can be understood only in terms of relativistic quantum theory; the result for an electron, as is well known from atomic physics, is

$$\mu_e = \frac{e\hbar}{2M_e}$$

which, since the spin quantum number is $\frac{1}{2}$, corresponds to $g_s = 2$. Measurements corroborate this result. For nucleons, however, measurements give

$$\mu_p = 2.7925 \frac{e\hbar}{2M_p}$$
$$\mu_n = -1.9128 \frac{e\hbar}{2M_p} \tag{1-8}$$

which corresponds to g_s values equal to 2 times the numerical factors in (1-8). These results, and their contrast with the results for an electron, lead one to

† The statement given here is not quite accurate: it is the maximum component of L in any direction that is equal to $l\hbar$.