2D and 3D
Image Analysis by Moments

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Presents recent significant and rapid developments in the field of 2D and 3D image analysis

2D and 3D Image Analysis by Moments, is a unique compendium of moment-based image analysis, which includes traditional methods and also reflects the latest developments of the field.

The book presents a survey of 2D and 3D moment invariants with respect to similarity and affine spatial transformations and to image blurring and smoothing by various filters. The book comprehensively describes the mathematical background and theorems about the invariants, but a large part is also devoted to practical usage of moments. Applications from various fields of computer vision, remote sensing, medical imaging, image retrieval, watermarking, and forensic analysis are demonstrated. Attention is also paid to efficient algorithms of moment computation.

Key features

- Presents a systematic overview of moment-based features used in 2D and 3D image analysis.
- Demonstrates invariant properties of moments with respect to various spatial and intensity transformations.
- Reviews and compares several orthogonal polynomials and respective moments.
- Describes efficient numerical algorithms for moment computation.
- Is a "classroom ready" textbook with a self-contained introduction to classifier design.
- Accompanying website contains around 300 lecture slides, Matlab codes, complete lists of the invariants, test images, and other supplementary material.

2D and 3D Image Analysis by Moments, is ideal for mathematicians, computer scientists, engineers, software developers, and Ph.D students involved in image analysis and recognition. With the addition of two introductory chapters on classifier design, the book may also serve as a self-contained textbook for graduate university courses on object recognition.

with website

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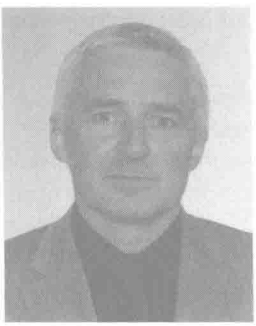

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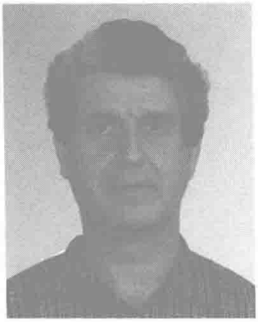
In 2007 Jan Flusser received the Award of the Chairman of the Czech Science Foundation for the best research project and won the Prize of the Czech Academy of Sciences for his contribution to image fusion theory. In 2010, he was awarded the SCOPUS 1000 Award presented by Elsevier. He received the Felber Medal of the Czech Technical University for excellent contribution to research and education in 2015. Personal webpage: `http://www.utia.cas.cz/people/flusser`



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Tomáš Suk has authored and coauthored more than thirty journal papers and fifty conference papers in these areas, including tutorials on moment invariants held at the conferences ICIP'07 and SPPRA'09. He also coauthored the monograph *Moments and Moment Invariants in Pattern Recognition* (Wiley, 2009). His publications have received about approximately citations. In 2002 he received the Otto Wichterle Premium of the Czech Academy of Sciences for young scientists. Personal webpage: `http://zoi.utia.cas.cz/suk`

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Preface

Seven years ago we published our first monograph on moments and their applications in image recognition: J. Flusser, T. Suk, and B. Zitová, *Moments and Moment Invariants in Pattern Recognition*, Wiley, 2009.

That book (referred to as MMIPR) was motivated by the need for a monograph covering theoretical aspects of moments and moment invariants and their relationship to practical image recognition problems. Long before 2009, object recognition had become an established discipline inside image analysis. Moments and moment invariants, introduced to the image analysis community in the early 1960s, have played a very important role as features in invariant recognition. Nevertheless, such a book had not been available before 2009[1].

The development of moment invariants after 2009 was even more rapid than before. In SCOPUS, which is probably the most widely-used publication database, we have received 16,000 search results as the response to the "image moment" keyword and 6,000 results of the "moment invariants" search[2]. There has been an overlap of about 2,000 papers, which results in 20,000 relevant papers in total. This huge number of entries illustrates how a large and important area of computer science has been formed by the methods based on image moments. In Figure 1 we can observe the development in time. A relatively slow growth in the last century was followed by a rapid increase of the number of publications in 2009–2010 (we believe that the appearance of MMIPR at least slightly contributed to the growing interest in moment invariants). Since then, the annual number of publications has slightly fluctuated, reaching another local maximum in 2014. In 2014, a new multi-authored book edited by G. A. Papakostas[3] appeared on the market. Although the editor did a very good job, this book suffers from a common weakness of multi-authored books – the topics of individual chapters had been selected by their authors according to their interest, which made some areas overstressed, while some others, remained unmentioned despite their importance. The Papakostas book reflects recent developments in some areas but can hardly be used as a course textbook.

[1] The very first moment-focused book by R. Mukundan and K. R. Ramakrishnan, *Moment Functions in Image Analysis*, World Scientific, 1998, is just a short introduction to this field. The second book by M. Pawlak, *Image Analysis by Moments: Reconstruction and Computational Aspects*, Wroclaw, Poland, 2006, is focused narrowly on numerical aspects of image moments, without providing a broader context of invariant image recognition and of practical applications.

[2] The search was performed within the title and abstract of the papers.

[3] G. A. Papakostas ed., *Moments and Moment Invariants – Theory and Applications*, Science Gate Publishing, 2014.

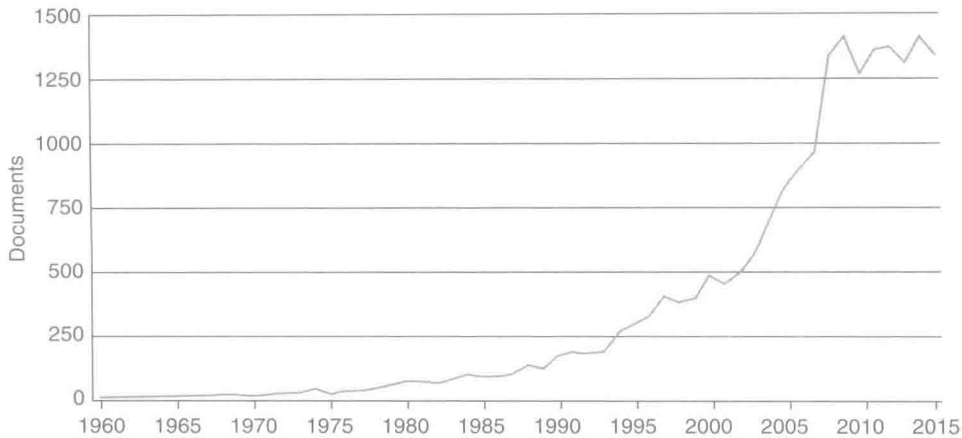

Figure 1 The number of moment-related publications as found in SCOPUS.

The great number of publications that have appeared since 2009 led us to the idea of writing a new, more comprehensive book on this topic. The abundant positive feedback we have received from the MMIPR readers and from our students was another factor which has strengthened our intentions. In 2014, the MMIPR was even translated into Chinese[4].

The main goal of the book you are now holding in your hands is to be a comprehensive monograph covering the current state of the art of moment-based image analysis and presenting the latest developments in this field in a consistent form. In this book, the reader will find a survey of all important theoretical results as well as a description of how to use them in practical image analysis tasks. In particular, our aims were

- To review the development of moments and moment invariants after 2009;
- To cover the area of 3D moments and invariants, which were mostly omitted in the MMIPR but have become important in the last few years;
- To present some of the latest unpublished results, especially in the field of blur invariants; and
- To provide readers with an introduction to a broader context, showing the role of the moments in image analysis and recognition and reviewing other relevant areas and approaches.

At the same time, we aimed to write a self-contained book. This led us to the decision (with the kind permission of Wiley) to include also the core parts of the MMIPR, of course in enhanced and extended/up-to-date form. Attentive readers may realize that about one half of this book was already treated in the MMIPR in some form, while the other half is original. In particular, Chapters 1, 2, 4, and 6 are completely or mostly original. Chapters 7, 8, 9, and 10 are substantially extended versions of their ancestors in the MMIPR, and Chapters 3 and 5 were adopted from the MMIPR after minor updates.

[4] Published by John Wiley & Univ. of Science and Technology of China Press, see `http://library.utia.cas.cz/separaty/2015/ZOI/flusser-0444327-cover.jpg`.

The book is based on our deep experience with moments and moment invariants gained from twenty-five years of research in this area, from teaching graduate courses on moment invariants and related fields at the Czech Technical University and at the Charles University, Prague, Czech Republic, and from presenting several tutorials on moments at major international conferences.

The target readership includes academic researchers and R&D engineers from all application areas who need to recognize 2D and 3D objects extracted from binary/graylevel/color images and who look for invariant and robust object descriptors, as well as specialists in moment-based image analysis interested in a new development on this field. Last but not least, the book is also intended for university lecturers and graduate students of image analysis and pattern recognition. It can be used as textbook for advanced graduate courses on Invariant Pattern Recognition. The first two chapters can be even utilized as supplementary reading to undergraduate courses on Pattern Recognition and Image Analysis.

We created an accompanying website at `http://zoi.utia.cas.cz/moment_invariants2` containing selected Matlab codes, the complete lists of the invariants, the slides for those who wish to use this book for educational purposes, and Errata (if any). This website is free for the book readers (the password can be found in the book) and is going to be regularly updated.

Acknowledgements

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Prague, Czech Republic, November 2015.

About the companion website

Don't forget to visit the companion website for this book:

http://www.wiley.com/go/flusser/2d3dimageanalysis

There you will find valuable material designed to enhance your learning, including:

- 300 slides
- Matlab codes

Scan this QR code to visit the companion website

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