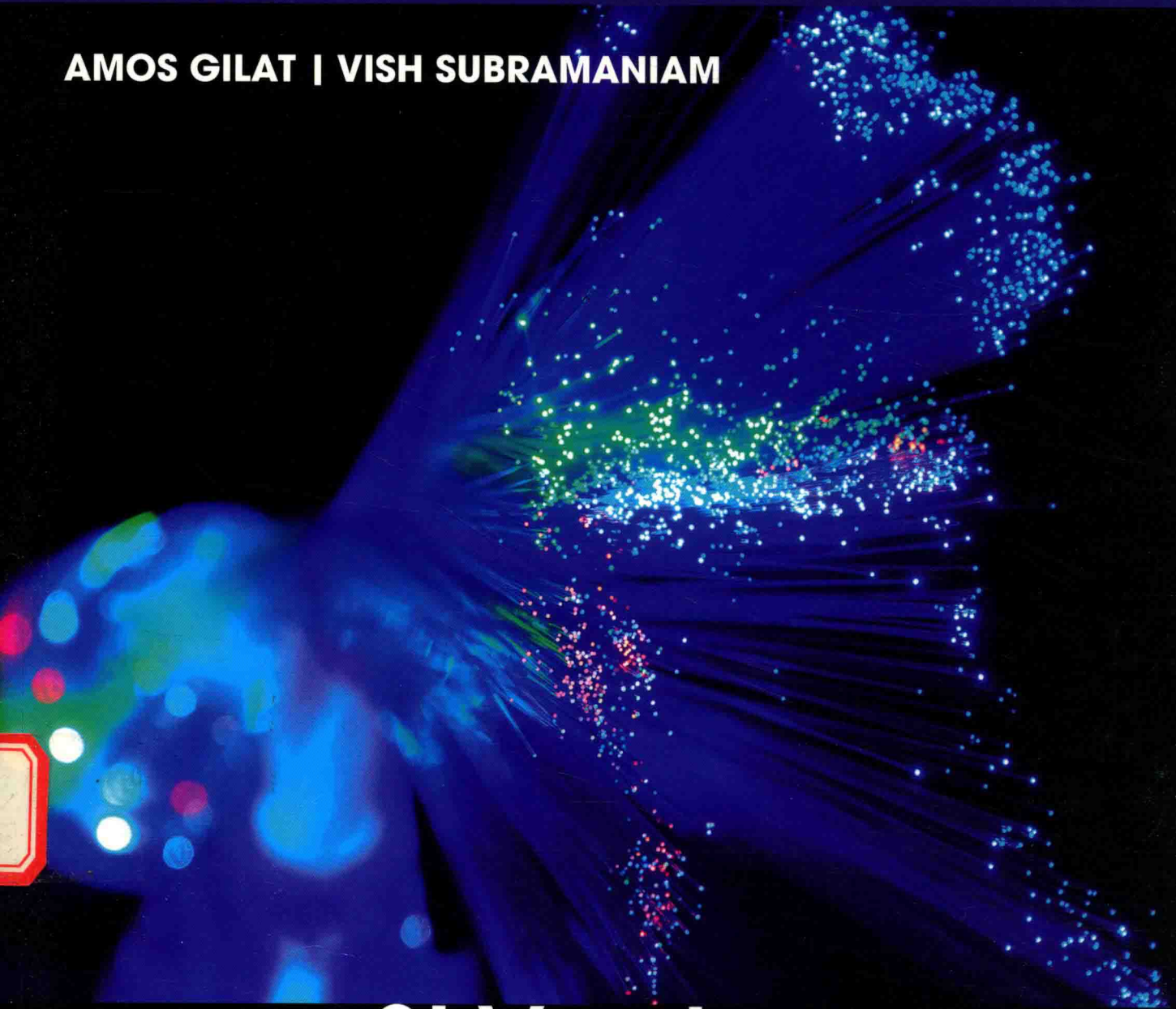


SECOND EDITION

Numerical Methods

An Introduction with Applications Using MATLAB®

AMOS GILAT | VISH SUBRAMANIAM



SI Version

Numerical Methods

An Introduction with Applications using MATLAB[®]

Second Edition
SI Version

Amos Gilat
Vish Subramaniam

Department of Mechanical Engineering
The Ohio State University



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Preface

This textbook is intended for a first course in numerical methods for students in engineering and science, typically taught in the second year of college. The book covers the fundamentals of numerical methods from an applied point of view. It explains the basic ideas behind the various methods and shows their usefulness for solving problems in engineering and science.

In the past, a numerical methods course was essentially mathematical, emphasizing numerical analysis and theory. More recently, due to the availability of powerful desktop computers and computing software that is both affordable and powerful, the content and nature of a first course in numerical methods for engineering and science students are changing. The emphasis is shifting more and more toward applications and toward implementing numerical methods with ready-to-use tools. In a typical course, students still learn the fundamentals of numerical methods. In addition, however, they learn computer programming (or improve their programming skills if they have already been introduced to programming), and use advanced software as a tool for solving problems. MATLAB is a good example of such software. It can be used by students to write their own programs, and can be used as a tool for solving problems using its built-in functions. One of the objectives of a course in numerical methods is to prepare students in science and engineering for future courses in their areas of specialization (and their future careers) where they will have to use computers for solving problems.

Main objectives of the book

To teach the fundamentals of numerical methods, with emphasis on the most essential methods.

To provide students with the opportunity to enhance their programming skills using the MATLAB environment to implement algorithms.

To teach the use of MATLAB as a tool (using its built-in functions) for solving problems in science and engineering, and for checking the results of any programs students write themselves.

Features/pedagogy of the book

- This book is written in simple, clear, and direct language. Frequently, bullets and a list of steps, rather than lengthy text, are used to list facts and details of a specific subject.
- Numerous illustrations are used for explaining the principles of the numerical methods.
- Many of the examples and end-of-chapter problems involve realistic problems in science and engineering.
- MATLAB is integrated within the text and in the examples. A light colored background is used when MATLAB syntax is displayed.
- Annotating comments that explain the commands are posted alongside the MATLAB syntax.
- MATLAB's built-in functions that are associated with the numerical methods are presented in detail.
- The homework problems at the end of the chapters are divided into three groups:
 - (a) **Problems to be solved by hand:** Problems related to improving understanding of numerical methods. In these problems the students are asked to answer questions related to the fundamentals of numerical methods, and to carry out a few steps of the numerical methods by hand.
 - (b) **Problems to be programmed in MATLAB:** Problems designed to provide the opportunity to improve programming skills. In these problems students are asked to use MATLAB to write computer programs (script files and user-defined functions) implementing various numerical methods.
 - (c) **Problems in math, science, and engineering:** Problems in science and engineering that have to be solved by using numerical methods. The objective is to train the students to use numerical methods for solving problems they can expect to see in future courses or in practice. Students are expected to use the programs that are presented in the book, programs that they write, and the built-in functions in MATLAB.

Organization of the book

Chapter 1: The first chapter gives a general introduction to numerical methods and to the way that computers store numbers and carry out numerical operations. It also includes a section on errors in numerical solutions and a section on computers and programming.

Chapters 2 through 8: The seven chapters present the various numerical methods in an order that is typically followed in a first course on numerical methods. These chapters follow the format explained next.

Organization of a typical chapter

An itemized list of the topics that are covered in the chapter is displayed below the title of the chapter. The list is divided into **core** and **complementary** topics. The **core topics** are the most essential topics related to the subject of the chapter. The **complementary topics** include more advanced topics. Obviously, a division of topics related to one subject into core and complementary is subjective. The intent is to help instructors in the design of their course when there is not enough time to cover all the topics. In practicality, the division can be ignored in courses where all the topics are covered.

The first section of the chapter provides a general background with illustrative examples of situations in the sciences and engineering where the methods described in the chapter are used. This section also explains the basic ideas behind the specific class of numerical methods that are described in the chapter. The following sections cover the core topics of the chapter. Next, a special section discusses the built-in functions in MATLAB that implement the numerical methods described in the chapter, and how they may be used to solve problems. The later sections of the chapter cover the complementary topics.

The order of topics

It is probably impossible to write a text book where all the topics follow an order that is agreed upon by all instructors. In the present book, the main subjects are in an order that is typical in a first course in numerical methods. Chapter 2 covers solution of nonlinear equations. It mostly deals with the solution of a single equation, which is a simple application of numerical methods. The chapter also includes, as a complementary topic, a section on the solution of a system of nonlinear equations. Chapter 3 deals with the solution of a system of linear equations. A complementary topic in this chapter deals with eigenvalue problems. Chapter 4 covers curve fitting and interpolation, and Chapters 5 and 6 cover differentiation and integration, respectively. Finally, solution of ordinary differential equations (ODE) is presented in the last two chapters. Chapter 7 deals with the solution of initial-value problems (first-order, systems, and higher-order) and Chapter 8 considers boundary-value problems.

The order of some of the topics is dictated by the subjects themselves. For example, differentiation and integration need to be covered before ordinary differential equations. It is possible, however, to cover the other subjects in different order than is in the book. The various chapters and sections in the book are written in a self-contained manner that make it easy for the instructor to cover the subjects in a different order, if desired.

MATLAB programs

This book contains many MATLAB programs. The programs are clearly identified as user-defined functions, or as script files. All the programs are listed in Appendix B. The programs, or the scripts, are written in a simple way that is easy to follow. The emphasis of these programs is on the basics and on how to program an algorithm of a specific numerical method. Obviously, the programs are not general, and do not cover all possible circumstances when executed. The programs are not written from the perspective of being shortest, fastest, or most efficient. Rather, they are written such that they are easy to follow. It is assumed that most of the students have only limited understanding of MATLAB and programming, and presenting MATLAB in this manner will advance their computing skills. More advanced users of MATLAB are encouraged to write more sophisticated and efficient programs and scripts, and compare their performance with the ones in the book.

SI Version

MATLAB: The SI version of the book is updated to MATLAB R2009b. All the programs have been modified and use anonymous functions instead of inline functions. In addition, function handles are used for passing functions into functions. Appendix A has been updated to the current version of MATLAB. It includes three new sections (A.8, A.9, and A.10) that cover anonymous functions, function functions, function handles, and subfunction.

Homework problems: Many new homework problems have been added to the SI version. Most are application problems in engineering and science. Most of the Chapters have 40 or more problems. Overall, more than 50% of the problems are new or have been revised.

Support material

The following is available on the instructor companion site at www.wiley.com/go/global/gilat):

- (a) for faculty who have adopted the text for use in their course, a fully worked solution manual, triple checked for accuracy.
- (b) suggested course syllabi with suggested assignments to help quickly integrate the text into your course.
- (c) conversion guides from other major numerical methods titles to show where each section of your current text is covered in this new text, helping you quickly convert from old to new.
- (d) electronic versions of all the figures and tables from the text, for creating lecture slides and quizzes/exams based on images from the book.
- (e) m-files of all the programs in the text.

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Amos Gilat (gilat.1@osu.edu)

Vish Subramaniam (subramaniam.1@osu.edu)

Columbus, Ohio

December 2010

To Yaela, Taly, and Edan

*To my parents, Dr. K. S. Venkateswaran & Seethalakshmy Venkateswaran,
and Deepa, Priya, and Sonya*

Brief Table of Contents

Preface	iii
Chapter 1	<i>Introduction</i> 1
Chapter 2	<i>Solving Nonlinear Equations</i> 23
Chapter 3	<i>Solving a System of Linear Equations</i> 65
Chapter 4	<i>Curve Fitting and Interpolation</i> 153
Chapter 5	<i>Numerical Differentiation</i> 211
Chapter 6	<i>Numerical Integration</i> 249
Chapter 7	<i>Ordinary Differential Equations: Initial–Value Problems</i> 293
Chapter 8	<i>Ordinary Differential Equations: Boundary–Value Problems</i> 377
Appendix A	<i>Introductory MATLAB</i> 415
Appendix B	<i>MATLAB Programs</i> 453
Index	457

Contents

Preface iii

Chapter 1 *Introduction* 1

- 1.1 Background 1
- 1.2 Representation of Numbers on a Computer 4
- 1.3 Errors in Numerical Solutions 10
 - 1.3.1 Round-Off Errors 10
 - 1.3.2 Truncation Errors 13
 - 1.3.3 Total Error 14
- 1.4 Computers and Programming 15
- 1.5 Problems 18

Chapter 2 *Solving Nonlinear Equations* 23

- 2.1 Background 23
- 2.2 Estimation of Errors in Numerical Solutions 25
- 2.3 Bisection Method 27
- 2.4 Regula Falsi Method 30
- 2.5 Newton's Method 32
- 2.6 Secant Method 37
- 2.7 Fixed-Point Iteration Method 40
- 2.8 Use of MATLAB Built-In Functions for Solving Nonlinear Equations 43
 - 2.8.1 The *fzero* Command 44
 - 2.8.2 The *roots* Command 45
- 2.9 Equations with Multiple Solutions 45
- 2.10 Systems of Nonlinear Equations 47
 - 2.10.1 Newton's Method for Solving a System of Nonlinear Equations 48
 - 2.10.2 Fixed-Point Iteration Method for Solving a System of Nonlinear Equations 52
- 2.11 Problems 54

Chapter 3 *Solving a System of Linear Equations* 65

- 3.1 Background 65
 - 3.1.1 *Overview of Numerical Methods for Solving a System of Linear Algebraic Equations* 66
- 3.2 Gauss Elimination Method 68
 - 3.2.1 *Potential Difficulties When Applying the Gauss Elimination Method* 76
- 3.3 Gauss Elimination with Pivoting 78
- 3.4 Gauss–Jordan Elimination Method 81
- 3.5 LU Decomposition Method 84
 - 3.5.1 *LU Decomposition Using the Gauss Elimination Procedure* 86
 - 3.5.2 *LU Decomposition Using Crout's Method* 87
 - 3.5.3 *LU Decomposition with Pivoting* 94
- 3.6 Inverse of a Matrix 94
 - 3.6.1 *Calculating the Inverse with the LU Decomposition Method* 95
 - 3.6.2 *Calculating the Inverse Using the Gauss–Jordan Method* 97
- 3.7 Iterative Methods 98
 - 3.7.1 *Jacobi Iterative Method* 99
 - 3.7.2 *Gauss-Seidel Iterative Method* 99
- 3.8 Use of MATLAB Built-In Functions for Solving a System of Linear Equations 102
 - 3.8.1 *Solving a System of Equations Using MATLAB's Left and Right Division* 102
 - 3.8.2 *Solving a System of Equations Using MATLAB's Inverse Operation* 103
 - 3.8.3 *MATLAB's Built-In Function for LU Decomposition* 104
 - 3.8.4 *Additional MATLAB Built-In Functions* 105
- 3.9 Tridiagonal Systems of Equations 107
- 3.10 Error, Residual, Norms, and Condition Number 112
 - 3.10.1 *Error and Residual* 112
 - 3.10.2 *Norms and Condition Number* 114
- 3.11 Ill-Conditioned Systems 117
- 3.12 Eigenvalues and Eigenvectors 121
 - 3.12.1 *The Basic Power Method* 124
 - 3.12.2 *The Inverse Power Method* 128
 - 3.12.3 *The Shifted Power Method* 129
 - 3.12.4 *The QR Factorization and Iteration Method* 129
 - 3.12.5 *Use of MATLAB Built-In Functions for Determining Eigenvalues and Eigenvectors* 139
- 3.13 Problems 141

Chapter 4	<i>Curve Fitting and Interpolation</i>	153
4.1	Background	153
4.2	Curve Fitting with a Linear Equation	155
4.2.1	Measuring How Good Is a Fit	155
4.2.2	Linear Least-Squares Regression	157
4.3	Curve Fitting with Nonlinear Equation by Writing the Equation in a Linear Form	161
4.4	Curve Fitting with Quadratic and Higher-Order Polynomials	165
4.5	Interpolation Using a Single Polynomial	170
4.5.1	Lagrange Interpolating Polynomials	172
4.5.2	Newton's Interpolating Polynomials	176
4.6	Piecewise (Spline) Interpolation	183
4.6.1	Linear Splines	183
4.6.2	Quadratic Splines	185
4.6.3	Cubic Splines	189
4.7	Use of MATLAB Built-In Functions for Curve Fitting and Interpolation	196
4.8	Curve Fitting with a Linear Combination of Nonlinear Functions	198
4.9	Problems	201
Chapter 5	<i>Numerical Differentiation</i>	211
5.1	Background	211
5.2	Finite Difference Approximation of the Derivative	213
5.3	Finite Difference Formulas Using Taylor Series Expansion	218
5.3.1	Finite Difference Formulas of First Derivative	218
5.3.2	Finite Difference Formulas for the Second Derivative	223
5.4	Summary of Finite Difference Formulas for Numerical Differentiation	225
5.5	Differentiation Formulas Using Lagrange Polynomials	227
5.6	Differentiation Using Curve Fitting	228
5.7	Use of MATLAB Built-In Functions for Numerical Differentiation	228
5.8	Richardson's Extrapolation	230
5.9	Error in Numerical Differentiation	233
5.10	Numerical Partial Differentiation	235
5.11	Problems	238

Chapter 6 *Numerical Integration* 249

- 6.1 Background 249
 - 6.1.1 *Overview of Approaches in Numerical Integration* 250
- 6.2 Rectangle and Midpoint Methods 252
- 6.3 Trapezoidal Method 254
 - 6.3.1 *Composite Trapezoidal Method* 255
- 6.4 Simpson's Methods 258
 - 6.4.1 *Simpson's 1/3 Method* 258
 - 6.4.2 *Simpson's 3/8 Method* 261
- 6.5 Gauss Quadrature 263
- 6.6 Evaluation of Multiple Integrals 269
- 6.7 Use of MATLAB Built-In Functions for Integration 270
- 6.8 Estimation of Error in Numerical Integration 272
- 6.9 Richardson's Extrapolation 274
- 6.10 Romberg Integration 277
- 6.11 Improper Integrals 280
 - 6.11.1 *Integrals with Singularities* 280
 - 6.11.2 *Integrals with Unbounded Limits* 281
- 6.12 Problems 282

Chapter 7 *Ordinary Differential Equations: Initial-Value Problems* 293

- 7.1 Background 293
- 7.2 Euler's Methods 298
 - 7.2.1 *Euler's Explicit Method* 298
 - 7.2.2 *Analysis of Truncation Error in Euler's Explicit Method* 302
 - 7.2.3 *Euler's Implicit Method* 306
- 7.3 Modified Euler's Method 309
- 7.4 Midpoint Method 312
- 7.5 Runge–Kutta Methods 313
 - 7.5.1 *Second-Order Runge–Kutta Methods* 314
 - 7.5.2 *Third-Order Runge–Kutta Methods* 318
 - 7.5.3 *Fourth-Order Runge–Kutta Methods* 319
- 7.6 Multistep Methods 325
 - 7.6.1 *Adams–Bashforth Method* 326
 - 7.6.2 *Adams–Moulton Method* 327

7.7	Predictor–Corrector Methods	328
7.8	System of First-Order Ordinary Differential Equations	330
7.8.1	Solving a System of First-Order ODEs Using Euler’s Explicit Method	332
7.8.2	Solving a System of First-Order ODEs Using Second-Order Runge–Kutta Method (Modified Euler Version)	332
7.8.3	Solving a System of First-Order ODEs Using the Classical Fourth-Order Runge–Kutta Method	339
7.9	Solving a Higher-Order Initial Value Problem	340
7.10	Use of MATLAB Built-In Functions for Solving Initial-Value Problems	345
7.10.1	Solving a Single First-Order ODE Using MATLAB	346
7.10.2	Solving a System of First-Order ODEs Using MATLAB	352
7.11	Local Truncation Error in Second-Order Range–Kutta Method	355
7.12	Step Size For Desired Accuracy	356
7.13	Stability	360
7.14	Stiff Ordinary Differential Equations	362
7.15	Problems	365

Chapter 8 *Ordinary Differential Equations: Boundary-Value Problems* 377

8.1	Background	377
8.2	The Shooting Method	380
8.3	Finite Difference Method	388
8.4	Use of MATLAB Built-In Functions for Solving Boundary Value Problems	398
8.5	Error and Stability in Numerical Solution of Boundary Value Problems	403
8.6	Problems	405

Appendix A *Introductory MATLAB* 415

A.1	Background	415
A.2	Starting with MATLAB	415
A.3	Arrays	420
A.4	Mathematical Operations with Arrays	425
A.5	Script Files	430
A.6	Plotting	432
A.7	User-Defined Functions and Function Files	434
A.8	Anonymous Functions	436
A.9	Function functions	438

A.10 Subfunctions	441
A.11 Programming in MATLAB	443
A.11.1 Relational and Logical Operators	443
A.11.2 Conditional Statements, if-else Structures	444
A.11.3 Loops	447
A.12 Problems	448

Appendix B *MATLAB Programs* 453

Index	457
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Chapter 1

Introduction

Core Topics

Representation of numbers on a computer (1.2).

Computers and programming (1.4).

Errors in numerical solutions, round-off errors and truncation errors (1.3).

1.1 BACKGROUND

Numerical methods are mathematical techniques used for solving mathematical problems that cannot be solved or are difficult to solve analytically. An analytical solution is an exact answer in the form of a mathematical expression in terms of the variables associated with the problem that is being solved. A numerical solution is an approximate numerical value (a number) for the solution. Although numerical solutions are an approximation, they can be very accurate. In many numerical methods, the calculations are executed in an iterative manner until a desired accuracy is achieved.

For example, Fig. 1-1 shows a block of mass m being pulled by a force F applied at an angle θ . By applying equations of equilibrium, the relationship between the force and the angle is given by:

$$F = \frac{\mu mg}{\cos \theta + \mu \sin \theta} \quad (1.1)$$

where μ is the friction coefficient and g is the acceleration due to gravity. For a given value of F , the angle that is required for moving the block can be determined by solving Eq. (1.1) for θ . Equation (1.1), however, cannot be solved analytically for θ . Using numerical methods, an approximate solution can be determined for specified accuracy. This means that when the numerical solution for θ is substituted back in Eq. (1.1), the value of F that is obtained from the expression on the right-hand side is not exactly equal to the given value of F , but is very close.

Numerical techniques for solving mathematical problems were developed and used hundreds and even thousands of years ago. Implementation of the numerical techniques was difficult since the calculations had to be carried out by hand or by use of simple mechanical

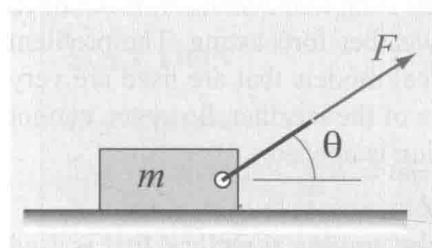


Figure 1-1: Motion of a block on a surface with friction.