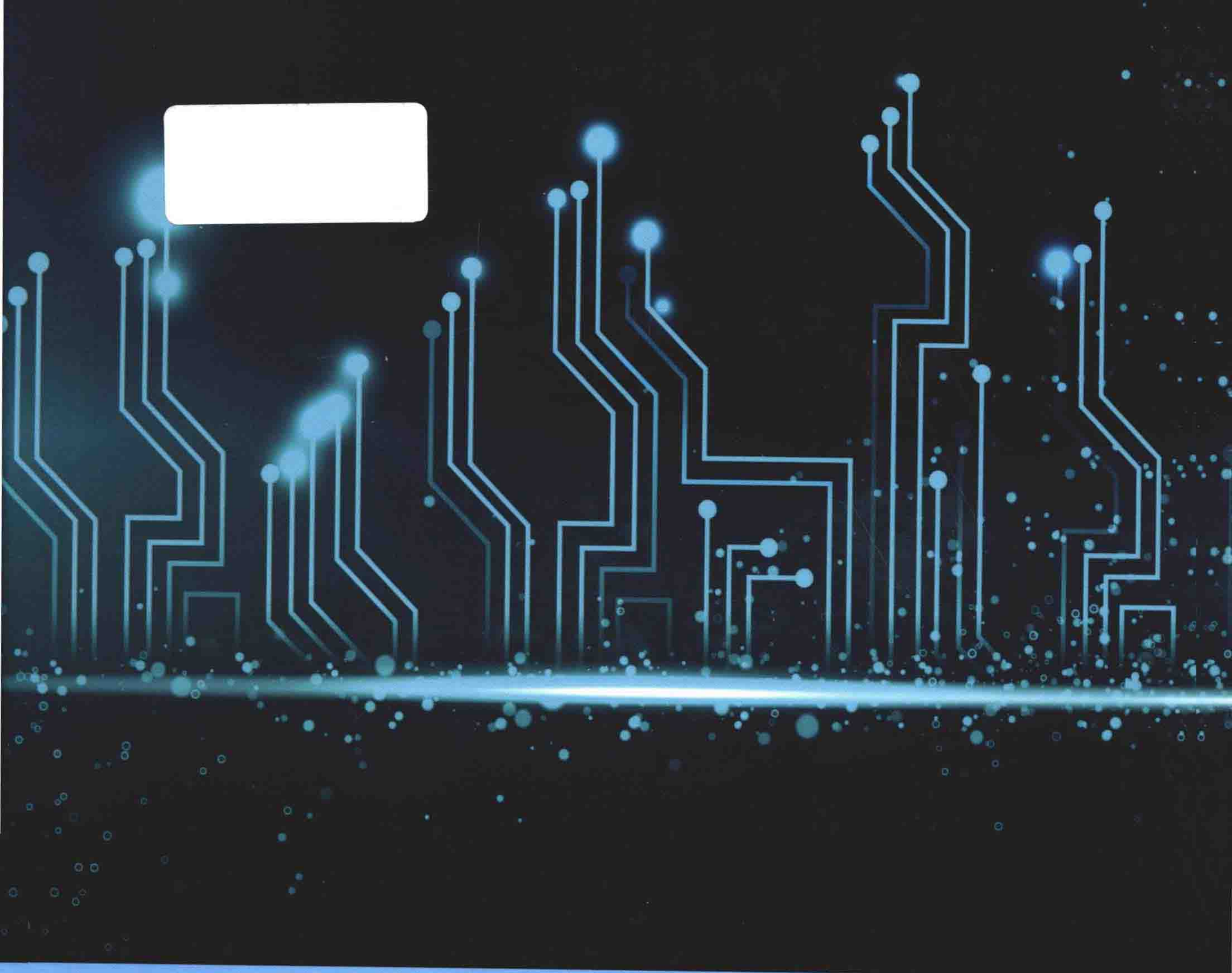

Random Wireless Networks

An Information Theoretic Perspective

Rahul Vaze

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To my son Niraad;
this book was written while babysitting him.

Preface

In addition to the traditional cellular wireless networks, in recent past, many other wireless networks have gained widespread popularity, such as sensor networks, military networks, and vehicular networks. In a sensor network, a large number of sensors are deployed in a geographical area for monitoring physical parameters (temperature, rainfall), intrusion detection, animal census, etc., while in a military network, heterogenous military hardware interconnects to form a network in a battlefield, and vehicular networks are being deployed today for traffic management, emergency evacuations, and efficient routing. For efficient scalability, these new wireless networks are envisaged to be self-configurable with no centralized control, sometimes referred to as *ad hoc* networks.

The decentralized mode of operation makes it easier to deploy these networks, however, that also presents with several challenges, such as creating large amount of interference, large overheads for finding optimal routes, complicated protocols for cooperation and coordination. Because of these challenges, finding the performance limits, both in terms of the amount of information that can be carried across the network and ensuring connectivity in the wireless network, is a very hard problem and has remained unsolved in its full generality.

From an information-theoretic point of view, where we are interested in finding the maximum amount of information that can be carried across the network, one of the major bottlenecks in wireless network is the characterization of interference. To make use of the spatial separation between nodes of the wireless network, multiple transmitters communicate at the same time, creating interference at other receivers. The arbitrary topology of the network further compounds the problem by directly affecting the signal interaction or interference profile. Thus, one of the several trade-offs in wireless networks is the extent of spatial reuse viz-a-viz the interference tolerance. Another important trade-off is the relation between the radio range (distance to which each node can transmit) of sensor nodes and the connectivity of the wireless network. Small radio range leads to isolated nodes, while larger radio ranges result in significant interference at the neighboring receivers affecting connectivity.

Over the last decade and a half, these trade-offs have been addressed in a variety of ways, with exact answers derived for *random* wireless networks, where nodes of the wireless network are located uniformly at random in a given area of interest. The primary reason for assuming random location for nodes is the applicability of rich mathematical tools from stochastic geometry, percolation theory, etc. that provide significant mathematical foundation and allow derivation of concrete results. This book ties up the different ideas introduced for understanding the performance limits of random wireless networks and presents a complete overview on the advances made from an information-theoretic (capacity limits) point of view.

In this book, we focus on two capacity metrics for random wireless networks, namely, the transmission and the throughput capacity, that have been defined to capture the successful number of bits that can be transported across the network. We present a comprehensive analysis of transmission capacity and throughput capacity of random wireless networks. In addition, using the tools from percolation theory, we also discuss the connectivity and percolation properties of random wireless networks, which impact the routing and large-scale connectivity in wireless networks. The book is presented in a cohesive and easy to follow manner, however, without losing the mathematical rigor. Sufficient background and critical details are provided for the advanced mathematical concepts required for solving these problems.

The book is targeted at graduate students looking for an easy and rigorous introduction to the area of information/communication theory of random wireless networks. The book also quantifies the effects of network layer protocols (e.g., automatic repeat requests (ARQs)), physical layer technologies such as multiple antennas (MIMO), successive interference cancelation, information-theoretic security, on the performance of wireless networks. The book is accessible to anyone with a background in basic calculus, probability theory, and matrix theory.

The book starts with an introduction to the signal processing, information theory, and communication theory fundamentals of a point-to-point wireless communication channel. Specifically, a quick overview of the concept of Shannon capacity, outage formulation, basic information-theoretic channels, basics of multiple antenna communication, etc. is provided that lays sufficient background for the rest of the book.

The book is divided into two parts, the first part exclusively deals with single-hop wireless networks, where each source–destination communicates directly with each other, while in the second part, we focus only on the multi-hop wireless networks, where source–destination pairs are out of each others' communication range and use multiple other nodes (called relays) for communication.

For the first part, we begin by deriving analytical expressions for the transmission capacity for a single-hop model with various scheduling protocols such as ALOHA, CSMA, guard-zone based, etc. Next, we discuss in detail the effect of using multiple antennas on the transmission capacity of a random wireless network and derive the optimal role of multiple antennas. We then extend our setup and present performance analysis of random wireless networks under a two-way communication model that allows for bidirectional communication between two nodes. We close the first part of the book by applying stochastic geometry tools to derive a tractable performance analysis of a cellular wireless network in terms of critical measures such as connection probability, average rate, etc. which is extremely useful for practicing engineers.

The second part of the book starts by extending the transmission capacity framework to a multi-hop wireless network, where we derive the transmission capacity expression and find the optimal value of several key parameters relevant to the multi-hop communication model. Then, we give a brief introduction to percolation theory results for both the discrete and the continuum case. The background on percolation theory sets up the platform for deriving several important results for random wireless networks, such as finding the optimal radio range for connectivity, formation of large connected clusters under different connection models, and most importantly for finding tight scaling bounds on the throughput capacity.

We then present the seminal result of Gupta and Kumar which shows that the throughput capacity of a random wireless network scales as square root of the number of nodes. Finally, we discuss the concept of hierarchical cooperation in a wireless network which is used to show that the throughput capacity can scale linearly with the number of nodes.

This book is an effort to present the several disparate ideas developed for deriving capacity of a random wireless network in a unified framework. For effective understanding, extensive effort is made to explain the physical interpretation of all results. As an attempt to reach out to a wider audience, effects of practical communication models, such as cellular networks, two-way communication (downlink/uplink) feedback constraints, modern communication techniques (such as multiple antenna nodes, interference cancelation and avoidance, cognitive radios), are also analyzed and discussed in sufficient detail.

Most of the ideas/results presented in this book are not more than a decade old and have not yet found a consolidated treatment. The presentation is kept short and lucid with sufficient detail and rigor. For clarity, at instances, places simplified proofs of the original results are provided.

Acknowledgments

I would like to thank everyone who have made this book possible. Starting with Srikanth Iyer, D. Yogeshwaran, Aditya Gopalan, Chandra Murthy, Radhakrishna Ganti, Sibiraj Pillai, Siddharth Banerjee, all have made detailed comments on my various drafts, which undoubtedly made the book more readable. Their critical comments have also shaped the structure and content. I would also like to thank Kaibin Huang, who urged me to write this as a textbook that he could use in his course. I do not know yet whether it will serve his purpose. Help from undergraduate internship students Vivek Bagaria, Dheeraj Narasimha, Jainam Doshi, Rushil Nagda, Siddharth Satpathi, Ajay Krishnan, and Navya Prem in proofreading is gratefully acknowledged. Their comments were valuable in making the book accessible to readers with little or no prior background. The review comments by the two referees helped immensely in changing the structure of the book. Their comments especially helped in the reorganization and pruning of the book to keep a clear and sharp focus.

Notation

$\mathbf{A}$	Matrix A
$\mathbf{A}(i,j)$	(i,j)th entry of matrix A
$\mathbf{a}$	vector a
$\mathbf{a}(i)$	ith element of vector $\mathbf{a}$
$\mathbf{a}^{\dagger}$	Conjugate transpose of vector $\mathbf{a}$
$\mathbb{R}$	Set of real numbers
$\mathbb{C}$	Set of complex numbers
$\mathbb{R}^d$	Set of real numbers in d dimensions
$\mathbb{C}^{m\times n}$	Matrices with m rows and n columns over the set of complex numbers
$\mathbb{P}(A)$	Probability of event A
$\mathbb{E}$	Expectation operator
$\#(A)$	Number of elements in set A
$\nu(A)$	Lebesgue measure of set A
o	Origin in $\mathbb{R}^d$
d_{xy}	Distance between nodes x and y
α	Path-loss exponent for wireless propagation
$\mathbf{B}(x,r)$	Disc of radius r centered at x
ϕ	Null set
$\|a\|$	Absolute value of a
Φ	A Poisson point process
p	ALOHA access probability
λ	Density of nodes of the network
μ	Density of blockages/obstacles in the network
ϵ	Outage probability constraint
Λ	Density measure of nodes of the network
γ	Interference suppression parameter
ρ	Random variable representing the random radius in the Gilbert's random disc model
$\Gamma(t)$	$\int_0^{\infty} x^{t-1}\exp^{-x}\, dx$
$\jmath$	$\sqrt{-1}$
$A \propto B$	$A = cB$, where c is a constant
SNR	Signal-to-noise-ratio
SIR	Signal-to-interference-ratio
SINR	Signal-to-interference-plus-noise-ratio

β	Signal-to-interference-plus-ratio threshold for successful packet reception
B	Rate of transmission corresponding to SINR threshold β, $\beta = 2^B - 1$
M_n	Number of retransmissions required on hop n
N_h	Number of hops
M	Number of end-to-end retransmissions required $\sum_{n=1}^{N_h} M_n = M$
$\mathsf{t}(n)$	Per-node throughput capacity
$\mathsf{T}(n)$	Network wide throughput capacity
C	Transmission capacity
C_{tw}	Two-way transmission capacity
C_d	Delay normalized transmission capacity
C_s	Spatial progress capacity
AWGN	Additive white Gaussian noise
N_0	Variance of the AWGN
$\mathcal{N}(\mathsf{m}, \mathsf{var})$	Normal distribution with mean m and variance var
$\mathcal{CN}(\mathsf{m}, \mathsf{var})$	Complex normal distribution with mean m and variance var
$\mathbf{1}_n$	Indicator variable for node n
$\chi^2(2m)$	Chi-square distribution with m degrees of freedom
$X \sim Y$	Random variable X has distribution Y
$f(n) = \Omega(g(n))$	If $\exists\, k > 0,\ n_0,\ \forall\, n > n_0,\ \|g(n)\|k \leq \|f(n)\|$
$f(n) = \mathcal{O}(g(n))$	If $\exists\, k > 0,\ n_0,\ \forall\, n > n_0,\ \|f(n)\| \leq \|g(n)\|k$
$f(n) = \Theta(g(n))$	If $\exists\, k_1,\ k_2 > 0,\ n_0,\ \forall\, n > n_0,\ \|g(n)\|k_1 \leq \|f(n)\| \leq \|g(n)\|k_2$
$f(n) = \mathrm{o}(g(n))$	If $\lim_{n\to\infty} \frac{f(n)}{g(n} = 0$

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