

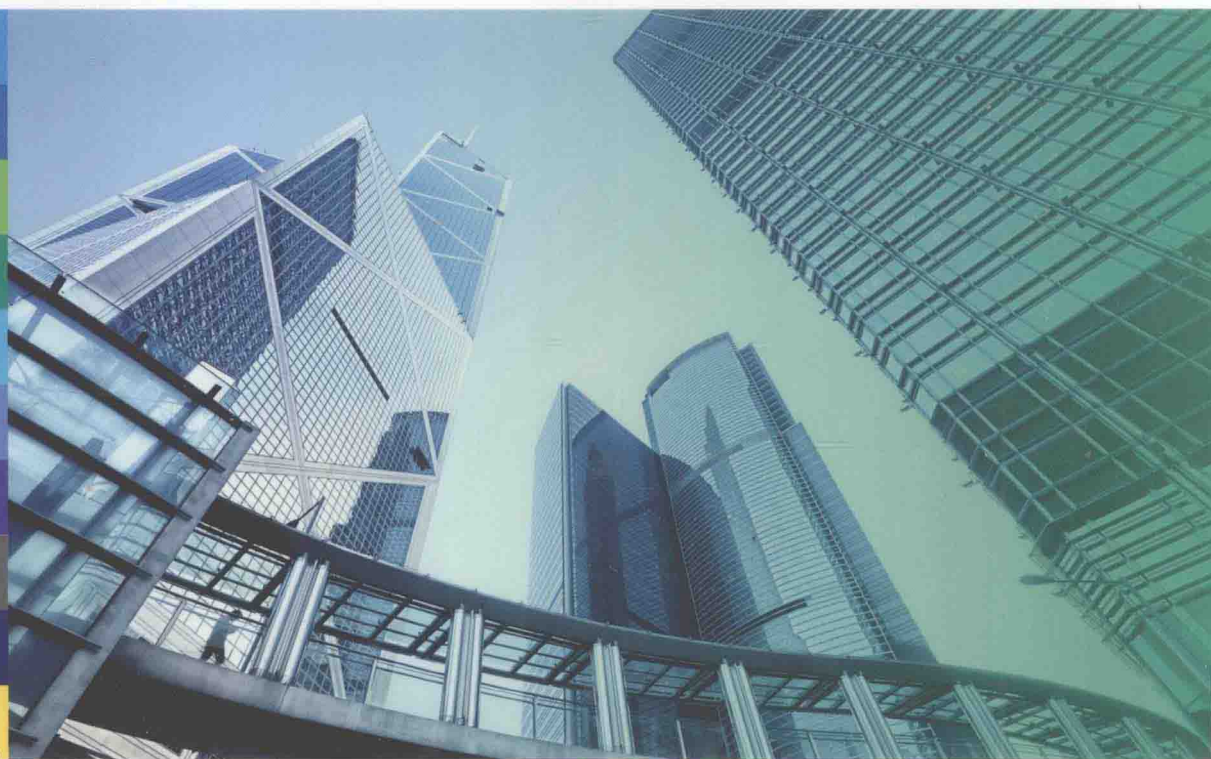


CFA Institute

CFA INSTITUTE INVESTMENT SERIES

QUANTITATIVE INVESTMENT ANALYSIS

Third Edition



Richard A. DeFusco, CFA ■ Dennis W. McLeavey, CFA
Jerald E. Pinto, CFA ■ David E. Runkle, CFA

Foreword by Mark J. P. Anson, PhD, CFA

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WILEY

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Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

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ISBN 978-1-119-10422-3 (Hardcover)
ISBN 978-1-119-10459-9 (ePDF)
ISBN 978-1-119-10460-5 (ePub)

Printed in the United States of America.
10 9 8 7 6 5 4 3 2 1

FOREWORD

“Central limits,” “probability distributions,” “hypothesis test”—investors have a bit of trouble generating enthusiasm for such terms. Yet, they should be enthusiastic because every investor needs these tools to analyze, compete, and succeed in today’s economic environment. The financial markets and the participants in them become increasingly sophisticated every year. So, at times, it seems like you need a PhD in mathematics just to keep up with the markets.

Fortunately, a PhD is not necessary to succeed. In fact, the financial market battlefield is littered with the credentials of highly educated individuals who have failed spectacularly despite their intense education. Nonetheless, the better equipped you are with the basic tools of financial calculus, the better your chance of success.

Quantitative Investment Analysis provides the necessary utensils for success. In this volume, you will find all the statistical gadgets you need to be a confident and knowledgeable investor. Math need not be a four letter word. It can make your wealth analysis sharper, your investment theme more precise, your portfolio construction more successful.

Furthermore, this book is chock full of examples, practice problems (with answers!), charts, tables, and graphs that bring home in clear detail the concepts and tools of financial calculus. Whether you are a novice investor or an experienced practitioner, this book has something for you. In fact, as I read the book in preparation for writing this foreword, I kept getting unconsciously pulled into the examples; unwittingly, I became engaged in the book before I knew it. But that effect is part of the beauty of this book: It is an easy-to-read and easy-to-use handbook. I wanted to know more with each example I read. I know that you, too, will find that this book stimulates your curiosity while having the same ease of use that I found. Enjoy!

MARK J. P. ANSON, PhD, CFA, CAIA, CPA
President & Chief Investment Officer
Acadia Capital
Bass Family Office

PREFACE

We are pleased to bring you *Quantitative Investment Analysis, Third Edition*, which focuses on key tools that are needed for today's professional investor. In addition to classic time value of money, discounted cash flow applications, and probability material, the text covers advanced concepts such as correlation and regression that ultimately figure into the formation of hypotheses for purposes of testing. The text teaches critical skills that challenge many professionals, including the ability to distinguish useful information from the overwhelming quantity of available data.

The content was developed in partnership by a team of distinguished academics and practitioners, chosen for their acknowledged expertise in the field, and guided by CFA Institute. It is written specifically with the investment practitioner in mind and is replete with examples and practice problems that reinforce the learning outcomes and demonstrate real-world applicability.

The CFA Program Curriculum, from which the content of this book was drawn, is subjected to a rigorous review process to assure that it is:

- Faithful to the findings of our ongoing industry practice analysis
- Valuable to members, employers, and investors
- Globally relevant
- Generalist (as opposed to specialist) in nature
- Replete with sufficient examples and practice opportunities
- Pedagogically sound

The accompanying workbook is a useful reference that provides Learning Outcome Statements, which describe exactly what readers will learn and be able to demonstrate after mastering the accompanying material. Additionally, the workbook has summary overviews and practice problems for each chapter.

We hope you will find this and other books in the CFA Institute Investment Series helpful in your efforts to grow your investment knowledge, whether you are a relatively new entrant or an experienced veteran striving to keep up to date in the ever-changing market environment. CFA Institute, as a long-term committed participant in the investment profession and a not-for-profit global membership association, is pleased to provide you with this opportunity.

ACKNOWLEDGMENTS

We would like to thank Eugene L. Podkaminer, CFA, for his contribution to revising coverage of multifactor models. We are indebted to Professor Sanjiv Sabherwal for his painstaking work in updating self-test examples in all chapters except the chapter on multifactor models. His contribution was essential to the fresh look of this third edition.

We are indebted to Wendy L. Pirie, CFA, Gregory Siegel, CFA, and Stephen E. Wilcox, CFA, for their help in verifying the accuracy of the text. Margaret Hill, Wanda Lauziere, and Julia MacKesson and the production team at CFA Institute provided essential support through the various stages of production. We thank Robert E. Lamy, CFA, and Christopher B. Wiese, CFA, for their encouragement and oversight of the production of a third edition.

ABOUT THE CFA INSTITUTE INVESTMENT SERIES

CFA Institute is pleased to provide you with the CFA Institute Investment Series, which covers major areas in the field of investments. We provide this best-in-class series for the same reason we have been chartering investment professionals for more than 45 years: to lead the investment profession globally by setting the highest standards of ethics, education, and professional excellence.

The books in the CFA Institute Investment Series contain practical, globally relevant material. They are intended both for those contemplating entry into the extremely competitive field of investment management as well as for those seeking a means of keeping their knowledge fresh and up to date. This series was designed to be user friendly and highly relevant.

We hope you find this series helpful in your efforts to grow your investment knowledge, whether you are a relatively new entrant or an experienced veteran ethically bound to keep up to date in the ever-changing market environment. As a long-term, committed participant in the investment profession and a not-for-profit global membership association, CFA Institute is pleased to provide you with this opportunity.

THE TEXTS

Corporate Finance: A Practical Approach is a solid foundation for those looking to achieve lasting business growth. In today's competitive business environment, companies must find innovative ways to enable rapid and sustainable growth. This text equips readers with the foundational knowledge and tools for making smart business decisions and formulating strategies to maximize company value. It covers everything from managing relationships between stakeholders to evaluating merger and acquisition bids, as well as the companies behind them. Through extensive use of real-world examples, readers will gain critical perspective into interpreting corporate financial data, evaluating projects, and allocating funds in ways that increase corporate value. Readers will gain insights into the tools and strategies used in modern corporate financial management.

Equity Asset Valuation is a particularly cogent and important resource for anyone involved in estimating the value of securities and understanding security pricing. A well-informed professional knows that the common forms of equity valuation—dividend discount modeling, free cash flow modeling, price/earnings modeling, and residual income modeling—can all be reconciled with one another under certain assumptions. With a deep understanding of the underlying assumptions, the professional investor can better understand what other investors assume when calculating their valuation estimates. This text has a global orientation, including emerging markets.

Fixed Income Analysis has been at the forefront of new concepts in recent years, and this particular text offers some of the most recent material for the seasoned professional who is not a fixed-income specialist. The application of option and derivative technology to the once staid province of fixed income has helped contribute to an explosion of thought in this area. Professionals have been challenged to stay up to speed with credit derivatives, swaptions, collateralized mortgage securities, mortgage-backed securities, and other vehicles, and this explosion of products has strained the world's financial markets and tested central banks to provide sufficient oversight. Armed with a thorough grasp of the new exposures, the professional investor is much better able to anticipate and understand the challenges our central bankers and markets face.

International Financial Statement Analysis is designed to address the ever-increasing need for investment professionals and students to think about financial statement analysis from a global perspective. The text is a practically oriented introduction to financial statement analysis that is distinguished by its combination of a true international orientation, a structured presentation style, and abundant illustrations and tools covering concepts as they are introduced in the text. The authors cover this discipline comprehensively and with an eye to ensuring the reader's success at all levels in the complex world of financial statement analysis.

Investments: Principles of Portfolio and Equity Analysis provides an accessible yet rigorous introduction to portfolio and equity analysis. Portfolio planning and portfolio management are presented within a context of up-to-date, global coverage of security markets, trading, and market-related concepts and products. The essentials of equity analysis and valuation are explained in detail and profusely illustrated. The book includes coverage of practitioner-important but often neglected topics, such as industry analysis. Throughout, the focus is on the practical application of key concepts with examples drawn from both emerging and developed markets. Each chapter affords the reader many opportunities to self-check his or her understanding of topics.

One of the most prominent texts over the years in the investment management industry has been Maginn and Tuttle's *Managing Investment Portfolios: A Dynamic Process*. The third edition updates key concepts from the 1990 second edition. Some of the more experienced members of our community own the prior two editions and will add the third edition to their libraries. Not only does this seminal work take the concepts from the other readings and put them in a portfolio context, but it also updates the concepts of alternative investments, performance presentation standards, portfolio execution, and, very importantly, individual investor portfolio management. Focusing attention away from institutional portfolios and toward the individual investor makes this edition an important and timely work.

The New Wealth Management: The Financial Advisor's Guide to Managing and Investing Client Assets is an updated version of Harold Evensky's mainstay reference guide for wealth managers. Harold Evensky, Stephen Horan, and Thomas Robinson have updated the core text of the 1997 first edition and added an abundance of new material to fully reflect today's investment challenges. The text provides authoritative coverage across the full spectrum of wealth management and serves as a comprehensive guide for financial advisors. The book expertly blends investment theory and real-world applications and is written in the same thorough but highly accessible style as the first edition.

All books in the CFA Institute Investment Series are available through all major booksellers. And, all titles are available on the Wiley Custom Select platform at <http://customselect.wiley.com/> where individual chapters for all the books may be mixed and matched to create custom textbooks for the classroom.

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CHAPTER 1

THE TIME VALUE OF MONEY

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LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

- interpret interest rates as required rates of return, discount rates, or opportunity costs;
- explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk;
- calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding;
- solve time value of money problems for different frequencies of compounding;
- calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows;
- demonstrate the use of a time line in modeling and solving time value of money problems.

1. INTRODUCTION

As individuals, we often face decisions that involve saving money for a future use, or borrowing money for current consumption. We then need to determine the amount we need to invest, if we are saving, or the cost of borrowing, if we are shopping for a loan. As investment analysts, much of our work also involves evaluating transactions with present and future cash flows.

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