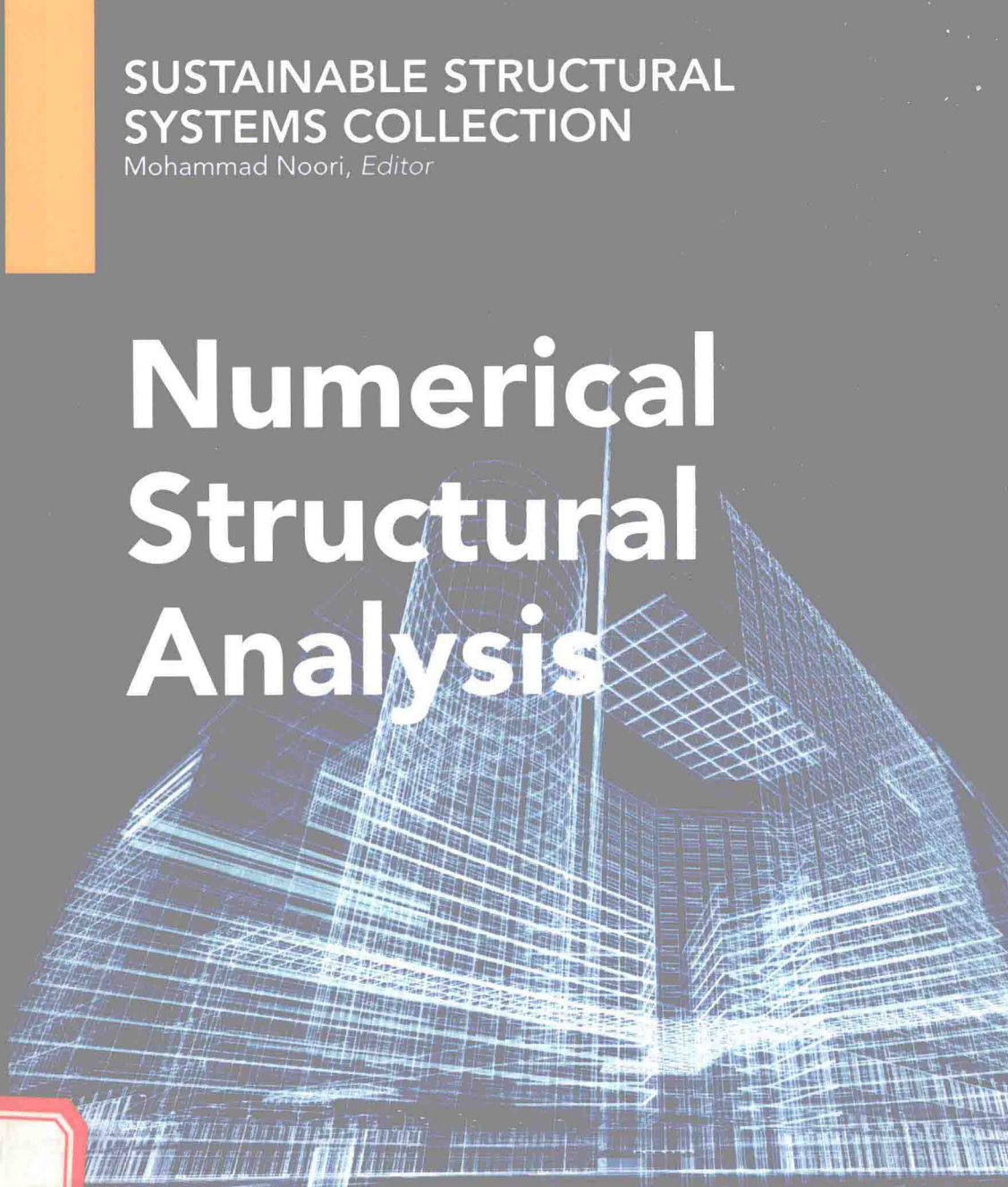


SUSTAINABLE STRUCTURAL
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Mohammad Noori, *Editor*

Numerical Structural Analysis



Steven O'Hara
Carisa H. Ramming



MOMENTUM PRESS
ENGINEERING

NUMERICAL STRUCTURAL ANALYSIS

STEVEN E. O'HARA
CARISA H. RAMMING



MOMENTUM PRESS

MOMENTUM PRESS, LLC, NEW YORK

Numerical Structural Analysis

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First published by Momentum Press®, LLC
222 East 46th Street, New York, NY 10017
www.momentumpress.net

ISBN-13: 978-1-60650-488-8 (print)
ISBN-13: 978-1-60650-489-5 (e-book)

Momentum Press Sustainable Structural Systems Collection

DOI: 10.5643/9781606504895

Cover and Interior design by Exeter Premedia Services Private Ltd.,
Chennai, India

10 9 8 7 6 5 4 3 2 1

Printed in the United States of America

NUMERICAL STRUCTURAL ANALYSIS

ABSTRACT

As structural engineers move further into the age of digital computation and rely more heavily on computers to solve problems, it remains paramount that they understand the basic mathematics and engineering principles used to design and analyze building structures. The analysis of complex structural systems involves the knowledge of science, technology, engineering, and math to design and develop efficient and economical buildings and other structures. The link between the basic concepts and application to real world problems is one of the most challenging learning endeavors that structural engineers face. A thorough understanding of the analysis procedures should lead to successful structures.

The primary purpose of this book is to develop a structural engineering student's ability to solve complex structural analysis problems that they may or may not have ever encountered before. The book will cover and review numerical techniques to solve mathematical formulations. These are the theoretical math and science principles learned as prerequisites to engineering courses, but will be emphasized in numerical formulation. A basic understanding of elementary structural analysis is important and many methods will be reviewed. These formulations are necessary in developing the analysis procedures for structural engineering. Once the numerical formulations are understood, engineers can then develop structural analysis methods that use these techniques. This will be done primarily with matrix structural stiffness procedures. Both of these will supplement both numerical and computer solutions. Finally, advanced stiffness topics will be developed and presented to solve unique structural problems. These include member end releases, nonprismatic, shear, geometric, and torsional stiffness.

KEY WORDS

adjoint matrix, algebraic equations, area moment, beam deflection, carry-over factor, castigliano's theorems, cofactor matrix, column matrix,

complex conjugate pairs, complex roots, conjugate beam, conjugate pairs, convergence, diagonal matrix, differentiation, distinct roots, distribution factor, eigenvalues, elastic stiffness, enke roots, extrapolation, flexural stiffness, geometric stiffness, homogeneous, identity matrix, integer, integration, interpolation, inverse, joint stiffness factor, linear algebraic equations, lower triangular matrix, matrix, matrix minor, member end release, member relative stiffness factor, member stiffness factor, moment-distribution, non-homogeneous, non-prismatic members, partial pivoting, pivot coefficient, pivot equation, polynomials, principal diagonal, roots, rotation, rotational stiffness, row matrix, second-order stiffness, shear stiffness, slope-deflection, sparse matrix, square matrix, stiffness matrix, structural flexibility, structural stiffness, symmetric transformation, torsional stiffness, transcendental equations, transformations, transmission, transposed matrix, triangular matrix, upper triangular matrix, virtual work, visual integration

LIST OF FIGURES

Figure 1.1.	Incremental search method.	11
Figure 1.2.	Refined incremental search method.	12
Figure 1.3.	Bisection method.	14
Figure 1.4.	Method of false position or linear interpolation.	16
Figure 1.5.	Secant method.	18
Figure 1.6.	Newton–Raphson method or Newton’s tangent.	19
Figure 1.7.	Newton–Raphson method or Newton’s tangent.	20
Figure 1.8.	Newton–Raphson method or Newton’s tangent.	20
Figure 1.9.	Newton’s second order method.	21
Figure 1.10.	Newton’s second order method.	23
Figure 1.11.	Graeffe’s root squaring method.	27
Figure 3.1.	Trapezoidal rule.	98
Figure 3.2.	Romberg integration.	100
Figure 3.3.	Simpson’s rule.	104
Figure 3.4.	Example 3.10 Simple beam with difference operator.	124
Figure 3.5.	Example 3.10 Simple beam with difference operator.	124
Figure 3.6.	Example 3.10 Simple beam with difference operator.	125
Figure 3.7.	Example 3.11 Fixed beam with difference operator.	129
Figure 3.8.	Numeric modeling with difference operators.	132
Figure 3.9.	Example 3.12 Column buckling with difference operator.	134
Figure 3.10.	Numeric modeling with partial difference operators.	140
Figure 3.11.	Example 3.19 Plate bending.	141
Figure 3.12.	Example 3.19 Plate bending.	142
Figure 3.13.	Example 3.19 Plate bending.	142

Figure 4.1.	Coordinate systems.	148
Figure 4.2.	Example 4.1 Rotation, α .	149
Figure 4.3.	Example 4.2 Rotation, β .	150
Figure 4.4.	Example 4.3 Rotation, γ .	151
Figure 4.5.	Transformation locations.	153
Figure 4.6.	Orthogonal forces.	153
Figure 4.7.	Transformation effects.	154
Figure 4.8.	Example 4.5 Area moment.	156
Figure 4.9.	Example 4.5 Area moment.	156
Figure 4.10.	Example 4.5 Area moment.	157
Figure 4.11.	Example 4.5 Area moment.	157
Figure 4.12.	Conjugate versus real supports.	159
Figure 4.13.	Conjugate versus real beams.	159
Figure 4.14.	Example 4.6 Conjugate beam.	160
Figure 4.15.	Example 4.6 Conjugate beam.	161
Figure 4.16.	Example 4.8 Virtual work.	162
Figure 4.17.	Example 4.8 Virtual work.	162
Figure 4.18.	Example 4.9 Visual integration.	163
Figure 4.19.	Example 4.9 Visual integration.	164
Figure 4.20.	Example 4.9 Visual integration.	164
Figure 4.21.	Example 4.10 Castigliano's second theorem.	166
Figure 4.22.	Example 4.10 Castigliano's second theorem.	166
Figure 4.23.	Example 4.11 Castigliano's second theorem.	167
Figure 4.24.	Example 4.12 Slope-deflection.	169
Figure 4.25.	Example 4.13 Moment-distribution.	172
Figure 4.26.	Elastic stiffness.	175
Figure 4.27.	Example 4.15 θ_{iy} stiffness.	176
Figure 4.28.	Example 4.15 θ_{iy} stiffness.	176
Figure 4.29.	Example 4.16 Δ_{iz} stiffness.	178
Figure 4.30.	Example 4.17 θ_{iz} stiffness.	181
Figure 4.31.	Example 4.18 Δ_{iy} stiffness.	183
Figure 4.32.	Example 4.18 Δ_{iy} stiffness.	184
Figure 4.33.	Example 4.19 Global joint stiffness.	190

Figure 4.34.	Example 4.19 Global joint stiffness.	191
Figure 4.35.	Example 4.20 Global joint stiffness.	197
Figure 5.1.	Example 5.1 Δ_{xz} end release.	206
Figure 5.2.	Example 5.2 θ_{xy} end release.	209
Figure 5.3.	Example 5.2 θ_{yx} end release.	210
Figure 5.4.	Example 5.2 θ_{xy} end release.	211
Figure 5.5.	Example 5.3 Member stiffness.	218
Figure 5.6.	Example 5.4 Non-prismatic member stiffness.	230
Figure 5.7.	Example 5.5 Non-prismatic member stiffness.	235
Figure 5.8.	Example 5.6 Non-prismatic member stiffness.	236
Figure 5.9.	Example 5.7 Non-prismatic member stiffness.	237
Figure 5.10.	Example 5.8 Shear stiffness.	239
Figure 5.11.	Example 5.9 Shear stiffness.	241
Figure 5.12.	Example 5.10 Shear stiffness.	243
Figure 5.13.	Example 5.11 Shear stiffness.	245
Figure 5.14.	Example 5.12 Shear area.	249
Figure 5.15.	Example 5.13 Shear area.	250
Figure 5.16.	Geometric stiffness.	251
Figure 5.17.	Example 5.14 Geometric stiffness.	252
Figure 5.18.	Example 5.15 Geometric stiffness.	255
Figure 5.19.	Example 5.16 Geometric stiffness.	258
Figure 5.20.	Example 5.17 Geometric stiffness.	260

LIST OF TABLES

Table 1.1.	Synthetic division	7
Table 1.2.	Example 1.5 Synthetic division	7
Table 1.3.	Example 1.5 Synthetic division	8
Table 1.4.	Example 1.5 Synthetic division	8
Table 1.5.	Example 1.5 Synthetic division	8
Table 1.6.	Example 1.6 Synthetic division	9
Table 1.7.	Example 1.6 Synthetic division	9
Table 1.8.	Example 1.6 Synthetic division	10
Table 1.9.	Example 1.6 Synthetic division	10
Table 1.10.	Example 1.7 Incremental search method	11
Table 1.11.	Example 1.8 Refined incremental search method	13
Table 1.12.	Example 1.9 Bisection method	15
Table 1.13.	Example 1.10 Method of false position	17
Table 1.14.	Example 1.11 Secant method	18
Table 1.15.	Example 1.12 Newton–Raphson method	21
Table 1.16.	Example 1.13 Newton’s second order method	24
Table 1.17.	Graeffe’s root squaring method	26
Table 1.18.	Example 1.14 Graeffe’s root squaring method—real and distinct roots	30
Table 1.19.	Example 1.15 Graeffe’s root squaring method—real and equal roots	34
Table 1.20.	Example 1.16 Graeffe’s root squaring method—real and complex roots	37
Table 1.21.	Bairstow’s method	39
Table 2.1.	Example 2.4 Cramer’s rule	58

Table 2.2.	Example 2.5 Cofactor method	61
Table 2.3.	Example 2.6 Method of adjoints	62
Table 2.4.	Example 2.7 Gaussian elimination	65
Table 2.5.	Example 2.8 Gaussian elimination	67
Table 2.6.	Example 2.10 Improved Gaussian–Jordan elimination method	74
Table 2.7.	Example 2.11 Cholesky decomposition method	77
Table 2.8.	Example 2.11 Cholesky decomposition method	78
Table 2.9.	Example 2.11 Cholesky decomposition method	78
Table 2.10.	Example 2.11 Cholesky decomposition method	78
Table 2.11.	Example 2.12 Error equations	81
Table 2.12.	Example 2.12 Error equations	82
Table 2.13.	Example 2.12 Error equations	83
Table 2.14.	Example 2.13 Matrix inversion method	84
Table 2.15.	Example 2.13 Matrix inversion method	85
Table 2.16.	Example 2.13 Matrix inversion method	85
Table 2.17.	Example 2.13 Matrix inversion method	86
Table 2.18.	Example 2.14 Gauss–Seidel iteration method	88
Table 2.19.	Example 2.16 Faddeev–Leverrier method	92
Table 2.20.	Example 2.17 Power method	94
Table 3.1.	Example 3.1 Trapezoidal rule	98
Table 3.2.	Example 3.1 Trapezoidal rule	99
Table 3.3.	Example 3.2 Romberg integration	102
Table 3.4.	Example 3.2 Romberg integration	102
Table 3.5.	Example 3.2 Romberg integration	102
Table 3.6.	Example 3.2 Romberg integration	102
Table 3.7.	Example 3.3 Simpson’s one-third rule	107
Table 3.8.	Example 3.3 Simpson’s one-third rule	107
Table 3.9.	Example 3.4 Simpson’s one-third and three-eighths rules	108
Table 3.10.	Example 3.4 Simpson’s one-third and three-eighths rules	108
Table 3.11.	Gaussian quadrature	110
Table 3.12.	Example 3.5 Gaussian quadrature	110
Table 3.13.	Example 3.5 Gaussian quadrature	111

Table 3.14.	Example 3.5 Gaussian quadrature	111
Table 3.15.	Example 3.6 Double integration by Simpson's one-third rule	113
Table 3.16.	Example 3.7 Double integration by Gaussian quadrature	115
Table 3.17.	Example 3.19 Plate bending	143
Table 3.18.	Example 3.19 Plate bending	143
Table 3.19.	Example 3.19 Plate bending	144
Table 3.20.	Example 3.19 Plate bending	145
Table 4.1.	Example 4.13 Moment-distribution	173
Table 4.2.	Example 4.14 Moment-distribution	174
Table 4.3.	Example 4.20 Global joint stiffness	199
Table 4.4.	Example 4.20 Global joint stiffness	200
Table 4.5.	Example 4.20 Global joint stiffness	200
Table 4.6.	Example 4.20 Global joint stiffness	202
Table 4.7.	Example 4.20 Global joint stiffness	202
Table 4.8.	Example 4.20 Global joint stiffness	203
Table 4.9.	Example 4.20 Global joint stiffness	203
Table 4.10.	Example 4.20 Global joint stiffness	203
Table 5.1.	Release codes—X-Z system	214
Table 5.2.	Release codes—X-Y system	217
Table 5.3.	Example 5.3 Member stiffness, member 1	220
Table 5.4.	Example 5.3 Member stiffness, member 4	221
Table 5.5.	Example 5.3 Member stiffness, member 5	222
Table 5.6.	Example 5.3 Member stiffness, member 3	223
Table 5.7.	Example 5.3 Member stiffness, member 2	224
Table 5.8.	Example 5.3 Member stiffness	225
Table 5.9.	Example 5.3 Member stiffness	226
Table 5.10.	Example 5.3 Member stiffness	227
Table 5.11.	Example 5.3 Member stiffness	228
Table 5.12.	Example 5.3 Member stiffness	228
Table 5.13.	Example 5.3 Member stiffness	228
Table 5.14.	Example 5.3 Member stiffness	229
Table 5.15.	Example 5.3 Member stiffness	229
Table 5.16.	Example 5.13 Shear area	251

ACKNOWLEDGMENTS

Our sincere thanks go to Associate Professor Christopher M. Papadopoulos, PhD, Department of Engineering Science and Materials University of Puerto Rico, Mayagüez.

We would also like to thank the ARCH 6243 – Structures: Analysis II, Spring 2014 class: Kendall Belcher, Conner Bowen, Harishma Donthineni, Gaurang Malviya, Alejandro Marco Perea, Michael Nachreiner, Sai Sankurubhuktha, Timothy Smith, Nuttapong Tanasap, Ignatius Vasant, and Lawrence Wilson.

A special thanks to Nicholas Prather for his assistance with figures.

CONTENTS

LIST OF FIGURES	xi
LIST OF TABLES	xv
ACKNOWLEDGMENTS	xix
1 ROOTS OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS	1
1.1 Equations	1
1.2 Polynomials	2
1.3 Descartes' Rule	3
1.4 Synthetic Division	7
1.5 Incremental Search Method	10
1.6 Refined Incremental Search Method	12
1.7 Bisection Method	13
1.8 Method of False Position or Linear Interpolation	15
1.9 Secant Method	17
1.10 Newton–Raphson Method or Newton's Tangent	18
1.11 Newton's Second Order Method	21
1.12 Graeffe's Root Squaring Method	24
1.13 Bairstow's Method	38
References	46
2 SOLUTIONS OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS USING MATRIX ALGEBRA	47
2.1 Simultaneous Equations	47
2.2 Matrices	48
2.3 Matrix Operations	52
2.4 Cramer's Rule	56
2.5 Method of Adjoints or Cofactor Method	57

2.6	Gaussian Elimination Method	63
2.7	Gauss–Jordan Elimination Method	68
2.8	Improved Gauss–Jordan Elimination Method	72
2.9	Cholesky Decomposition Method	73
2.10	Error Equations	78
2.11	Matrix Inversion Method	80
2.12	Gauss–Seidel Iteration Method	86
2.13	Eigenvalues by Cramer’s Rule	87
2.14	Faddeev–Leverrier Method	90
2.15	Power Method or Iteration Method	91
	References	95
3	NUMERICAL INTEGRATION AND DIFFERENTIATION	97
3.1	Trapezoidal Rule	97
3.2	Romberg Integration	99
3.3	Simpson’s Rule	104
3.4	Gaussian Quadrature	109
3.5	Double Integration by Simpson’s One-Third Rule	112
3.6	Double Integration by Gaussian Quadrature	114
3.7	Taylor Series Polynomial Expansion	116
3.8	Difference Operators by Taylor Series Expansion	118
3.9	Numeric Modeling with Difference Operators	123
3.10	Partial Differential Equation Difference Operators	136
3.11	Numeric Modeling with Partial Difference Operators	140
	References	145
4	MATRIX STRUCTURAL STIFFNESS	147
4.1	Matrix Transformations and Coordinate Systems	147
4.2	Rotation Matrix	148
4.3	Transmission Matrix	153
4.4	Area Moment Method	155
4.5	Conjugate Beam Method	158
4.6	Virtual Work	161
4.7	Castigliano’s Theorems	165
4.8	Slope-Deflection Method	168

4.9	Moment-Distribution Method	170
4.10	Elastic Member Stiffness, X-Z System	174
4.11	Elastic Member Stiffness, X-Y System	181
4.12	Elastic Member Stiffness, 3-D System	186
4.13	Global Joint Stiffness	187
	References	204
5	ADVANCED STRUCTURAL STIFFNESS	205
5.1	Member End Releases, X-Z System	205
5.2	Member End Releases, X-Y System	214
5.3	Member End Releases, 3-D System	217
5.4	Non-prismatic Members	229
5.5	Shear Stiffness, X-Z System	239
5.6	Shear Stiffness, X-Y System	243
5.7	Shear Stiffness, 3-D System	247
5.8	Geometric Stiffness, X-Y System	251
5.9	Geometric Stiffness, X-Z System	258
5.10	Geometric Stiffness, 3-D System	260
5.11	Geometric and Shear Stiffness	262
5.12	Torsion	265
5.13	Sub-structuring	266
	References	268
	ABOUT THE AUTHORS	271
	INDEX	273

ROOTS OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS

In structural engineering, it is important to have a basic knowledge of how computers and calculators solve equations for unknowns. Some equations are solved simply by algebra while higher order equations will require other methods to solve for the unknowns. In this chapter, methods of finding roots to various equations are explored. The *roots* of an equation are defined as values of x where the solution of an equation is true. The most common roots are where the value of the function is zero. This would indicate where a function crosses an axis. Roots are sometimes *complex roots* where they contain both a real number and an imaginary unit.

1.1 EQUATIONS

Equations are generally grouped into two main categories, algebraic equations and transcendental equations. The first type, an *algebraic equation*, is defined as an equation that involves only powers of x . The powers of x can be any real number whether positive or negative. The following are examples of algebraic equations:

$$8x^3 - 3x^2 + 5x - 6 = 0$$

$$\frac{1}{x} + 2\sqrt{x} = 0$$

$$x^{1.25} - 3\pi = 0$$

The second type is *transcendental equations*. These are non-algebraic equations or functions that transcend, or cannot be expressed in terms of