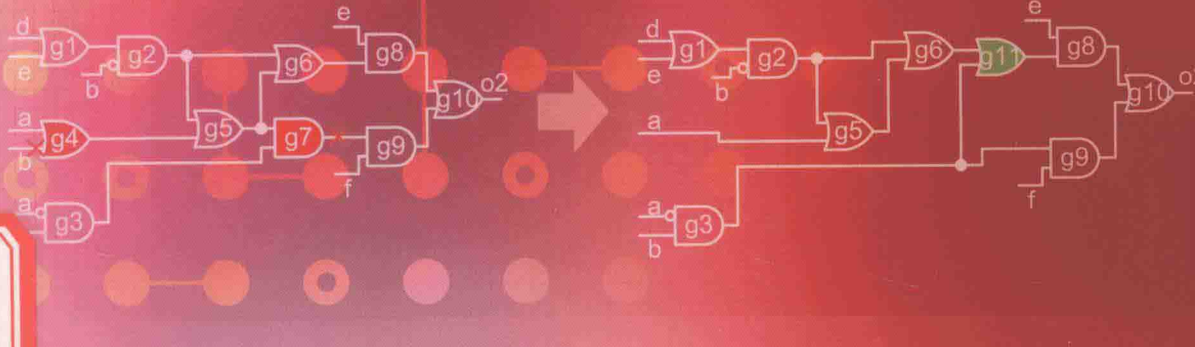
Tak-Kei Lam
Wai-Chung Tang
Xing Wei
Yi Diao
David Yu-Liang Wu

Boolean Circuit Rewiring

Bridging Logical and Physical Designs



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BOOLEAN CIRCUIT REWIRING:

BRIDGING LOGICAL AND PHYSICAL DESIGNS

Tak-Kei Lam

The Chinese University of Hong Kong, Hong Kong, P. R. China

Wai-Chung Tang

Queen Mary University of London, UK

Xing Wei

Easy-Logic Technology Ltd. Hong Kong, Hong Kong, P. R. China

Yi Diao

Easy-Logic Technology Ltd. Hong Kong, Hong Kong, P. R. China

David Yu-Liang Wu

Easy-Logic Technology Ltd. Hong Kong, Hong Kong, P. R. China

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Preface

Our group has been working on wire-based logic restructuring (rewiring) for over a decade. Over the years, we have published numerous conference and journal papers on rewiring. As a recent major milestone, we have developed a rewiring scheme that reaches a near-complete rewiring rate (96%). This result demonstrates the high power of this kind of logic transformation techniques and the great potential of applying them on modern electronic design automation (EDA) tools.

Because of the aggressive and continuous scaling down of transistor sizes, to 45, 22 nm, and even below 16 nm, wires have become a dominant factor affecting circuit performance. Hence, rewiring is particularly suitable for today's nanometer technologies.

We could not find a book that focuses on and discusses rewiring techniques. Since rewiring techniques have become much more practical in nanometer technologies, we felt there was a need to publish a reference book to provide readers with the key ideas.

This book is of introductory to intermediate level. We hope this book will help in popularizing science, and the readers will find this book interesting and informative.

Tak-Kei Lam, Wai-Chung Tang, Xing Wei,
Yi Diao, and David Yu-Liang Wu

Introduction

The concepts of various major rewiring techniques are explained throughout the book gradually. First, readers will be presented with the basic ideas of rewiring. Next, the technical details of each kind of rewiring technique will be discussed in detail. Finally, the applications of rewiring techniques in various electronic design automation (EDA) areas will be introduced.

Intended Audience

Students studying computer/electronic engineering, academic staff, and even EDA engineers are the intended readers of this book. The readers should have some basic knowledge of Boolean algebra, logic gates, and graph theory. For readers without the related advanced knowledge, essential concepts will be introduced and explained throughout the book.

Type Conventions

The following conventions are used in this book:

- Mathematical symbols and names of circuit elements, such as a and α, are typeset in this font.
- `Codes are typeset in this font.`

Acknowledgments

Many people have contributed to this book in the forms of research output, implementations of algorithms, suggestions for content, and, last but not least, simply being encouraging. This book could never have been completed without their generous effort. We are very grateful to the following people for all they have done:

- The authors of RAMBO, REWIRE, RAMFIRE, GBAW, IRRA, NAR, ECR, FECR, CECR, SPFD-based and all other rewiring techniques.
- The authors of the typesetting system LaTeX and the plugins.

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1

Preliminaries

1.1 Boolean Circuits

A Boolean variable is a variable whose value can only be either 0 (false) or 1 (true) or unknown. Every Boolean variable has two literals. They are the normal form and the negation/complement of the variable. The negation of a variable always evaluates to the opposite value of the variable. Suppose v is a Boolean variable; then its negation is $\bar{v}$. When v is 1, $\bar{v}$ is 0; when v is 0, $\bar{v}$ is 1. The literals of variable v are then v and $\bar{v}$.

A function consisting of Boolean variables is known as a Boolean function. It is a mapping between Boolean spaces. For example, the function $f : B^m \rightarrow B^n$ is a mapping between the input space of m Boolean variables and the output space of n Boolean variables. We use $f(x_1, x_2, \ldots, x_{m-1}, x_m)$ to indicate the input variables or input values of the Boolean function f.

The mapping between Boolean spaces is achieved by Boolean operators. The basic Boolean operators (operations) AND, OR, NOT, XOR, and XNOR are denoted as $\cdot$, $+$, $\sim$, $\oplus$, and $\bar{\oplus}$, respectively, in this book. We may omit the symbol $\cdot$ for clarity. The behavior of the basic operators is listed in Table 1.1. Complex Boolean operators can be derived from these basic operators. In fact, only AND and NOT, or only OR and NOT, are sufficient to derive all other Boolean operations.

An example of Boolean function is $f(a, b) = a \cdot b$, which computes the logical conjunction of variables a and b. A Boolean function may contain literals. The Boolean function $f(a, b) = a \cdot \bar{b}$ is such an example that computes the logical conjunction of variable a and the negation of variable b. It may be surprising for readers who are not familiar with Boolean algebra to see a function $f(a, b, c) = a \cdot b$. This function is actually nothing special but is normal and valid. It just means that, among the three variables, the value of variable c is "don't care." That is to say, the value of c can be either 0 or 1, and $f(a, b, c) = ab = ab\bar{c} + abc$. For another example, the function $f(a, b, c) = (a + b)$ can be expanded into $f(a, b, c) = (a + b) = (a + b)\bar{c} + (a + b)c$.

Observability don't cares (ODCs) (Damiani and De Micheli 1990) of a Boolean variable are the conditions under which the variable is not affecting any of the primary outputs. For example, if an input i of an AND gate has the controlling value 0, its set of other inputs J are unobservable no matter what values they have. The ODC of J is $\bar{i}$. Satisfiability don't cares

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Table 1.1 Behavior of the basic Boolean operators

Operator	When will it returns true?
AND $\cdot$	All of its operands are true
OR $+$	Any one of its operands is true
NOT $\sim$	Its operand is false
XOR $\oplus$	Both of its operands have different values
XNOR $\bar{\oplus}$	Both of its operands have the same values

(SDCs) of a circuit node represent the local input patterns at the node that cannot be generated by the node's fanins. As a trivial example of SDC, if we connect all inputs of a two-input AND gate to a common signal, the values of its inputs can never be $\{1, 0\}$ or $\{0, 1\}$.

Many rules in ordinary algebra, such as commutative addition and multiplication, associative addition and multiplication, and variable distribution, can be applied into Boolean algebra. Therefore, function $f(a, b, c) = (a + b) = (a + b)\bar{c} + (a + b)c = a\bar{c} + b\bar{c} + ac + bc$. For each of the conjunction term, it can be expanded by connecting it with all combinations of the literals of the missing variables by conjunction. Some additional important rules that are obeyed in Boolean algebra only include $a \cdot a = a$ and $a + a = a$. Regarding our example, it can be expanded as follows:

$$\begin{aligned}
f(a, b, c) &= (a + b) \\
&= (a + b)\bar{c} + (a + b)c \\
&= (a\bar{c} + b\bar{c}) + (ac + bc) \\
&= (ab\bar{c} + a\bar{b}\bar{c} + ab\bar{c} + \bar{a}b\bar{c}) + (abc + a\bar{b}c + abc + \bar{a}bc) \\
&= ab\bar{c} + a\bar{b}\bar{c} + \bar{a}b\bar{c} + abc + a\bar{b}c + \bar{a}bc
\end{aligned}$$

Other rules can be derived from the basic rules easily. Since Boolean algebra is a vast area of study, even the elementary topics can cover a whole book. In this book, we shall not cover every detail.

Boolean functions can be realized in hardware using logic gates. A Boolean circuit or Boolean network is composed of gates and implements some Boolean functions. We simply use circuits or networks to represent Boolean circuits when the meaning is clear in the context. Figure 1.1 illustrates the logic gates implementing the basic Boolean functions. An example circuit composed of AND, OR, and NOT gates is shown in Figure 1.2(b). For gate e, its inputs are a and b, so it is implementing the function $a + b$. Regarding gate f, its function is $a \cdot \bar{b}$.

A less famous Boolean operator is the cofactor. The cofactor of a Boolean function $f(x_1, x_2, \ldots, x_{n-1}, x_n)$ with respect to a variable x_k is $f|_{x_k} = f(x_1, x_2, \ldots, x_{k-1}, 1, x_{k+1}, \ldots, x_{n-1}, x_n)$. Suppose $f = ab + c$, $f|_a = b + c$, and $f|_c = ab + 1 = 1$. Similarly, the cofactor of a Boolean function $f(x_1, x_2, \ldots, x_{n-1}, x_n)$ with respect to the complement of a variable x_k is $f|_{\overline{x_k}} = f(x_1, x_2, \ldots, x_{k-1}, 0, x_{k+1}, \ldots, x_{n-1}, x_n)$. Suppose $f = ab + c$, $f|_{\bar{a}} = 0 \cdot b + c = c$. Every Boolean function can be expressed using Shannon's expansion. For example, $f(x) = x \cdot f|_x + \bar{x} \cdot f|_{\bar{x}}$. An example of a function with multiple inputs is $f(x, y, z, w) = xyf|_{xy} + \bar{x}yf|_{\bar{x}y} + x\bar{y}f|_{x\bar{y}} + \bar{x}\bar{y}f|_{\bar{x}\bar{y}}$

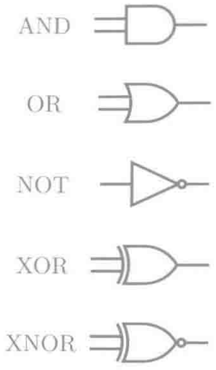
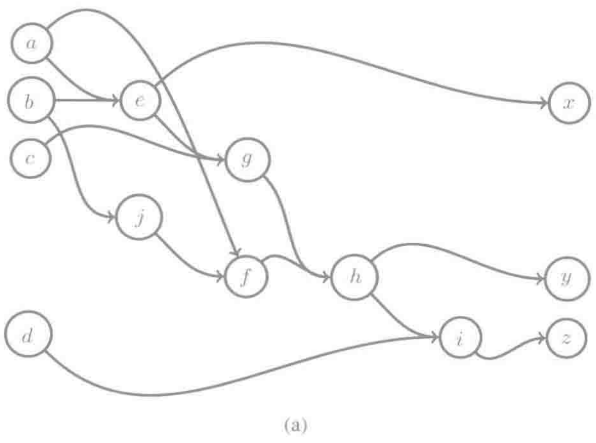


Figure 1.1 Basic logic gates



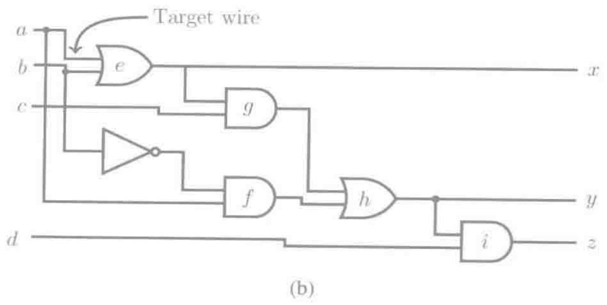


Figure 1.2 Directed acyclic graph representation of a circuit. (a) A directed acyclic graph; (b) a Boolean circuit

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