DIGITAL IMAGE PROCESSING AND ANALYSIS

Edited by

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FOREWORD

A NATO advanced Study Institute took place at Bonas from June 14th to June 25th 1976 on "Digital Image Processing and Analysis". This book is the lasting result of a successful meeting, where the best specialists of the field could exchange their ideas and results.

The papers are arranged so as to present first the more general and tutorial articles and then the more specific ones on applications.

The general topics cover two dimensional transforms, techniques of image restoration, recursive filters, segmentation and analysis of image parts, some points of view from psychology and physiology, and problems of software and processing.

The application fields concerned are remote sensing, medical applications, TV image compression,and optical character recognition.

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Two Dimensional Unitary Transforms

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Two dimensional unitary transforms have found three major applications in image processing. Transforms have been utilized to extract features from images. For example, in the Fourier transform the "d.c." term is proportional to the average image brightness, and the high frequency terms give an indication of the amplitude and orientation of edges within an image. Another application is transform image coding in which a bandwidth reduction is achieved by discarding or grossly quantizing low magnitude transform coefficients. Dimensionality reduction in computation is a third application. Stated simply, those transform coefficients that are small may be excluded from processing operations such as filtering without much loss in processing performance.

1. Unitary Transform Operators

A unitary transformation is a specific type of linear transformation in which the linear operator is exactly invertible and the operator kernel satisfies certain orthogonality conditions <1,2>. The forward unitary transform of the N x N image array F(j,k) results in an N x N transformed image array as defined by

$$\mathcal{F}(j,k) = \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} F(j,k)\, A(j,k;u,v) \qquad (1\text{-}1)$$

where A(j,k;u,v) represents the forward transform kernel. A reverse or inverse transformation provides a mapping from the transform domain to the image space as given by

$$F(j,k) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} \mathcal{F}(u,v)\, B(j,k;u,v) \qquad (1\text{-}2)$$

where B(j,k;u,v) denotes the inverse transform kernel.

The transformation is said to be separable if its kernels can be written in the form

$$A(j,k;u,v) = A_C(j,u)\, A_R(k,v) \qquad (1\text{-}3a)$$

$$B(j,k;u,v) = B_C(j,u)\, B_R(k,v) \qquad (1\text{-}3b)$$

where the kernel subscripts indicate row and column one dimensional transform operations. A separable two dimensional unitary transform can be computed in two steps. First, a one dimensional transform is taken along each column of the image yielding

$$P(u,k) = \sum_{j=0}^{N-1} F(j,k)\, A_C(j,u) \qquad (1\text{-}4)$$

Next, a second one dimensional unitary transform is taken along each column of P(u,k) giving

$$\mathcal{F}(u,v) = \sum_{k=0}^{N-1} P(u,k)\, A_R(k,v) \tag{1-5}$$

Unitary transforms can be conveniently expressed in vector space form <3>. Let $\underline{F}$ and $\underline{f}$ denote the matrix and vector representations of the image array, and let $\mathcal{F}$ and $\underline{\mathit{f}}$ be the matrix and vector forms of the transformed image. Then, the two dimensional unitary transform written in vector form is given by

$$\underline{\mathit{f}} = \underline{A}\,\underline{f} \tag{1-6}$$

where $\underline{A}$ is the forward transformation matrix. The reverse transform is

$$\underline{f} = \underline{B}\,\underline{\mathit{f}} \tag{1-7}$$

where $\underline{B}$ represents the inverse transformation matrix. It is obvious then that

$$\underline{B} = \underline{A}^{-1} \tag{1-8}$$

For a unitary transformation the matrix inverse is given by

$$\underline{A}^{-1} = \underline{A}^{*T} \tag{1-9}$$

and $\underline{A}$ is said to be a unitary matrix. A real unitary matrix is called an orthogonal matrix. For such a matrix

$$\underline{A}^{-1} = \underline{A}^{T} \tag{1-10}$$

If the transform kernels are separable such that

$$\underline{A} = \underline{A}_C \otimes \underline{A}_R \tag{1-11}$$

where $\underline{A}_R$ and $\underline{A}_C$ are row and column unitary transform matrices, then the transformed image matrix can be obtained from the image matrix by

$$\underline{\mathcal{F}} = \underline{A}_C \underline{F} \underline{A}_R^T \qquad (1\text{-}12a)$$

The inverse transform is given by

$$\underline{F} = \underline{B}_C \underline{\mathcal{F}} \underline{B}_R^T \qquad (1\text{-}12b)$$

where $\underline{B}_C = \underline{A}_C^{*T}$ and $\underline{B}_C = \underline{A}_C^{*T}$.

Separable unitary transforms can also be expressed in a hybrid series - vector space form as a sum of vector outer products. Let $\underline{a}_C(j)$ and $\underline{a}_R(k)$ represent columns j and k of the unitary matrices $\underline{A}_C^T$ and $\underline{A}_R^T$, respectively. Then, it is easily verified that

$$\underline{\mathcal{F}} = \sum_{j=1}^{N} \sum_{k=1}^{N} F(j,k)\, \underline{a}_C(j)\, \underline{a}_R^T(k) \qquad (1\text{-}13a)$$

Similarly,

$$\underline{F} = \sum_{u=1}^{N} \sum_{v=1}^{N} \mathcal{F}(u,v)\, \underline{b}_C(u)\, \underline{b}_R^T(v) \qquad (1\text{-}13b)$$

where $\underline{b}_C(u)$ and $\underline{b}_R(v)$ denote columns u and v of the unitary matrices $\underline{B}_C^T$ and $\underline{B}_R^T$, respectively. The vector outer products of eq.(14) form a series of matrices, called basis matrices, which provide matrix decompositions of the image matrix $\underline{F}$ or its unitary transformation $\underline{\mathcal{F}}$.

There are several ways in which a unitary transformation may be viewed. An image transformation can be interpreted as a decomposition of the image data into a generalized two dimensional spectrum <4>. Each spectral component in the transform domain corresponds to the amount of energy of the spectral function within the original image. In this context the concept of frequency may now be generalized to include transformations of functions other than sine and cosine waveforms. This type of generalized spectral analysis is useful in the investigation of specific decompositions which are best suited for particular classes of images. Another way to visualize an image transformation is to consider the transformation as a multi-dimensional rotation of coordinates. One of the major properties of a unitary transformation is that measure is preserved. For example, the mean square difference between two images is equal to the mean square difference between the transforms of the image. A third approach to the visualization of image transformation is to consider eq.(2) as a means of synthesizing an image with a set of two dimensional mathematical functions B(j,k;u,v) for a fixed transform domain coordinate (u,v). In this interpretation, the kernel B(j,k;u,v) is called a two dimensional basis function and the transform coefficient $\mathcal{F}(u,v)$ is the amplitude of the basis function required in the synthesis of the image.

2. Fourier Transform

The discrete two dimensional Fourier transform of an image array is defined in series form as <5-1Ø>

$$\mathcal{F}(u,v) = \frac{1}{N}\sum_{j=0}^{N-1}\sum_{k=0}^{N-1} F(j,k)\exp\left\{\frac{-2\pi i}{N}(uj+vk)\right\} \qquad (2\text{-}1a)$$

where $i=\sqrt{-1}$, and the discrete inverse transform is given by

$$F(j,k) = \frac{1}{N}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \mathcal{F}(u,v)\exp\left\{\frac{2\pi i}{N}(uj+vk)\right\} \qquad (2\text{-}1b)$$

The indices (u,v) are called the spatial frequencies of the transformation, in analogy with continuous Fourier transforms. It should be noted that eq.(1) is not a universal definition; some definitions place all scaling constants in the inverse transform equation, while still others employ a reversal in the sign of the kernels.

Since the transform kernels are separable and symmetric, the two dimensional transform can be computed as sequential row and column one dimensional transforms. The basis functions of the transform are complex exponentials which may be decomposed into sine and cosine components. That is,

$$A(j,k;u,v) = \exp\{-\frac{2\pi i}{N}(uj+vk)\} = \cos\{\frac{2\pi}{N}(uj+vk)\} - i\sin\{\frac{2\pi}{N}(uj+vk)\} \quad (2\text{-}2a)$$

$$B(j,k;u,v) = \exp\{\frac{2\pi i}{N}(uj+vk)\} = \cos\{\frac{2\pi}{N}(uj+vk)\} + i\sin\{\frac{2\pi}{N}(uj+vk)\} \quad (2\text{-}2b)$$

The Fourier transform plane possesses many interesting structural properties. The spectral component at the origin of the Fourier domain

$$\mathcal{F}(0,0) = \frac{1}{N}\sum_{j=0}^{N-1}\sum_{k=0}^{N-1} F(j,k) \quad (2\text{-}3)$$

is equal to N times the spatial average of the image plane. The transform array is periodic in the sense that

$$\mathcal{F}(u+mN, v+nN) = \frac{1}{N}\sum_{j=0}^{N-1}\sum_{k=0}^{N-1} F(j,k)\exp\{\frac{-2\pi i}{N}(uj+vk)\}\exp\{-2\pi i(mj+nk)\} \quad (2\text{-}4)$$

for $m,n=0,\pm 1,\pm 2,\ldots$

If the image array represents a luminance field, F(j,k) will be a real positive function. However, its Fourier transform will, in general, be complex. Since the transform domain contains 2N components, the real and imaginary, or phase and magnitude components, of each coefficient, it might be thought that the Fourier transformation causes an increase in dimensionality. This, however, is not the case because $\mathcal{F}(u,v)$ exhibits a property of conjugate symmetry. That is

$$\mathcal{F}(u,v) = \mathcal{F}^{*}(-u+mN, -v+nN) \tag{2-5}$$

for $m,n=0,\pm1,\pm2,\ldots$. As a result of the conjugate symmetry property, almost one-half of the transform domain samples are redundant, i.e., they can be generated from other transform samples.

Figure 1 contains a photograph of an original image of 256 x 256 pixels and 256 grey levels. In the photograph of the Fourier transform components shown in Figure 1b a point is displayed if the logarithm of the magnitude of a component exceeds a threshold value.

The Fourier transform written in series form in eq.(1) may be redefined in vector space form as

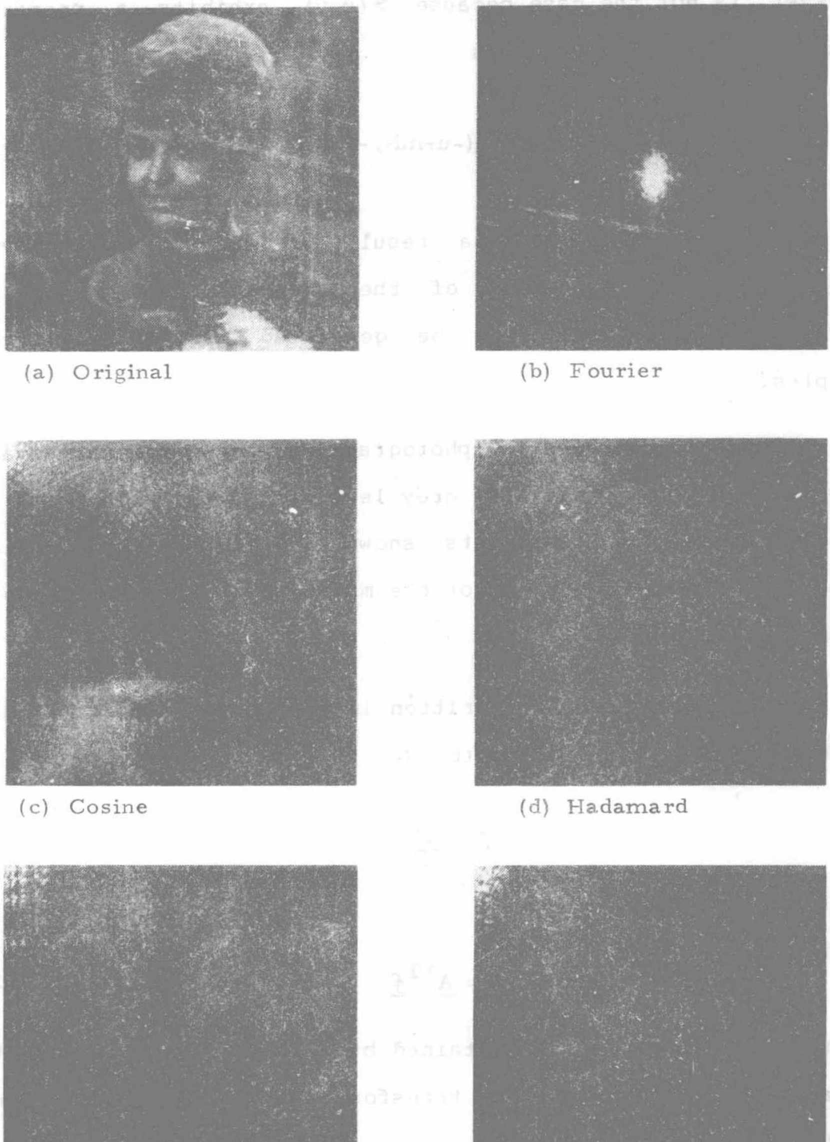
$$\underline{\mathcal{f}} = \underline{A}\underline{f} \tag{2-6a}$$

and

$$\underline{f} = \underline{A}^{*T}\underline{\mathcal{f}} \tag{2-6b}$$

where $\underline{f}$ and $\underline{\mathcal{f}}$ are vectors obtained by column scanning the matrices $\underline{F}$ and $\underline{\mathcal{F}}$, respectively. The transformation matrix $\underline{A}$ can be written in direct product form as

$$\underline{A} = \underline{A}_C \otimes \underline{A}_R \tag{2-7}$$



(a) Original (b) Fourier

(c) Cosine (d) Hadamard

(e) Haar (f) Slant

Figure 1. Unitary transforms of a picture