Sumit Ganguly
Ramesh Krishnamurti (Eds.)

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Algorithms and Discrete Applied Mathematics

First International Conference, CALDAM 2015
Kanpur, India, February 8–10, 2015
Proceedings

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Preface

The First International Conference on Algorithms and Discrete Applied Mathematics was held during February 8–10, 2015, at the Indian Institute of Technology, Kanpur, India. This event was organized by the Department of Computer Science and Engineering, Indian Institute of Technology Kanpur. The workshop covered a diverse range of topics on algorithms and discrete mathematics, comprising computational geometry, algorithms including approximation algorithms, graph theory and computational complexity. This volume contains 26 contributed papers presented during CALDAM 2015. There were 58 submissions from 10 countries. These submissions were carefully reviewed by the Program Committee members with the help of external reviewers. Pavol Hell and C.R. Subramanian delivered excellent invited talks whose abstracts are included in this volume.

We would like to thank the authors for contributing high-quality research papers to the workshop. We express our thanks to the Program Committee members and the external reviewers for their active participation in reviewing the papers. We thank Springer for publishing the proceedings in the reputed *Lecture Notes in Computer Science* series. We thank our invited speakers Pavol Hell and C.R. Subramanian. We thank the Organizing Committee chaired by Surender Baswana and the CSE IIT Kanpur technical team B.M. Shukla, Meeta Bagga, Nagendra Yadav and Adarsh Jagannatha from CSE IIT Kanpur, for the smooth functioning of the workshop. We thank the chair of the Steering Committee, Subir Ghosh, for his active help, support and guidance throughout. We thank our sponsor Google Inc. for their financial support. Finally, we thank the EasyChair conference management system which was very effective in handling the entire reviewing process.

November 2014

Sumit Ganguly
Ramesh Krishnamurti

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Invited Talk (Abstracts)

Obstruction Characterizations in Graphs and Digraphs

Pavol Hell

School of Computing Science, Simon Fraser University, Canada

Abstract. Some of the nicest characterizations of graph families are stated in terms of obstructions – forbidden induced subgraphs or other substructures. A typical example characterizes interval graphs by the absence of asteroidal triples and induced cycles of length greater than three. I will discuss similar obstruction characterizations for classes of digraphs. The obstructions are novel, but similar in spirit to asteroidal triples. Surprisingly, these obstructions permit new characterizations even for undirected graphs. In particular, I will describe the first obstruction characterization of circular arc graphs, and a corresponding certifying polynomial time recognition algorithm for this graph class. The digraph results are joint with Arash Rafiey, Jing Huang, and Tomás Feder, and the circular arc graph characterization and algorithm results are joint with Juraj Stacho and Mathew Francis.

Probabilistic Arguments in Graph Coloring

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Abstract. Probabilistic arguments has come to be a powerful way to obtain bounds on various chromatic numbers. It plays an important role not only in obtaining upper bounds but also in establishing the tightness of these upper bounds. It often calls for the application of various (often simple) ideas, tools and techniques (from probability theory) like moments, concentration inequalities, known estimates on tail probabilities and various other probability estimates. A number of examples illustrate how this approach can be a very useful tool in obtaining chromatic bounds. For many of these bounds, no other approach for obtaining them is known so far. In this talk, we illustrate this approach with some specific applications to graph coloring. On the other hand, several specific applications of this approach have also motivated and led to the development of powerful tools for handling discrete probability spaces. An important tool (for establishing the tightness results) is the notion of random graphs. We do not necessarily present the best results (obtained using this approach) since the main purpose is to provide an introduction to the power and simplicity of the approach. Many of the examples and the results are already known and published in the literature and have also been improved further.

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Probabilistic Arguments in Graph Coloring (Invited Talk)

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Abstract. Probabilistic arguments has come to be a powerful way to obtain bounds on various chromatic numbers. It plays an important role not only in obtaining upper bounds but also in establishing the tightness of these upper bounds. It often calls for the application of various (often simple) ideas, tools and techniques (from probability theory) like moments, concentration inequalities, known estimates on tail probabilities and various other probability estimates. A number of examples illustrate how this approach can be a very useful tool in obtaining chromatic bounds. For many of these bounds, no other approach for obtaining them is known so far. In this talk, we illustrate this approach with some specific applications to graph coloring. On the other hand, several specific applications of this approach have also motivated and led to the development of powerful tools for handling discrete probability spaces. An important tool (for establishing the tightness results) is the notion of random graphs. We do not necessarily present the best results (obtained using this approach) since the main purpose is to provide an introduction to the power and simplicity of the approach. Many of the examples and the results are already known and published in the literature and have also been improved further.

1 Introduction

We mostly focus on simple undirected graphs $G = (V, E)$. A proper k-coloring of G is any map $f : V \to [k]$ which satisfies $f(u) \neq f(v)$ for every $uv \in E$. Here, $[k]$ denotes $\{1, \ldots, k\}$. The least value of k for which G admits a proper k-coloring is known as the chromatic number of G and is denoted by $\chi(G)$. The chromatic index of G (denoted by $\chi'(G)$) is the least value of k such that there is a map $f : E \to [k]$ satisfying $f(uv) \neq f(uw)$ for every $uv, uw \in E$ with $v \neq w$. It is well-known (from the theorems of Brooks and Vizing) that $\chi(G) \leq d + 1$ and $d \leq \chi'(G) \leq d + 1$ for any graph G with maximum degree d.

Several variants of vertex/edge colorings have also been introduced and studied as these notions model several situations arising in practice. Some of these variants impose further restrictions on the standard notion of colorings mentioned before, while others generalize this standard notion. For example, an acyclic vertex k-coloring is a proper k-coloring in which every cycle of G is colored with three or more colors. Eqivalently, the union of any two color classes

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induces a forest. The acyclic chromatic number $a(G)$ is the least k for which G admits such a coloring. When the union is further restricted to be a star forest, it is called a star coloring with star chromatic number being the corresponding invariant. Acyclic and star colorings were introduced by Grünbaum [2]. Acyclic and star colorings model certain partition problems arising in sparse matrix computation [26]. Further examples of such restrictions could be proper colorings in which the union of any two color classes induces (i) a partial 2-tree or (ii) a planar graph, etc.

An example of a generalization is a list coloring f in which each $u \in V$ is provided with a list L_u of colors and f satisfies $f(u) \in L_u$ for each u and $f(u) \neq f(v)$ for every $uv \in E$. Such a coloring is referred to as a proper $\mathcal{L}$-coloring where $\mathcal{L} = \{L_u : u \in V\}$. The least value of k such that G admits a $\mathcal{L}$-coloring for every $\mathcal{L}$ satisfying $|L_u| \geq k$ (for every $u \in V$) is known as the choice number or list chromatic number of G and is denoted by $ch(G)$. Clearly, $\chi(G) \leq ch(G)$ for any G.

List colorings naturally model scheduling problems where one needs to find a conflict-free schedule of resources to users where a resource can be assigned only to a user who is willing to accept that resource. List colorings also naturally arise in constructive proofs of chromatic bounds. To prove the existence of a k-coloring, one starts with an initial partial proper coloring which colors a subset of vertices and extends this partial coloring in an iterative fashion to a full coloring of V. Any such extension is essentially about proving the existence of a list coloring where any uncolored vertex is forbidden to use colors used on its colored neighbors.

Total coloring is another generalization. A total k-coloring is a k-coloring f of $V \cup E$ with colors from $[k]$ such that (i) f restricted to V (or E) is a proper vertex (or edge) coloring and (ii) $f(u) \neq f(uv)$ for every $uv \in E$. The least k for which G admits a total k-coloring is known as the total chromatic number of G and is denoted by $\chi_T(G)$. A trivial bound is $\chi_T(G) \leq \chi(G) + \chi'(G)$.

Given a coloring notion, one is interested in knowing whether the associated chromatic number can be bounded by a function g of one or more of other graph invariants like maximum degree $\Delta(G)$, maximum clique size $\omega(G)$ or the standard chromatic number $\chi(G)$? In addition, one is interested in actually obtaining such a function g and also in determining how tight the estimate provided by g is ? To establish tightness, one needs to obtain implicit or explicit examples of graphs where any such coloring will require a number of colors which is closer to the estimate provided by g. Probabilistic arguments is a very useful way to obtain bounds and also to establish their tightness.

1.1 Constrained Colorings

For an illustration, consider the example of b-frugal colorings. For a given $b \geq 1$, a proper k-coloring f of G is a b-frugal coloring if for every $u \in V$ and $j \in [k]$, at most b neighbors of u are colored j. The least k for which such a coloring exists, is known as the b-frugal chromatic number of G and is denoted by $\chi_b^{frug}(G)$. By introducing randomness, one can show that for every $b \geq 1$, we have

$$\chi_b^{frug}(G) \leq 16d^{b+1/b} \quad \ldots\ldots \quad (A)$$

for every G of maximum degree d. Hind, et.al. [8] obtained this bound with e^3 in place of 16. The idea is as follows : Define $k = 16d^{b+1/b}$. Choose uniformly at random a k-coloring $f : V \to [k]$. Define a collection of bad events as follows :

(*i*) For every $uv \in E$, $\mathcal{E}_{uv}$ happens if $f(u) = f(v)$.
(*ii*) For every $u, S \subseteq N(u)$ with $|S| = b+1$, $\mathcal{E}_{u,S}$ happens if $f(v) = f(w)$ for every $v, w \in S$.

The random choice f is proper and b-frugal if none of these events (referred to as bad events) happens. Note that the claimed bound (A) is true if it can be shown that, **(C1) :** with positive probability f satisfies none of the bad events. **(C1)** can be established by applying a powerful probability tool known as Local Lemma (discovered by Erdös and Lovasz [6]) (see also [17]). This tool is very useful in situations where we have a collection of bad events each of which is independent of all but at most a "small" number of other events. The precise meaning of "small" will vary with events and are implicitly captured by a set of inequalities (involving probabilities of bad events and the number of influential events) which need to be satisfied.

The choice and definition of bad events play an important role in simplifying and shortening the proof arguments. Another important factor is in obtaining tight bounds on the number of events influencing a given bad event. Good estimates of probabilities also play an important role.

Using probabilitic arguments (particularly, Local Lemma), good upper bounds (many of them are also tight) have been obtained on various constrained chromatic numbers. For a sample, we suggest the reader to [7] (acyclic chromatic number), [14] (star chromatic numbers), [25, 10] (acyclic chromatic index), [13] (generalized acyclic chromatic number), [11] (k-intersection chromatic index), [5] (chromatic number bounds for graphs with sparse neighborhoods).

Recently, Aravind and Subramanian [22] have generalized the notions of acyclic chromatic number, frugal chromatic number, etc. to a generic notion of $(j, \mathcal{F})$-colorings and and obtained upper bounds (in terms of Δ) on the associated chromatic numbers. Here, $j \geq 2$ and $\mathcal{F}$ is a family of connected j-colorable graphs with each member being a graph on more than j vertices. A $(j, \mathcal{F})$-coloring of G is a proper vertex coloring such that the union of any j color classes induces a subgraph which is free of any copy of any member of $\mathcal{F}$. The $(j, \mathcal{F})$-chromatic number of G (denoted by $\chi_{j,\mathcal{F}}(G)$) is the least k used on any such coloring. This notion specializes to star colorings if we set $j = 2$ and $\mathcal{F} = \{P_4\}$. It specializes to b-frugal colorings if we set $j = 2$ and $\mathcal{F} = \{K_{1,b+1}\}$.

In a related work [21], they also considered the edge analogues of $(j, \mathcal{F})$-colorings and obtained nearly tight (within a constant multiplicative factor) upper bounds on the associated chromatic indices. For both vertex and edge colorings, it was also shown that several such restrictions (with each specified by a pair $(j_i, \mathcal{F}_i)$) can be simultaneously enforced on the colorings. As a sample consequence, it follows that graphs of maximum degree d can be properly edge colored with $O(d)$ colors so that all of the following restrictions can be simultaneously enforced : (i) union of any four matchings forms a partial 2-tree, (ii) union of any 16 matchings forms a 5-degenerate graph, (iii) union of any $\frac{k^2-k+2}{2}$ (for fixed k) matchings forms a k-colorable graph, etc. The upper bounds of [22, 21] were based on probabilistic arguments employing Local Lemma.

In a subsequent work [23], the authors improved the bound of [22] (for the special case of $j = 2$) to $O\left(d^{\frac{m}{m-1}}\right)$. Here, m denotes the minimum number of edges in any member of $\mathcal{F}$. This bound is also nearly tight in view of the lower bounds in derived in [22] for the case $j = 2$. In this work, a connection between $(j, \mathcal{F})$-chromatic numbers and oriented chromatic numbers was also established. It also presented an improvement of the previously best bound of $2d^2 2^d$ (due to Kostochka, Sopena and Zhu [16]) to $16kd2^d$ on the oriented chromatic number. Here, d refers to the maximum degree and k denotes the degeneracy.

The oriented chromatic number $\chi_o(G)$ of G is the least K such that for every orientation $\boldsymbol{G}$ of $E(G)$, there is an oriented graph (without self-loops) $\boldsymbol{H}$ on K vertices admitting a homomorphism $f : \boldsymbol{G} \rightarrow \boldsymbol{H}$. The improvement is basically due to a more careful analysis of the proof of the bound obtained in [16]. It works by proving the existence of a tournament T on K vertices in which for every $I \subseteq V(T), |I| = i \leq d$ and for every $a \in \{IN, OUT\}^i$, there are at least $kd + 1$ vertices in $V(T) \setminus I$ all having a as the set of orientations into I. The existence was proved by choosing a random tournament on K vertices and showing that it satisfies the required conditions with a positive probability.

1.2 Randomized Coloring Procedure

Sometimes, to obtain strong and tight chromatic bounds, proving the existence of a coloring does not reduce to the probabilistic analysis of one random experiment. It may call for repeated random choices and analyses to arrive at the conclusion. In other words, it reduces to building a desired coloring in an incremental fashion by starting with an initial partial coloring and extending it to a full coloring. Each such extension will involve random choices and their analyses.

One paradigm that has been successfully employed in deriving several chromatic bounds is the following procedure (referred to as *randomized coloring procedure* in [12]) :

- Each uncolored vertex u picks a color randomly from the list of colors available to it. This coloring may not be proper. In that case, uncolor any recently colored vertex whose choice conficts with the random choice or pre-assigned color of any neighbor.

An application of this procedure initially helps us in obtaining a proper partial coloring that takes care of at least the colored vertices. This forces us to prune the list of colors available to uncolored vertices. However, one needs to ensure that each such list is of "sufficient" size to ensure that one can proceed further and obtain a full coloring of desired type. This calls for analyzing the randomized coloring procedure using some of the various concentration inequalities (like Azuma's martingale inequality, Talagrand's inequality) and tail bounds. Some striking applications of this procedure (and its variants) are the list chromatic index bounds (obtained by Kahn [3]), list chromatic bounds for triangle-free graphs (by Kim [4]), list coloring constants (by Reed and Sudakov [24]), total chromatic number bounds (by Hind, Molloy and Reed [9]), frugal coloring and weighted equitable coloring bounds (by Srinivasan and Pemmaraju [12]).

1.3 List Coloring

As mentioned before, we have $\chi(G) \leq ch(G)$ for any G. How large can $ch(G)$ be compared to $\chi(G)$. It can be shown that $ch(K_{n,n}) = \Theta(\ln n)$ and hence one cannot hope to bound $ch(G)$ by only a function of $\chi(G)$. One can easily see that $ch(G) \leq d(G) + 1 \leq \Delta(G) + 1$ for any G. Here, $d(G)$ denotes the degeneracy of G. As the example of $K_{n,n}$ shows, even these bounds can be quite far from $ch(G)$.

The following upper bound is well-known (as has been observed by several researchers using probabilistic arguments)

$$(B1:)\ \ ch(G) \leq c\chi(G)(\ln n)$$

for any G, where $c > 0$ is a constant. An asymptotic improvement of this bound was obtained by Noga Alon in [1] for very special classes of graphs. It was shown that $ch(K_{r*m}) \leq cr(\ln m)$ for every $r, m \geq 2$. Here, K_{m*r} denotes a complete r-partite graph with each part being of size m. For those m satisfying $m = n^{o(1)}$ where $n = rm$, this leads to an asymptotic improvement. This bound was also established to be tight within a constant multiplicative factor for every $r, m \geq 2$. The proof is based on probabilistic arguments. Consider the unique optimal coloring $(V_1, \ldots, V_r)$ of K_{r*m}. If $r \leq \sqrt{n}$, then $m \geq r$ and hence $(B1)$ can be applied. When $r \geq m$, let $L = \cup_u L_u$ and consider the bipartition $L = L_1 \cup L_2$ formed by a uniformly random choice of $h : L \to \{1, 2\}$. Prune each L_u into $L_u \cap L_1$ or $L_u \cap L_2$ depending on whether $u \in \cup_{j \leq r/2} V_j$ or otherwise. It can be shown (using Chernoff bounds on tail probabilities) that this random pruning of lists does not reduce the list sizes by too much. Now it reduces to proving the bound for $K_{\frac{r}{2}*m}$. Continuing in this way, one reaches a stage where $(B1)$ can be applied. The assumption of equal part sizes played a very important role in this proof.

Later, Subramanian [18] extended the proof arguments of [1] to work for any G. Precisely, it was shown that

$$(B2:)\quad ch(G) \leq c\chi(G)\left(\ln\frac{n}{\chi} + 1\right)$$

for any G. The main difficulty in extending the arguments was the potentially highly non-uniform part sizes in any optimal coloring of G under consideration. Hence, to obtain a proof based on inductive reduction on the value of χ, a simple uniform bipartition of L will not work and one needs to choose a random