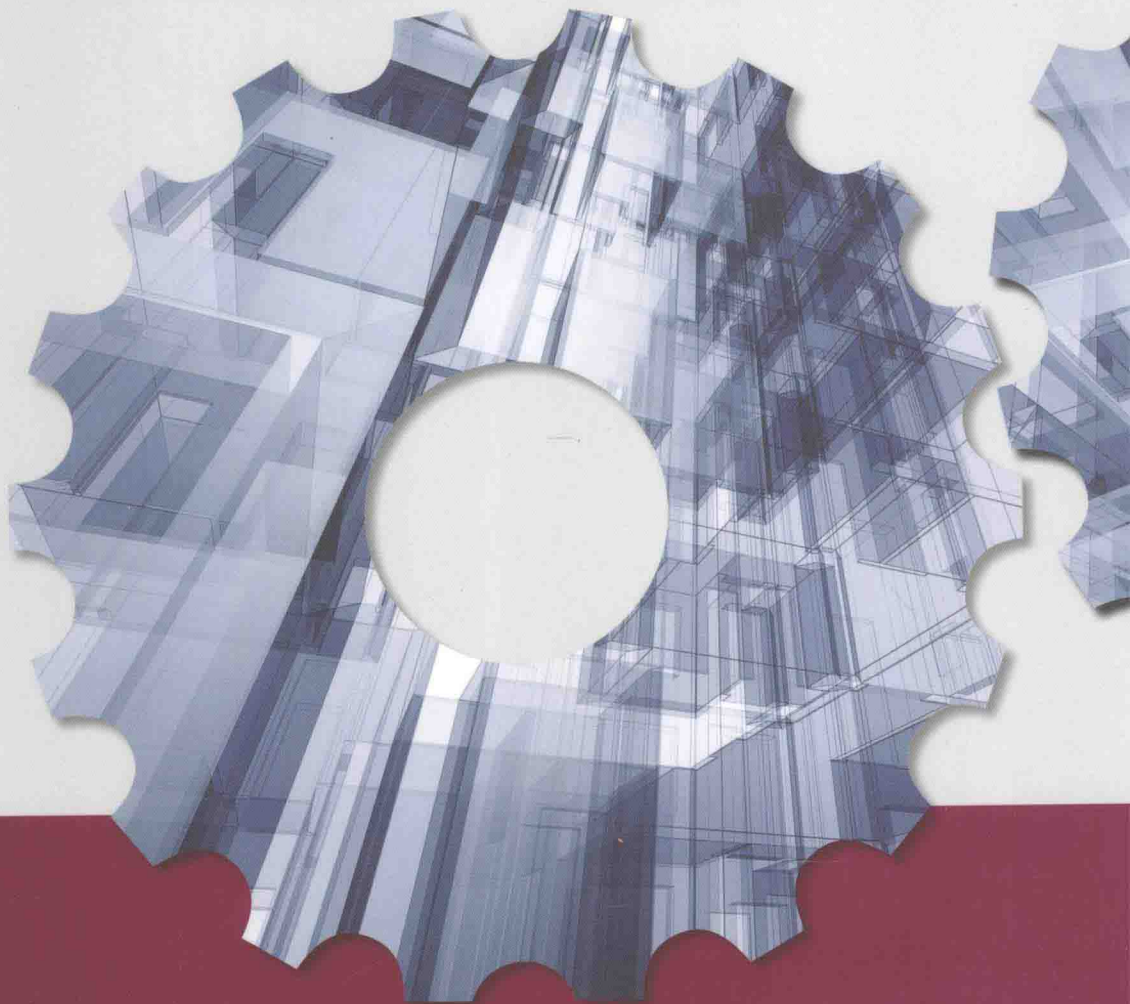


fourth edition

# Fuzzy Logic



WITH ENGINEERING APPLICATIONS

TIMOTHY J. ROSS



WILEY

# FUZZY LOGIC WITH ENGINEERING APPLICATIONS

**Fourth Edition**

**Timothy J. Ross**

*University of New Mexico, USA*

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*This book is dedicated to the memory of my loving mother, Phyllis; to my dear aunts, Glee, Ruth, and Virginia; and to my beloved cousin, Carol, all of whom have departed in the last five years; and to my partner, Laura, who is giving me much optimism for the future. And, to my dear US/Brazilian friends, Cassiano and Kenya, for their unconditional friendship since 2010. Finally, to my new grandson, Jackson, I give you my best wishes in life.*

# About the Author

**Timothy J. Ross** is Professor and Regents' Lecturer of Civil Engineering at the University of New Mexico. He received his PhD in Civil Engineering from Stanford University, his MS from Rice University, and his BS from Washington State University. Professor Ross has held previous positions as Senior Research Structural Engineer, Air Force Weapons Laboratory, from 1978 to 1986; and Vulnerability Engineer, Defense Intelligence Agency, from 1973 to 1978. Professor Ross has authored more than 150 publications and has been active in the research and teaching of fuzzy logic since 1983. He is the founding Co-Editor-in-Chief of the *International Journal of Intelligent and Fuzzy Systems*, the co-editor of *Fuzzy Logic and Control: Software and Hardware Applications*, and the co-editor of *Fuzzy Logic and Probability Applications: Bridging the Gap*. His sabbatical leaves in 1994–1995 at the US Army Environmental Policy Institute, Atlanta, Georgia; in 2001–2002 at the University of Calgary, Alberta, Canada; in 2008–2009 at Gonzaga University in Spokane, Washington; and most recently in 2015 at the Air Force Research Laboratory at Kirtland AFB, New Mexico, have resulted in the education of numerous additional students, faculty, and research professionals in the subject of fuzzy logic because he transferred this technology to all of these institutions. Ross continues to be active in applying fuzzy logic in his areas of research: decision support systems, reliability theory, data fusion, and structural engineering.

# Preface to the Fourth Edition

My primary motivations for writing the fourth edition of this text have been to (1) reduce the length of the text, (2) correct the errata discovered since the publication of the third edition, and (3) introduce limited new material for the readers. The first motivation has been accomplished by eliminating some sections that are rarely taught in the classroom by various faculty using this text and by eliminating some sections that do not add to the utility of the text as a tool to learn basic fundamentals of the subject.

Since the first edition was published, in 1995, the technology of fuzzy set theory and its application to systems, using fuzzy logic, has moved rapidly. Developments in other theories such as possibility theory and evidence theory (both being elements of a larger collection of methods under the rubric “generalized information theories”) have shed more light on the real virtues of fuzzy logic applications, and some developments in machine computation have made certain features of fuzzy logic much more useful than in the past. In fact, it would be fair to state that some developments in fuzzy systems are quite competitive with other, linear algebra-based methods in terms of computational speed and associated accuracy.

There is some new material which is included in the fourth edition to try to capture some of the newer developments; the keyword here is *some* because it would be impossible to summarize or illustrate even a small fraction of the new developments of the last six years since the third edition was published. As with any book containing technical material, the third edition contained errata that have been corrected in this fourth edition. As with the first three editions, a solutions manual for all problems in the fourth edition and software can be obtained by qualified instructors who visit [www.wiley.com/go/ross/fuzzy4e](http://www.wiley.com/go/ross/fuzzy4e) and provide official documentation of their teaching status. In addition to the solutions manual, a directory of software will be made available to all student users and faculty of the text on this same website. Most of the software routines make use of the MATLAB platform, and most of the routines have been written by my students, except for the standard routines that exist as MATLAB functions.

As I discussed in the preface of the third edition, the axioms of a probability theory referred to as the *excluded middle* are again referred to in this edition as axioms—never as laws.

The operations due to De Morgan are also not referred to as laws, but as *principles* because these principles do apply to some (*not all*) uncertainty theories (e.g., probability and fuzzy). The *excluded middle axiom* (and its dual, the *axiom of contradiction*) are not *laws*; Newton produced *laws*, Kepler produced *laws*, Darcy, Boyle, Ohm, Kirchhoff, Bernoulli, and many others too numerous to list here all developed *laws*. *Laws* are mathematical expressions describing the immutable realizations of nature. Definitions, theorems, and axioms collectively can describe a certain axiomatic foundation describing a particular kind of theory, and nothing more; in this case, the *excluded middle* and other axioms can be used to describe a probability theory. Hence, if a fuzzy set theory does not happen to be *constrained* by an *excluded middle axiom*, it is not a *violation* of some immutable law of nature like Newton's laws; fuzzy set theory simply does not happen to have an axiom of the excluded middle; it does not need, nor is *constrained by*, such an axiom. In fact, as early as 1905 the famous mathematician L. E. J. Brouwer defined this excluded middle axiom as a *principle* in his writings; he showed that the *principle of the excluded middle* was inappropriate in some logics, including his own, which he termed *intuitionism*. Brouwer observed that Aristotelian logic is only a part of mathematics, the special kind of mathematical thought obtained if one restricts oneself to relations of the whole and part. Brouwer had to specify in which sense the principles of logic could be considered "laws" because within his intuitionistic framework thought did not follow any rules, and, hence, "law" could no longer mean "rule" (see the detailed discussions on this in Chapters 5 and 13). In this regard, I continue to take on the cause advocated by Brouwer more than a century ago.

Also in this fourth edition, as in previous editions, I do not refer to "fuzzy measure theory" but instead describe it as "monotone measure theory"; the reader will see this in the title of Chapter 13. The former phrase still causes confusion when referring to fuzzy set theory; we hope to help in ending this confusion. In Chapter 13, in describing the monotone measure,  $m$ , I use the phrase describing this measure as a "basic evidence assignment (bea)," as opposed to the early use of the phrase "basic probability assignment (bpa)." Again, we attempt to avoid confusion with any of the terms typically used in probability theory.

As with the first three editions, this fourth edition is designed for the professional and academic audience interested primarily in applications of fuzzy logic in engineering and technology. I have found that the majority of students and practicing professionals are interested in the applications of fuzzy logic to their particular fields. Hence, the text is written for an audience primarily at the senior undergraduate and first-year graduate levels. With numerous examples throughout, this text is written to assist the learning process of a broad cross section of technical disciplines. It is primarily focused on applications, but each of the chapters begin with the rudimentary structure of the underlying mathematics required for a fundamental understanding of the methods illustrated.

Chapter 1 introduces the basic concept of fuzziness and distinguishes fuzzy uncertainty from other forms of uncertainty. It also introduces the fundamental idea of set membership, thereby laying the foundation for all material that follows, and presents membership functions as the format used for expressing set membership. The chapter summarizes a historical review of uncertainty theories and reviews the idea of "sets as points" in an  $n$ -dimensional Euclidean space as a graphical analog in understanding the relationship between classical (crisp) and fuzzy sets. A new section in the chapter addresses the intuition of propagating uncertainty by showing an example that compares the results of propagating probabilities on the one hand,

or membership values on the other, through a simple nonlinear function. In this example there are some counterintuitive findings that readers will find both interesting and instructive.

Chapter 2 reviews classical set theory and develops the basic ideas of fuzzy sets. Operations, axioms, and properties of fuzzy sets are introduced by way of comparisons with the same entities for classical sets. Various normative measures to model fuzzy intersections (t-norms) and fuzzy unions (t-conorms) are summarized.

Chapter 3 develops the ideas of fuzzy relations as a means of mapping fuzziness from one universe to another. Various forms of the composition operation for relations are presented. Again, the epistemological approach in this chapter uses comparisons with classical relations in developing and illustrating fuzzy relations. Chapter 3 also illustrates methods to determine the numerical values contained within a specific class of fuzzy relations, called *similarity relations*. The section on a three-dimensional physical analogy of equivalence relations has been deleted.

Chapter 4 discusses the fuzzification of scalar variables and the defuzzification of membership functions. It introduces the basic features of a membership function and it discusses, briefly, the notion of interval-valued fuzzy sets. Defuzzification is necessary in dealing with the ubiquitous crisp (binary) world around us. The chapter details defuzzification of fuzzy sets and fuzzy relations into crisp sets and crisp relations, respectively, using lambda-cuts, and it describes a variety of methods to defuzzify membership functions into scalar values. Some of the defuzzification methods in the third edition have been deleted because they are seldom used in practice and because they are covered elsewhere in the literature. Examples of all methods are given in the chapter.

Chapter 5 introduces the precepts of fuzzy logic, again through a review of the relevant features of classical, or a propositional, logic. Various logical connectives and operations are illustrated. There is a thorough discussion of the various forms of the implication operation and the composition operation provided in this chapter. Three different inference methods, popular in the literature, are illustrated. Approximate reasoning, or reasoning under imprecise (fuzzy) information, is also introduced in this chapter. Basic IF–THEN rule structures are introduced and three graphical methods of inference are presented. The section on Natural Language has been shortened. A few more examples of the difficulties of using the axiom of the excluded middle are given in the summary of the chapter.

Chapter 6 provides several classical methods of developing membership functions, including methods that make use of the technologies of neural networks, genetic algorithms, and inductive reasoning.

Chapter 7 presents six automated methods that can be used to generate rules and membership functions from observed or measured input–output data. The procedures are essentially computational methods of learning. Examples are provided to illustrate each method. Many of the problems at the end of the chapter will require software; this software can be downloaded from [www.wiley.com/go/ross/fuzzy4e](http://www.wiley.com/go/ross/fuzzy4e).

Beginning the second category of chapters in the book highlighting applications, Chapter 8 continues with the rule-based format to introduce fuzzy nonlinear simulation and complex system modeling. In this context, nonlinear functions are seen as mappings of information “patches” from the input space to information “patches” of the output space, instead of the “point-to-point” idea taught in classical engineering courses. Fidelity of the simulation is illustrated with standard functions, but the power of the idea can be seen in systems too complex for



an algorithmic description. This chapter formalizes fuzzy associative memories (FAMs) as generalized mappings.

Chapter 9 develops fuzzy decision making by introducing some simple concepts in ordering, preference and consensus, and multiobjective decisions. It introduces the powerful concept of Bayesian decision methods by fuzzifying this classic probabilistic approach. This chapter illustrates the power of combining fuzzy set theory with probability to handle random and nonrandom uncertainty in the decision-making process.

Chapter 10 discusses a few fuzzy classification methods by contrasting them with classical methods of classification and develops a simple metric to assess the goodness of the classification, or misclassification. This chapter also summarizes classification using equivalence relations. It now has a section on pattern recognition, gleaned from the third edition. This section introduces a useful metric in pattern recognition using the algebra of fuzzy vectors. A single-feature and a multiple-feature procedure are summarized in the chapter. The section on image processing has been deleted because other books have extensive coverage of this area.

The chapter in the third edition on fuzzy arithmetic and fuzzy numbers has been deleted. A summary of Zadeh's extension principle and a few simple examples of fuzzy arithmetic are included in Chapter 12.

Chapter 11 introduces the field of fuzzy control systems. A brief review of control system design and control surfaces is provided. Simple example problems in control are provided. Two sections in this chapter are worth noting: fuzzy engineering process control and fuzzy statistical process control, with examples on both provided. A discussion of the comparison of fuzzy and classical control is contained in the chapter summary, and a few more examples of fuzzy control in industrial systems and applications are also included.

Chapter 12 has been extensively changed by including more information on genetically evolved fuzzy cognitive maps, and new sections on the extension principle and fuzzy arithmetic and on fuzzy data fusion are also detailed. Previous sections on fuzzy optimization and fuzzy agent-based methods are still contained in this chapter.

Finally, Chapter 13 enlarges the reader's understanding of the relationship between fuzzy uncertainty and random uncertainty (and other general forms of uncertainty, for that matter) by illustrating the foundations of monotone measures. The chapter discusses monotone measures in the context of evidence theory, possibility theory, and probability theory. The chapter has a section on methods to develop approximate possibility distribution functions derived from both data intervals and scalar point data.

Most of the text can be covered in a one-semester course at the senior undergraduate level. In fact, most science disciplines and virtually all math and engineering disciplines contain the basic ideas of set theory, mathematics, and deductive logic, which form the only knowledge necessary for a complete understanding of the text. For an introductory class, instructors may want to exclude some or all of the material covered in the last section of Chapter 6 (neural networks, genetic algorithms, and inductive reasoning), Chapter 7 (automated methods of generation), and any of the final three chapters: Chapter 11 (fuzzy control), Chapter 12 (miscellaneous fuzzy applications), and Chapter 13 on alternative measures of uncertainty. I consider the applications in Chapter 8 on simulations, Chapter 9 on decision making, and Chapter 10 on classification to be important in the first course on this subject. The other topics could be used either as introductory material for a graduate-level course or for additional coverage for graduate students taking the undergraduate course for graduate credit.

In terms of organization, the first eight chapters of the text develop the foundational material necessary to get students in a position where they can generate their own fuzzy systems. The last five chapters use the foundation material from the first eight chapters to present specific applications.

Most of the problems at the end of each chapter have been revised with different numbers, and there are many new problems that have been added to the text and some problems from the third edition deleted. In reducing the length of the book, some old problems have been deleted from many chapters. The problems in this text are typically based on current and potential applications, case studies, and education in intelligent and fuzzy systems in engineering and related technical fields. The problems address the disciplines of computer science, electrical engineering, manufacturing engineering, industrial engineering, chemical engineering, petroleum engineering, mechanical engineering, civil engineering, environmental engineering, and engineering management, and a few related fields such as mathematics, medicine, operations research, technology management, the hard and soft sciences, and some technical business issues. The references cited in the chapters are listed toward the end of each chapter. These references provide sufficient detail for those readers interested in learning more about particular applications using fuzzy sets or fuzzy logic. The large number of problems provided in the text at the end of each chapter allows instructors a sizable problem base to afford instruction using this text on a multise semester or multiyear basis, without having to assign the same problems term after term.

Again I wish to give credit to some of the individuals who have shaped my thinking about this subject since the first edition of 1995, and to others who by their simple association with me have caused me to be more circumspect about the use of the material contained in the text. Three colleagues at Los Alamos National Laboratory have continued to work with me on applications of fuzzy set theory, fuzzy logic, and generalized uncertainty theory: Dr. Greg Chavez (who wrote much of Chapter 7) and Drs. Jamie Langenbrunner and Jane Booker (retired), who both have worked extensively in an area known as *quantification of margins and uncertainty (QMU)* in assessing reliability of man-made systems; in this regard these three individuals have all explored the use of fuzzy logic and possibility theory in their work. I wish to acknowledge the organizational support of two individuals in the Brazilian Institute, Instituto de Pesquisas Energéticas e Nucleares (IPEN), in São Paulo, Brazil. These two researchers, Drs. Francisco Lemos and Antônio Barroso, through their invitations and travel support, have enabled me to train numerous Brazilian scientists and engineers in fuzzy logic applications in their own fields of work, most notably nuclear waste management, knowledge management, and risk assessment. My discussions with them have given me ideas about where fuzzy logic can impact new fields of inquiry.

I wish to thank two of my recent graduate students who have undertaken MS theses or PhD dissertations related to fuzzy logic and whose diligent work has assisted me in writing this new edition: Clay Phillips, Sandia National Laboratory, and Donald Lincoln, NStone Corporation. These former students have helped me with additional material that I have added in Chapter 12 and have helped discover some errata in this text. There have been numerous students over the past five years who have found much of the errata I have corrected; unfortunately, too numerous to mention in this brief preface. I want to thank them all for their contributions.

Four individuals need specific mention because they have contributed some sections to this text. I would like to thank specifically Dr. Jerry Parkinson for his contributions to Chapter 11, in the areas of chemical process control and fuzzy statistical process control; Dr. Greg Chavez for

his contributions in automated methods; Dr. Sunil Donald for his early work in possibility distributions in Chapter 13; and Dr. Jung Kim for his contribution in Chapter 13 of a new procedure to combine disparate interval data. I would like to acknowledge the work of two graduate students at the University of New Mexico: Rashid Ahmad who has helped in developing some equations and figures for one chapter in the text, and to Pradeep Paudel for the development of the solutions manual for the text.

One individual deserves my special thanks and praise, and that is Professor Mahmoud Taha, my colleague in Civil Engineering at the University of New Mexico. In the last five years he has continued with his work in fuzzy logic applications, pattern recognition, and applications using possibility theory; I am proud and grateful to have been his mentor. I am indebted to his collaborations, his quick adaptation in the application of these tools, and in being a proficient research colleague of mine.

I am grateful for support in the past from IPEN in Brazil to teach this subject in their facility in late 2012 and to work in the area of fuzzy cognitive maps and to the Fulbright Foundation of Brazil to support me for two summers in 2013–2014 to continue my work at the Pontificia Universidade Católica (PUC) do Rio de Janeiro, Brazil. Although most of my research at PUC was in bamboo engineering, I taught a graduate course there in fuzzy logic. Three individuals at PUC deserve special note: Professors Khosrow Ghavami and Raul Rosas da Silva, who both made my visit to Rio de Janeiro a most personally rewarding visit, and graduate student, Marco Antônio da Cunha, whose energy allowed me to advance my understanding of the use of cognitive maps in modeling the creation of a new field in bamboo engineering and in the use of agent-based models in a new field of enzymatic catalysis. Most recently (2015) on my sabbatical to the Air Force Research Laboratory, Space Vehicles Directorate, Kirtland AFB, New Mexico, I want to thank Messrs. Paul Zetocha and Apoorva Bhopale for their interest, and collaboration in, a new thrust area of research in fuzzy data fusion in the area of satellite orbital control.

With so many texts covering specific niches of fuzzy logic it is not possible to summarize all the important facets of fuzzy set theory and fuzzy logic in a single text. The hundreds of edited works and tens of thousands of archival papers show clearly that this is a rapidly growing technology, where new discoveries are being published every month. It remains my fervent hope that this introductory text will assist students and practicing professionals to learn, to apply, and to be comfortable with fuzzy set theory and fuzzy logic. I welcome comments from all readers to improve this text as a useful guide for the community of engineers and technologists who will become knowledgeable about the potential of fuzzy system tools for their use in solving the complex problems that challenge us each day.

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# 1

## Introduction

*It is the mark of an instructed mind to rest satisfied with that degree of precision which the nature of the subject admits, and not to seek exactness where only an approximation of the truth is possible.*

Aristotle, 384–322 B.C.  
Ancient Greek philosopher

*Precision is not truth.*

Henri E. B. Matisse, 1869–1954  
Impressionist painter

*All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life but only to an imagined celestial existence.*

Bertrand Russell, 1923  
British philosopher and Nobel Laureate

*We must exploit our tolerance for imprecision.*

Lotfi Zadeh, 1973  
Professor, Systems Engineering, UC–Berkeley

The preceding quotes, all of them legendary, have a common thread. That thread represents the relationship between precision and uncertainty. The more uncertainty in a problem, the less precise we can be in our understanding of that problem. It is ironic that the oldest quote is attributed to the philosopher who is credited with the establishment of Western logic—a binary logic that admits only the opposites of true and false, a logic that does not admit degrees of truth in between these two extremes. In other words, Aristotelian logic does not admit imprecision in truth. However, Aristotle’s quote is so appropriate today; it is a quote that admits uncertainty.



It is an admonishment that we should heed; we should balance the precision we seek with the uncertainty that exists. Most engineering texts do not address the uncertainty in the information, models, and solutions that are conveyed within the problems addressed therein. This text is dedicated to the characterization and quantification of uncertainty within engineering problems such that an appropriate level of precision can be expressed. When we ask ourselves why we should engage in this pursuit, one reason should be obvious: achieving high levels of precision costs time or money or both. Are we solving problems that require precision? The more complex a system is, the more imprecise or inexact is the information that we have to characterize that system. It seems, then, that precision and information and complexity are inextricably related in the problems we pose for eventual solution. However, for most of the problems that we face, the quote credited to Professor Zadeh suggests that we can do a better job in accepting some level of imprecision.

It seems intuitive that we should balance the degree of precision in a problem with the associated uncertainty in that problem. Hence, this text recognizes that uncertainty of various forms permeates all scientific endeavors, and it exists as an integral feature of all abstractions, models, and solutions. Hence, the intent of this book is to introduce methods to handle one of these forms of uncertainty in our technical problems, the form we have come to call *fuzziness*.

## The Case for Imprecision

Our understanding of most physical processes is based largely on imprecise human reasoning. This imprecision (when compared to the precise quantities required by computers) is nonetheless a form of information that can be quite useful to humans. The ability to embed such reasoning in hitherto intractable and complex problems is the criterion by which the efficacy of fuzzy logic is judged. Undoubtedly, this ability cannot solve problems that require precision, problems such as shooting precision laser beams more than tens of kilometers in space; milling machine components to accuracies of parts per billion; or focusing a microscopic electron beam on a specimen the size of a nanometer. The impact of fuzzy logic in these areas might be years away, if ever. But not many human problems require such precision, problems such as parking a car, backing up a trailer, navigating a car among others on a freeway, washing clothes, controlling traffic at intersections, judging beauty contestants, and a preliminary understanding of a complex system.

There are many simple examples in our culture that illustrate the lack of necessity for precision in much of what we do. There is a joke that is a good illustration about the lack of information contained in a precise number (Paulos, 1995). “A natural history museum guard told a visitor that the dinosaur on exhibit was 90,000,006 years old. Upon questioning about the specific number he used, the guard explained that he was told the dinosaur was 90,000,000 years old when he was hired, six years before! One can easily see the folly in adding a precise number to an imprecise number.

Another example follows us on a daily basis (Rocha, Massad, and Pereira, 2005). In food preparation the older manuals provide recipes that are appropriate enough for cooking delectable foods. A typical recipe calls for “*about* a cup” of this, a “*few* tablespoons” of that, a “*smidgen*” of something, “*four or five*” slices of something else, a “*couple* of medium-sized” other things and “*seasoning to taste*.” The recipe goes on to state that this will produce “*about* four servings.” This vagueness and ambiguity is not objectionable, but the arithmetic that comes from it is. In italicized print at the end of this older recipe, it’s affirmed that the content