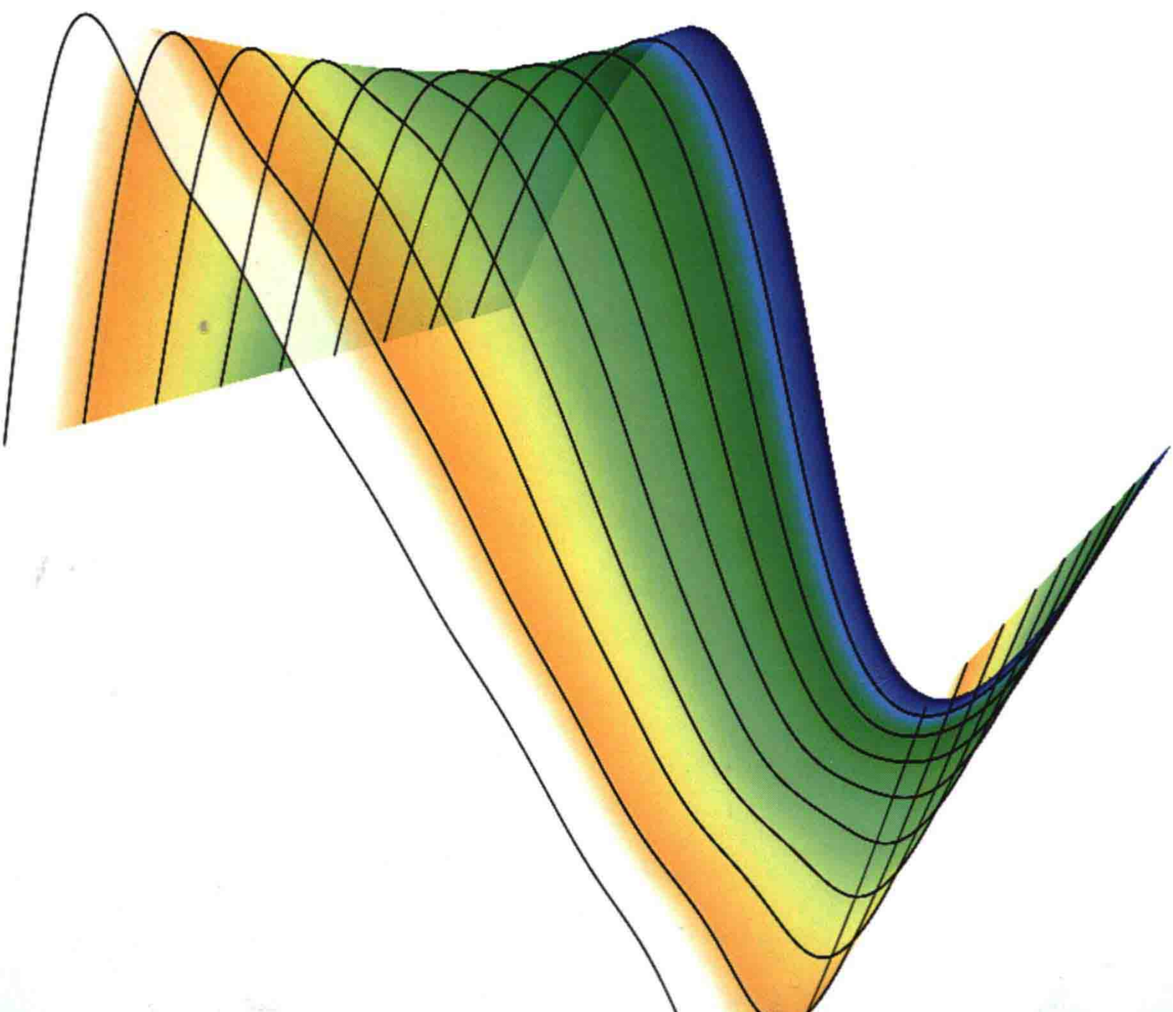
PETE SYMONS

Digital Waveform Generation



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Digital Waveform Generation

This concise overview of digital signal generation will introduce you to powerful, flexible and practical digital waveform generation techniques. These techniques, based largely on phase accumulation and phase–amplitude mapping, will enable you to generate sinusoidal and arbitrary waveforms in real-time with independently controlled waveshape, frequency, phase offset and amplitude, and to design and optimise bespoke digital waveform generation systems from scratch.

The book includes a review of key definitions, a brief explanatory introduction to classical analogue waveform generation and its basic conceptual and mathematical foundations, coverage of recursion, DDS, IDFT and dynamic waveshape and spectrum control, and a chapter dedicated to detailed examples of hardware design, accompanied by downloadable Mathcad models created to help you explore 'what if?' design scenarios. It is essential reading for practitioners in the digital signal processing community, and for students who want to understand and apply digital waveform synthesis techniques.

Pete Symons is a professional engineer with over 30 years' experience in the design of digital and analogue signal processing systems, and is Chief Engineer at Avalon Sciences Ltd. He has held numerous positions in industry, including Chief Engineer for the Special Projects Research Group at Thales UK, and Group Leader of the Electronics Research Group at the Atomic Weapons Establishment, UK. He is a Chartered Engineer, and a Fellow of the IET.

'Symons writes with admirable precision and attention to detail in describing techniques for arbitrary waveform generation based upon Direct Digital Synthesis principles. Newcomers to the topic and seasoned practitioners alike will benefit from the extensive tutorial content supporting the theoretical foundations, synthesis algorithms and processing architectures employed in the implementation of these powerful and flexible techniques. A notable feature is the inclusion of simulated performance results for several arbitrary waveform phase–amplitude mapping algorithms, illustrating the dependence of the amplitude spectrum and other performance metrics on a range of design and control parameters.'

Mike Meade, Open University

'This is a very readable signal processing book – comprehensive in scope, and thorough in presentation. Clear explanations, detailed illustrations, and practical implementation guidance make it a welcomed addition to the literature of modern-day digital waveform generation.'

Rick Lyons, Besser Associates

Preface

Most electronic design engineers, irrespective of being the 'analogue' or 'digital' variety, are occasionally faced with the task of designing an oscillatory signal generator with particular implementation constraints, control and performance requirements. These requirements might include extremely low distortion, unusual 'application specific' waveshape, wide frequency tuning range, low temperature drift, and so on. Historically, such a task will have been tackled with a wholly analogue design, possibly augmented by digital control, where extremely high levels of performance are evident in some cases. If we take high-end audio test instrumentation as an example, the now legendary Hewlett Packard HP8903B and Audio Precision AP1 audio test sets both use digitally controlled analogue state variable oscillators to generate extremely low distortion sine waves. The state variable analogue oscillator is effectively an analogue computer model designed to compute solutions of a second-order differential equation. A specific class of solution (under certain parametric conditions) is a continuous sinusoidal oscillation. These generators are outstanding examples of what can be achieved with innovative analogue design. However, the world is becoming increasingly digital and very high levels of digital processing power can be implemented at relatively low cost. Various 'all digital' waveform generation techniques are therefore now practicable; and when all of their advantages are weighed up against the disadvantages (*yes, digital processing is not necessarily a panacea to guarantee ideal performance*), they nearly always represent the best solution. This approach is reinforced, if not driven, by the ever-improving performance of commercial digital to analogue conversion (DAC) devices as measured by their spurious-free dynamic range (i.e. distortion) and bandwidth. It is not unreasonable to state that the integrated DAC is the foremost enabling technology for nearly all applications of digital waveform generation. Exceptions to this observation apply to purely digital signals, which exist as a discrete-time sequence of binary numbers representing the signal waveform samples.

Electronic signals are often described as *waveforms*, indicating the time domain image observed when the signal is measured with an oscilloscope. A signal produced by an electronic signal generator may be defined as a periodic (or aperiodic), time-varying voltage or current. The corresponding shape function can be described (or approximated to some level of accuracy) by a causal mathematical function. This definition suggests a generation method where the underlying mathematical function which describes the waveform is effectively computed in real time by analogue or digital means. The simplest and most widely used waveform is the sine wave, whose corresponding mathematical function may be computed by various well-known analogue methods, digital recursive algorithms and so-called '*direct digital synthesis*' (DDS). Classical non-sinusoidal, so-called 'function' waveforms include the sawtooth, triangle, square and pulse waveforms; each having a simple piecewise-linear discontinuous mathematical description. These waveforms can also be generated using an analogue computer model so as to have a waveshape which changes according to a waveshape modulation parameter. For example, a pulse waveform whose duty-cycle (i.e. the ratio of the waveform's 'on' time to the total waveform period) changes according to the value of some parameter, although the waveform's frequency remains constant. These waveforms suffice for many applications, but increasingly signals with user-defined waveshape (or spectrum) are required in complex electronic systems as test stimuli, excitation, control or modulation signals. We generalise signals with user-defined waveshape as *arbitrary waveforms* and this book is primarily concerned with investigating digital techniques for their generation. Further examples of arbitrary waveform application include:

- communication system carrier modulation;
- transducer excitation signal generation;
- medical and related imaging signal emulation;
- signal emulation in complex test instrumentation;
- radar return signal simulation;
- electronic warfare adaptive signal generation;
- control system diagnostic excitation;
- multi-tone audio system testing;
- computer music synthesis.

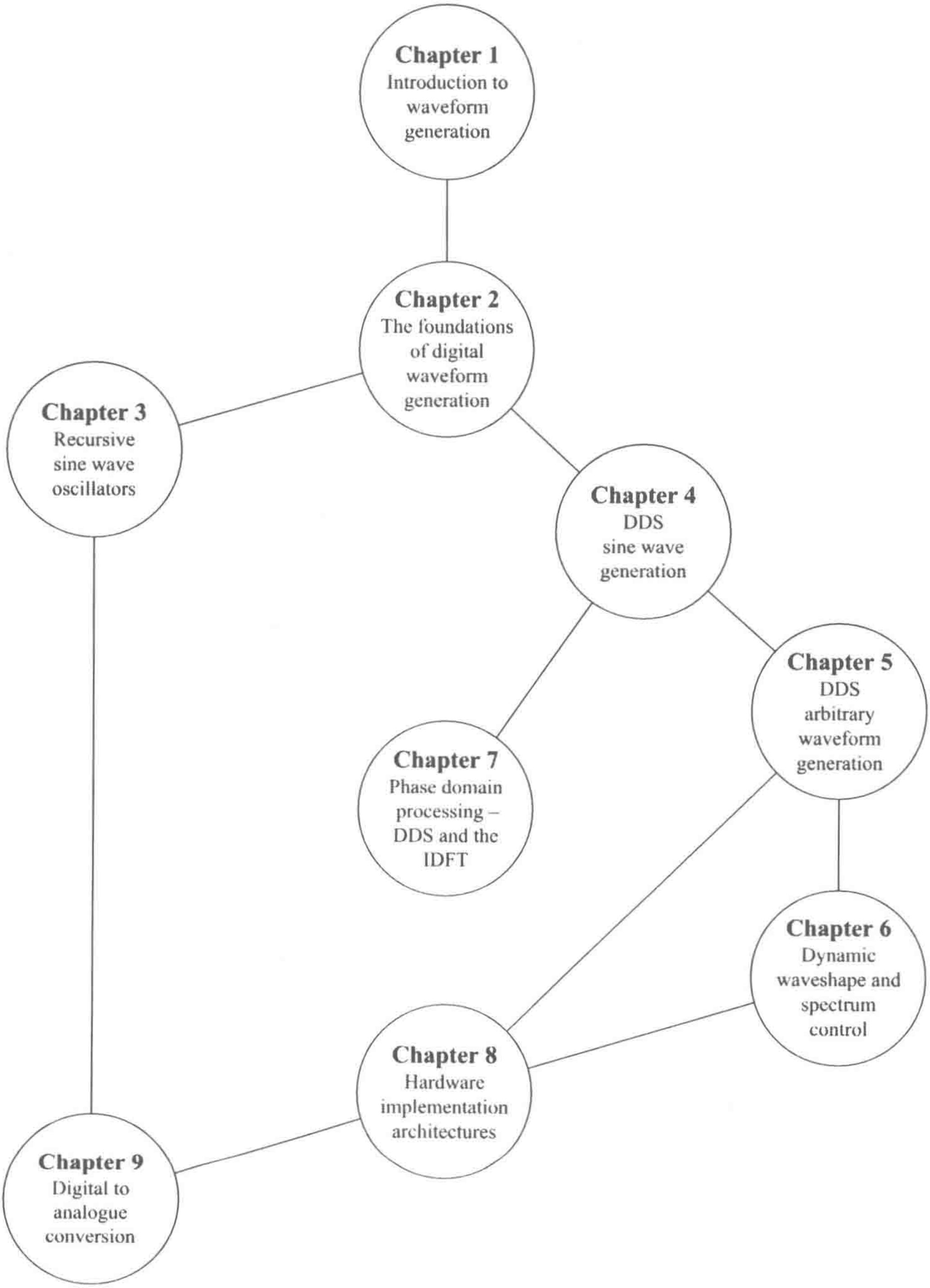
Many books have been written on signal generation by purely analogue means, such as phase shift and state variable oscillators or analogue computer function generators. Indeed, the first electronic signal generators, whether laboratory instrumentation or bespoke, were completely analogue systems, and only much later augmented by various forms of digital control. Over time, these techniques evolved to allow the generation of so-called 'function waveforms' (e.g. triangle, sawtooth and pulse) in addition to the ubiquitous sine wave. Indeed, some novel designs were capable of generating rudimentary arbitrary waveforms by purely analogue means according to a piecewise-linear *shape* specification. The HP3314A is one example of such a commercial laboratory signal generator. However, analogue techniques have always been beset by the same fundamental problems – thermal instability, the need for periodic recalibration, lack of control flexibility and limited waveform shape or corresponding spectrum specification.

Before the emergence of DSP techniques, non-sinusoidal waveforms were generated by analogue computation models such as the voltage controlled function generator. Subsequently, digital *control* of key parameters was realised by using a digital to analogue convertor (DAC) to translate from the digital to analogue domain, allowing computer control of the signal generation process. Today, digital signal generation is a cornerstone function in many modern analogue and DSP systems. Novel DSP techniques enable the generation of essentially any waveform shape, and provide independent control of waveshape, frequency, phase offset and amplitude. These techniques have evolved from the well-established methods of *phase accumulation* and *phase–amplitude mapping*, which have hitherto been widely applied to the generation of sine wave signals through DDS. Recursive digital techniques have also been applied to sine wave synthesis, but while having distinct advantages in some applications, they lack the flexibility of the phase accumulation approach. They are, however, well suited to many application areas due to their simplicity and low computational cost.

This book investigates signal generation techniques based on phase accumulation and phase–amplitude mapping. These 'all-digital' techniques enable the generation of sinusoidal and arbitrary waveforms with user-programmable waveshape, frequency, phase offset and amplitude. These powerful, yet flexible, techniques synthesise waveforms whose shape (and hence corresponding spectrum) can be defined according to a time or frequency domain specification. Furthermore, with appropriate phase–amplitude mapping, the synthesised waveshape can be varied according to a control parameter in real time. Multiple waveform generators are easily synchronised, and with programmable phase offsets generate polyphase waveforms or quadrature IQ sine wave constellations. The true utility of arbitrary waveform generation can be appreciated when we consider that a waveform's instantaneous amplitude may correspond to *any* parameter in a digital or analogue system, whether as a test or control signal. For example, to control the set point of a servo mechanism, the instantaneous amplitude or phase of a carrier signal or the forcing function is applied to evaluate a control system's response.

Motivation for this book has come in part from observations of the currently available signal generation literature. Many excellent works have been written on the digital generation of sinusoidal signals using DDS and related techniques. However, there is a notable deficiency in the area of non-sinusoidal or arbitrary waveform generation using purely digital techniques. Several published texts have dealt with waveform generation in the field of computer music, and these can be considered as special cases of the techniques described here. This book endeavours to address the gap by providing a treatise on both sinusoidal and arbitrary waveform generation that incorporates tutorial and hardware implementation descriptions at the functional block level. It is this latter material which is intended to assist the engineer or system architect who has the task of designing a bespoke waveform generator for a specific application. Synthesis algorithms are presented as signal-flow block descriptions and are deliberately abstracted from specific hardware or software implementation technologies (e.g. FPGA or DSP code). However, generic throughput enhancement techniques such as arithmetic pipelining and parallel memory architectures are discussed. Accordingly, it is assumed that practitioners who are suitably 'skilled in the art' can implement and optimise these signal flow descriptions in a hardware or software technology appropriate to their application. It is reasonable to suggest that many of the suggested hardware architectures, and the underlying synthesis techniques which they implement, are 'solutions looking for a problem'. Hopefully, this book will enable the reader to find these techniques a good home, or at the very least, provide food for thought.

We begin with a review of key definitions, historical context, classical analogue waveform generation and the conceptual and mathematical foundations pertinent to digitally generating electronic signals. The text then presents a detailed review of sine wave synthesis algorithms beginning with recursion and finally progressing to sinusoidal direct digital synthesis (DDS). This lays the essential foundations required for an understanding of phase accumulating arbitrary waveform generation – the focus of this book. The figure below illustrates the flow and interconnection between chapters. To supplement the written material and to assist the reader in 'what happens if' design simulation, Mathcad models are available that compute qualitative performance metrics for various configurations of the techniques presented. Mathcad models are also used to generate the graphical plot figures used throughout the book to assist communication of key ideas. These models are available for free download from www.cambridge.org/symons, and from the author's website www.petesymons.com/dwg.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I would wish to send my heartfelt thanks to Karen, my long-suffering wife, without whose love and encouragement this book would certainly not have been written; to Chris, Ben and Jenny for making me so proud; to my oldest friend Mike McNabb for providing useful feedback on writing style and always making me laugh; and finally, Hannah Eustace for her detailed critiques of several draft chapters from a non-technical perspective.

I dedicate this book to the memory of my father, Bob Symons, who both started and fuelled my love of electronics as a young boy who was always taking the back off the television set to see how it worked; and Bob Denson – my 'Uncle Bob' – simply the nicest and most decent person I have ever known.

Glossary of terms

(*Terms are grouped as used in the analysis.*)

A	waveform amplitude scaling factor.
B	post-DAC low-pass reconstruction filter -3 dB bandwidth.
b	number of amplitude sample bits.
C	waveform *crest factor* – ratio of waveform peak to RMS amplitude.
D	waveform *duty-cycle* – ratio of waveform 'on' time to period.
$\varepsilon_a(n)$	amplitude error sequence.
t	continuous-time variable.
f	waveform or signal frequency in hertz.
τ	waveform period equivalent to the reciprocal of waveform frequency in hertz.
f_0	fundamental frequency, typically of a harmonic series.
f_o	waveform generator output frequency.
$f(t)$	continuous-time instantaneous frequency in hertz.
f_s	sampling frequency (or rate) in hertz.
T	sample period (or interval) equivalent to the reciprocal of sampling frequency in hertz.
β_R	the oversampling ratio between the processing sample frequency f_s and an equivalent sample frequency corresponding to twice the bandwidth of the post-DAC reconstruction filter B.
$y(t)$ or $x(t)$	continuous-time waveform amplitude signal.
$X(f)$	continuous complex amplitude spectrum.
θ	waveform phase offset (typically in radians).
$\phi(t)$	continuous-time instantaneous phase.
$\phi_w(t)$	wrapped continuous-time instantaneous phase.
$\langle a \rangle_b$	'a modulo b' – the remainder after repeated subtraction of b from a until $(a-b) < b$.
(a, b)	greatest common divisor between a and b.
M	phase accumulator word length – 2^M possible phase states.
I	phase accumulator integer component word length.
F	phase accumulator fraction component word length.
$\phi(n)$	instantaneous discrete-time phase sequence.

$\phi_I(n)$	integer component of discrete-time instantaneous phase within a fractional phase representation.
$\phi_F(n)$	fraction component of discrete-time instantaneous phase within a fractional phase representation.
$\alpha(n)$	rational fraction interpretation of $\phi_F(n)$.
φ	phase increment.
$\varphi(n)$	discrete-time phase increment sequence.
Λ	number of sinusoidal DDS amplitude spectrum spurs.
$\vartheta(n)$	discrete-time phase dither sequence.
A_k	kth element of the IDFT harmonic amplitude vector.
θ_k	kth element of the IDFT harmonic phase offset vector.
H_k	kth element of an arbitrary waveform band-limiting amplitude response.
N_h	number of harmonics in a harmonic waveform specification.
N_s	number of samples in a particular computation.
$\mathrm{S}[a]$	ath element of a sinusoidal wavetable.
$\mathrm{W}[a]$	ath element of an arbitrary waveform wavetable.
L	wavetable length in samples.
$P_N(x)$	Nth-order Lagrange interpolation polynomial.
P	number of bits in a wavetable page address – 2^P wavetables.
$p(n)$	wavetable page address sequence.
$P_P(n)$	P-bit integer wavetable page index sequence component.
$\gamma(n)$	F'-bit fraction wavetable page index sequence component.

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