THE PHYSICS OF LASER FUSION

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PREFACE

Are fusion reactions feasible? The answer to this question may turn out to be of great significance for the future of mankind. It will determine the options open for the task of providing energy to sustain civilized life on the planet. Existing science cannot give a definitive answer to this question. Fusion reactions sustain the sun and have done so for times of the order of 10^9 years. Hydrogen bombs are fusion devices liberating the energy within microseconds. A controlled thermonuclear device for the use of fusion energy must work on a time scale in between these extremes. It must be of a suitable size and deliver a suitable amount of power, say between 1 MW and 10^4 MW per plant with a capital expenditure comparable to or less than that needed by its competitors. Thus even if the question of whether energy output of a fusion reaction can be made to exceed energy input in a terrestrial device is answered affirmatively, a number of technological and economic questions remain to which physics has no ready answer.

Yet, the enthusiasm and faith of the scientific community has commanded support for a large research effort in the fusion field. Broadly there are two approaches: magnetic confinement of hot plasma and inertial confinement. Inertial confinement schemes aim at compression of deuterium–tritium fuel to very high densities, so that the reaction can proceed extremely quickly, within pico- or nanoseconds, as a microexplosion.

This compression may be achieved by a flux of energetic particles (electrons or ions) or by means of laser light. This book is mainly devoted to laser driven fusion. It covers a whole range of interesting problems of physics; they range from Plasma Physics to Fluid Dynamics and the properties of plasma at the high densities, temperatures and pressures encountered in stellar matter. It seemed desirable to introduce novices in this field to relevant physics topics. In view of their wide range, it is hoped that even the specialist might find some chapters useful. They are written in such a way that they can be read independently, particularly by readers acquainted with the rest of the material. The book is thus written for physicists or engineers of divers backgrounds and aims at a consistent, easily readable presentation. Derivations of results will at least be sketched, leaving out details if they can easily be found in existing texts, which need not be consulted in order to comprehend this book.

Questions of a more technological nature are not tackled. The emphasis is on the understanding of the basic physical processes. They are rather involved and in order to get realistic quantitative theoretical predictions it is necessary to resort to numerical computations. It is unfortunately rather difficult to gain an insight into what is going on without guidance by analytic solutions of simplified processes which intermingle in the real situation. In this book, analytically soluble problems which are relevant to laser fusion are presented and solved. The results of full computer computations are reported as much as is possible within our limited space, and the physics underlying the computational codes is explained.

Perhaps a few words on how to read the mathematics in this book will be helpful. When some intermediate steps are left out, they are either too trivial or too long-winded. It is best to concentrate on the statement that a given equation makes the relationship which it establishes. The aim is throughout to make clear what kind of reasoning leads to the result and to quote results when the algebra is rather involved.

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CONTENTS

1

FUSION REACTIONS

In the early days of nuclear physics it was already known that energy may be released by reactions between light nuclei (fusion reactions) or by splitting a very heavy nucleus into two or more parts (fission reactions). In either case these energies are of the order of MeV per nucleon which is six orders of magnitude more than those available per atom of chemical reactions.

When the sum of the masses of light nuclei (e.g. protons and neutrons) exceeds the mass of the product nucleus by an amount δm, the energy W liberated by this fusion of nuclei is, by Einstein's relation, given by

$$W = \delta m . c^2$$

and it is released in the form of kinetic energy of the reaction products. For such a reaction to take place, the particles must come within range of the nuclear forces and this means that the Coulomb barrier has to be overcome by the kinetic energy available in the centre of mass system of the colliding particles. It was soon realized that bombardment of light-element targets with high-energy particle beams could not efficiently produce power, unless the energy, necessarily imparted to outer shell electrons in the collision process, was utilized. This means that the reacting particles must be confined at high density for a time sufficiently long for energy transfer to the nuclei to take place. This transfer is disorderly, i.e. the ions (or nuclei) are heated to a high temperature. At a given temperature, the reaction rate is proportional to the square of the density; the time during which confinement can be secured turns out to be limited to a small fraction of a second and, therefore, the density needed in order to achieve a useful power output is very high.

The temperature required for barrier penetration and the density required for a practical device will be determined below from data concerning reaction cross-sections. They represent conditions of matter known to exist in stars. The H-bomb first realized similar conditions on earth and the problem of its use for triggering a thermonuclear reaction was taken up and solved by E. Teller. The first release of man-made thermonuclear energy occurred in 1952 but the problem of how to control this release for the purpose of generating electric power is still with us. Early reviews by Post [1] and by Glasstone and Lovberg [2] may be consulted for an introduction to the main lines of research on controlled thermonuclear reactions which aim at a quasi-steady-state confinement by magnetic fields. The main difficulty of this approach is well known. It is necessary to produce a state of matter far from equilibrium where large pressure and temperature gradients cannot easily be maintained in the face of instabilities causing premature particle and energy loss. This book is concerned with the basic physics underlying another approach: the pulsed energy-release from a fuel pellet, with no external confinement during the very short time required for the fluid dynamic motion in the pellet during which it is compressed to a high density such that the reaction rate is high enough to lead to a useful output.

The problems associated with magnetic confinement thus cannot arise, but the method has problems of its own which will emerge in the further course of the exposition. But first we want to examine the conditions which must be realized to make this approach work.

To do this we must assemble information concerning the nuclear reactions involved. The basic reactions are

$$\mathrm{D + D \rightarrow {}^3He\,(0{\cdot}82\,MeV) + \mathit{n}\,(2{\cdot}45\,MeV)} \tag{1}$$

$$\mathrm{D + D \rightarrow T\,(1{\cdot}01\,MeV) + H\,(3{\cdot}02\,MeV)} \tag{2}$$

$$\mathrm{D + T \rightarrow {}^4He\,(3{\cdot}5\,MeV) + \mathit{n}\,(14{\cdot}1\,MeV)} \tag{3}$$

$$\mathrm{D + {}^3He \rightarrow {}^4He\,(3{\cdot}6\,MeV) + H\,(14{\cdot}7\,MeV)} \tag{4}$$

and the division of kinetic energy between the reaction products has been indicated. As will be seen below, the last reaction becomes interesting only at higher energies. There are other reactions involving lithium and boron isotopes which may be of interest at high temperatures, e.g.

$$\mathrm{B^{11} + H \rightarrow He + He + He}$$

which involves only charged reaction products and does not lead to radioactive contamination, but from a practical point of view, only the DD and the DT reactions have been considered. We shall see that the DT reaction is much more favourable because it occurs at lower energies than the DD reaction. On the other hand, deuterium occurs in nature with an abundance of 1 in 6500 atoms while tritium which is radioactive with a half-life of 12·26 yr has to be manufactured. The neutrons from the thermonuclear reactions could be slowed down in a suitable moderator containing lithium-6 to produce tritium by the reaction

$$^6\mathrm{Li} + {}_0n \rightarrow {}_2\mathrm{He} + {}_1\mathrm{T} + 4{\cdot}6\ \mathrm{MeV} \tag{5}$$

in the fusion plant itself.

The reaction rates can be estimated by means of theoretical considerations due to Houtermans *et al.* [3] and Gamov [4]. The energy needed to surmount the Coulomb barrier is given by $Z_A Z_B e^2/R_0$ in terms of the nuclear charges Z_A and Z_B of the reaction partners and the range R_0 of the nuclear forces between them. Taking this equal to a nuclear diameter 5.10^{-13} cm and with $e = 1{\cdot}6.10^{-19}$ C, this becomes $0{\cdot}28\, Z_A Z_B$ MeV. Thus classical theory would predict a minimum energy of 0·28 MeV for hydrogen isotopes. Quantum theory abolishes the sharp limit and there is no threshold. The probability of barrier penetration as a function of energy E is given by

$$\sigma_{AB}(E) \cong \frac{A}{E}\exp(-2^{3/2}\pi^2 m^{1/2} Z_A Z_B e^2/hE^{1/2}) \tag{6}$$

where m is the reduced mass $m = m_A m_B/(m_A + m_B)$, h is Planck's constant, and E is the total energy in the centre of the mass system of the nuclei A and B. The reaction rate in a hot gas with nuclei of type A and velocity $\mathbf{u}$ and of type B with velocity $\mathbf{v}$ is defined by the double integral over the velocity space spanned by $\mathbf{u}$ and $\mathbf{v}$

$$R_{AB} = \iint f(\mathbf{u})\, g(\mathbf{v})\, \sigma(w) w \, \mathrm{d}\mathbf{u}\, \mathrm{d}\mathbf{v} \tag{7}$$

where $f(\mathbf{u})$ and $g(\mathbf{v})$ are the respective velocity distributions functions such that the particle numbers in the volume elements $\mathrm{d}\mathbf{u}$ and $\mathrm{d}\mathbf{v}$ of velocity per unit volume of ordinary space are given by

$$\mathrm{d}n_A = f(\mathbf{u})\, \mathrm{d}\mathbf{u}$$
$$\mathrm{d}n_B = g(\mathbf{v})\, \mathrm{d}\mathbf{v} \tag{8}$$

and where w is the relative velocity

$$w = |\mathbf{u} - \mathbf{v}|.$$

For Maxwellian velocity distributions it follows from (6) that in a system where the nuclei B are at rest and the nuclei A have velocity v and kinetic energy $E = m_A v^2/2$;

$$R_{AB} = \langle \sigma v \rangle = (8\pi n_A n_B/m_A^2)(m/2\pi \mathrm{k}T)^{3/2} \int E\sigma_{AB}(E) \times \exp(-mE/m_A \mathrm{k}T)\, \mathrm{d}E. \tag{9}$$

The result (9) of folding the cross-section $\sigma(E)$ into the distribution function, "the velocity averaging" is, in the literature commonly called $\langle \sigma v \rangle$ or $\overline{\sigma v}$. $\overline{\sigma v}$.

The constant of formula (6) depends on R_0 and must thus be determined experimentally. The fusion cross-section may be written

$$\ln E\sigma_{AB} = A' - E^{-1/2} B', \tag{10}$$

the constants A' and B' have been determined from experiments, and the resulting reaction rates have been computed by several authors [5–8] who are not in complete agreement. The useful curves of Fig. 1 reproduced from

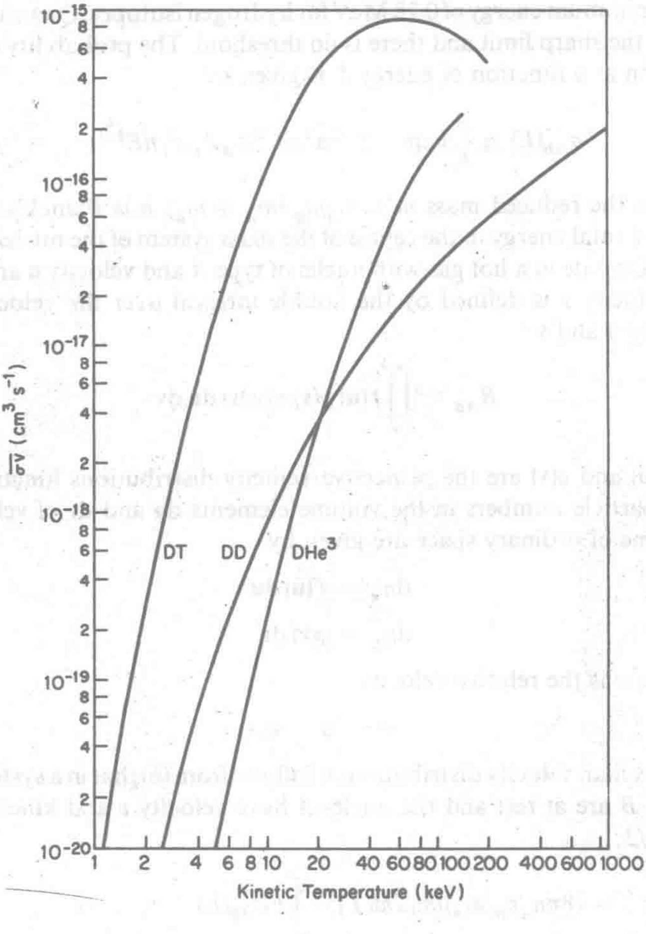

Fig. 1. Values of $\langle \sigma v \rangle$ based on Maxwellian distribution for DT, DD (total), and DHe^3 reactions.

[2] show the essential features. The temperature is expressed in keV (1 keV → $8{\cdot}6 \,.\, 10^7$ K) which is usual in the literature on fusion. The DT rate in the range up to 60 keV is orders of magnitude higher than the DD rate and at 9 keV it is already up to 10 % of the maximum rate.

Post remarks [1] that the "tail of the Maxwell distribution wags the dog." This is seen in Fig. 2 which shows the contributions of deuterons of various energies for the DD reaction (2) which is expressed as $\langle \sigma v \rangle = \int_0^\infty \phi(E)\, dE$.

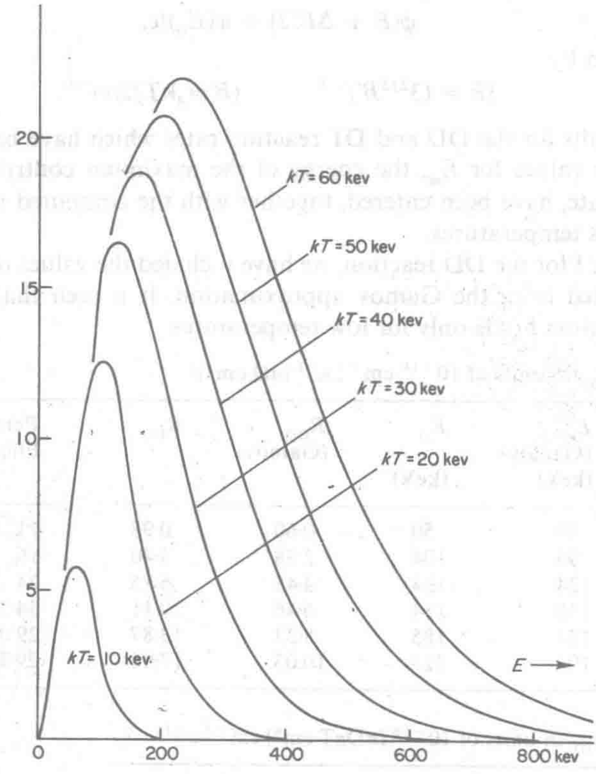

Fig. 2. Contribution to the reaction rate R_{DD} as a function of the deuteron energy E for kT = (left to right) 10, 20, 30, 50, 60 keV.

The DT rate behaves similarly. Figure 2 shows ϕ_{DD} for temperatures of 10–60 keV, in steps of 10 keV. It is seen that the reaction rate is in each case mainly due to particles belonging to the tail of the Maxwell distribution.

An approximation to the reaction rate can be obtained by means of a method due to Gamov. One can write, in first approximation

$$\int_0^\infty \phi(E)\,dE = \Delta E \phi(E_m), \tag{11}$$

because the integrand is of the form

$$\phi(E) = \exp\{-(mE/m_A kT) - B'E^{-1/2}\} \tag{12}$$

and has a sharp maximum as a function of E at $E = E_m = (B'm_A kT/2m)^{3/2}$. If ΔE is defined by

$$\phi(E + \Delta E/2) = \phi(E_m)/e, \tag{13}$$

ΔE is given by

$$\Delta E = (3^{2/3}B')^{1/2} \qquad (B'm_A kT/2m)^{5/6}. \tag{14}$$

In the results for the DD and DT reaction rates which have been tabulated below, the values for E_m, the energy of the maximum contribution to the reaction rate, have been entered, together with the computed reaction rates for various temperatures.

In Table I for the DD reaction, we have included the values of E_m and R_{DD} as computed from the Gamov approximation. It is seen that the Gamov approximation holds only for low temperatures.

TABLE I. R_{DD} in units of 10^{-18} cm^{-3} s^{-1} $(n\text{D cm}^3)^2$

kT (keV)	E_m (Gamov) (keV)	E_m (keV)	R_{DD} (Gamov)	R_{DD}	Percent Effective
10	59	50	0·60	0·98	11
20	94	104	2·38	3·40	16
30	124	124	4·45	6·75	24
40	150	134	6·46	10·11	34·2
50	174	185	8·33	13·87	29·6
60	196	225	10·03	17·96	29·7

TABLE II. R_{DT} in units of 10^{-18} $(n\text{D}n\text{T cm}^6)$ cm^{-3} s^{-1}

kT (keV)	E_m (keV)	R_{DT}	Percent effective
2	19·8	0·263	0·71
4	31·6	5·91	2·3
6	40·6	25·71	4·27
7	44·4	40·27	5·4
8	48	60·51	6·52
40	110	748	25

It is interesting to compute the mean energy $m_D\bar{u}^2/2$ of deuterons in the laboratory frame of reference which have relative energy $E = m_D W^2/2$ on collision. Statistical analysis gives the result

$$m_A\bar{u}^2/2 = \tfrac{1}{2}m_A W^2(m/m_A)^2 + \tfrac{3}{2}kT(m/m_B) \quad (15)$$

where m is the reduced mass.

Conversely, one can compute the mean relative energy on collision of deuterons with energy $m_D u^2/2$ in the laboratory system. The result is

$$m_A\overline{W}^2/2 = m_A u^2/2 + \tfrac{3}{2}kT(m_A/m_B). \quad (16)$$

Using these results, Tables III and IV can be computed for DD and DT reactions

TABLE III. DD reactions.

kT (keV)	E_m (keV)	$mD\bar{u}^2/2$ (keV)	$mDu^2/2$ (keV)
10	59	22	44
20	94	30	64
30	124	54	79
40	150	68	90
50	174	81	99
60	196	94	106

TABLE IV. DT reactions

kT (keV)	E_m (keV)	$mD\bar{u}^2/2$ (keV)	$mDu^2/2$ (keV)
2	19·8	8·35	17·8
4	31·6	14·1	27·6
6	40·6	18·2	34·6
7	44·4	20·2	37·4
8	48·0	22·1	40
40	110	63·8	70

The energies $m_D u^2/2$ have been computed from equation (16), where $m_A\overline{W}^2/2$ has been identified with E_m. They are thus the energies of deuterons which have a mean relative energy equal to the relative energy giving the maximum contribution to the reaction rate.

In Tables I and II there are columns headed "Percent Effective". These are the percentages $\Delta n_A/n_A . 100$ of deuterons with relative energy larger than E_m

computed from

$$\Delta n_A/n_A = n_B 4/\sqrt{\pi} \int_Z^\infty z^2 \exp(-z^2)\,dz \tag{17}$$

where the lower limit of the integral is given by $Z = (mW^2/2)^{1/2}$ and m is again the reduced mass.

Looking at these tables, one notes that ions with fairly low energies in the laboratory system have on average a high relative energy. This may be relevant when one is considering the effect on the reaction rate of accelerated deuterons inserted into a discharge. Looking at Table II one notes that in the case of the DT reaction, the effective percentages are rather low, again showing that relatively few accelerated deuterons injected into a plasma may produce reaction rates comparable to the purely thermonuclear ones.

Energy Loss and Radiation by Particles

Fusion devices operate at temperatures in the keV range. In this range matter is completely ionized, ions of low mass number are stripped of their orbital electrons; it consists of a gas of positively charged nuclei and an equivalent number of electrons which is referred to as a plasma. (More precisely an ionized gas is called a plasma if the Debye length (Chapter 4, equation (8)) is small compared to all other lengths of interest).

Hydrogen atoms are completely stripped and energy loss occurs in the form of *bremsstrahlung*, i.e. continuous radiation emitted by charged particles, mainly electrons, as a result of deflection by the Coulomb field of other charged particles. Ions of higher Z, if they are not completely stripped, are subjected to excitation processes contributing to radiation loss. These losses must be subtracted from the yield of the nuclear reactions and we shall compute the loss rate below.

The deflections suffered by protons and α-particles during their passage through the plasma are also of great interest. They limit their range in such a way that, at sufficiently high density, their energy may be deposited in the fusion pellet thus accelerating the burning process.

Rather than just quoting the relevant formulae we proceed to an intuitive derivation. The classical expression for the rate P_e at which energy is radiated by an accelerated electron is given by

$$P_e = \frac{2}{3}\frac{e^2}{c^3}a^2. \tag{18}$$

The acceleration a due to the Coulomb force between charged particles