



# Design of Modern Communication Networks

## Methods and Applications

Christofer Larsson



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*To Mária*



# Preface

Design of communication network is a complex task, as many of the problems involved are inherently difficult to solve. This leads to the necessity of using a collection of cleverly designed algorithms and methods rather than a single approach. The book is written on at least two levels. Firstly, specific problems related to network design form the main structure of the book. On a second level, various types of algorithms are used, discussed and modified in the course of the text. The structure gives views: one problem based, one method based.

The main intention is that the text will serve as a handbook in engineering with a focus on algorithms and their illustration by examples. On the other hand, rather than being an encyclopedia of algorithms and methods, the text should be possible to read as a discourse in techniques of network design. Even if mainly intended as a handbook, it may also be useful in academia as side literature in telecommunication education programs.

In practice, network design has been more of a handicraft than a science. This is no doubt due to the fact that almost all problems related to network design are  $\mathcal{NP}$ -hard, or in some other class of problems which are difficult to solve. However, these problems can often be solved approximately. When presenting approximation schemes in this book, the author has taken into account both the precision of the result and the complexity of the algorithm. A simpler, intuitive algorithm has sometimes been favored over more complex algorithms, sometimes even if the precision is lower.

In this text, we will use a mixture of analytical analysis, heuristics, approximations, randomization and common sense to solve difficult problems related to network design. Thus, we may not be able to answer the question “What is the optimal solution to this problem?”, but we may be able to answer the question “What is the probability that there is a better solution to this problem?”, and if the probability is low enough, we should be close to the answer of the first question. Also, if a proposed network is available to us we can assert its superiority or inferiority to solutions obtained using the methods presented in this book.

It is the hope of the author that the reader will experience the same fascinating journey it has been writing this book. Indeed, at the heart of network design lies the combination of methods, mixed with a fair dose of common sense and experience, needed to solve often very intricate problems. It is an exquisite example of applied mathematics: finding methods that might work!

Most proofs of mathematical theorems have been omitted in the text, as the focus is on application level rather than a theoretical level. These proofs can be found in the cited references. The intention is instead to provide “empirical proofs” of the methods by solving problems using different methods and thereby arriving at the same or similar results. A number of such comparative examples are provided. Proofs have been included in some cases when they provide details that are instructive for constructing an algorithm.

With network design we mean the initial planning or long-term modifications of networks on a rather large scale. The text does not discuss control or routing mechanisms which instantaneously have to react on failure or overload situations. Such aspect may be referred to as operational rather than design related.

The aim of the text is to be as technology independent as possible. That is the reason why there are very little description of actual network technologies such as STM, ATM, IP, and so on, their protocols and functionality. There is a lot of literature available on these topics, and the algorithms presented in this book should be possible to translate into technology specific terms rather easily.

As mathematical prerequisites, the reader is probably familiar with some combinatorics, optimization, fundamental probability theory, queueing theory and analysis.

The author would like to thank V.B. Iversen and F.P. Kelly for their comments and encouragement. He is also grateful to the many researchers and scientists that have made their interesting papers freely available on the Internet.

**Christofer Larsson,  
Bratislava.**

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