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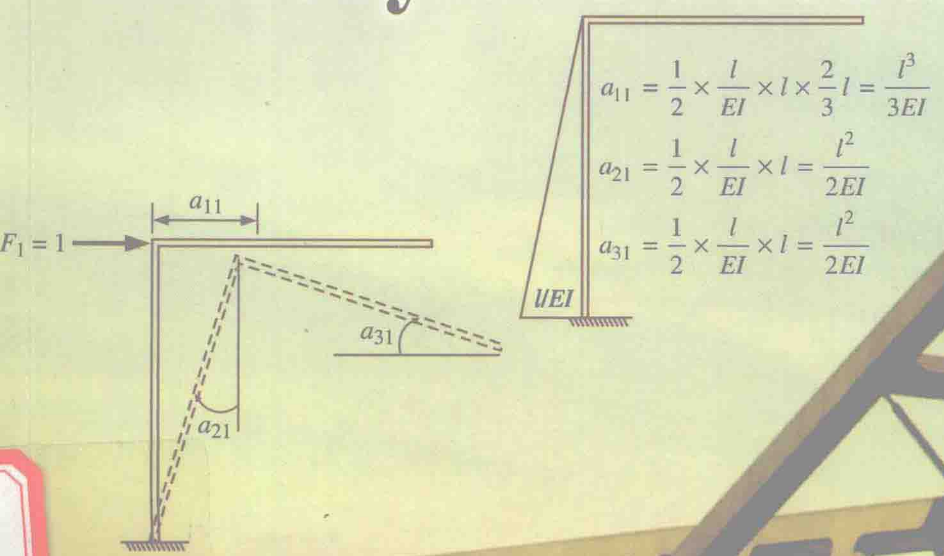
Matrix Methods of Structural Analysis

Theory and Problems



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MATRIX METHODS OF STRUCTURAL ANALYSIS
Theory and Problems

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MATRIX METHODS OF STRUCTURAL ANALYSIS: Theory and Problems
C. Natarajan and P. Revathi

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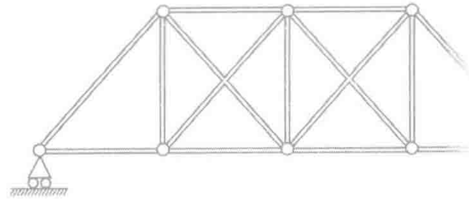
Matrix Methods of Structural Analysis

Dedicated to

our beloved parents



Foreword

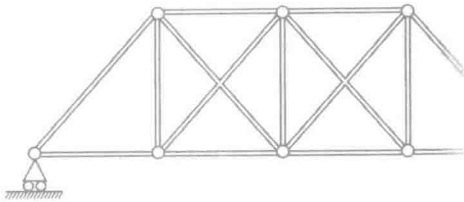
Engineering Structures play a crucial role in our system. Apart from construction and applications, the structural analysis approach has extended its wings to Aerospace, Transportation and many other applications. Structural Analysis is an excellent tool in the hands of a structural specialist. The skill on the subject comes handy for a specialist who is involved in the design and subsequent execution. During special cases of failure, these tools are used for arriving at the reasons for failure. This highly specialized subject requires an excellent understanding of the fundamentals and very special methods of analysis of various structures.

Dr. C. Natarajan has over two decades of experience in teaching, research and consultancy in this area. His lectures and recommendations have received highest appreciation from various students and professionals. He has also organized International Workshops and Seminars on the subject. This book which has been co-authored by Dr. P. Revathi of Pondicherry Engineering College is based on his extensive experience as well as the courses he has been offering to postgraduate students of National Institute of Technology, Tiruchirappalli for several years.

The presentation and organization of the book clearly reflect the intellectual input he has put in on this topic and his interaction with students. I am sure the book will be well received by academicians, graduate students and practicing engineers.

I wish the authors come out with more such books to add to the glory of their institutes.

Dr. Srinivasan Sundarrajan
Director
National Institute of Technology
Tiruchirappalli



Preface

The developments in computer technology and internet communications have revolutionised various aspects of science and engineering. Their influence on education has been phenomenal. At the root of all these developments and influence is the way to pose problems in computing and internet, and here 'Matrix Representation and Operations' have formed the bases on which engineering and scientific computing rest on. A major area which had received maximum attention both in terms of research and development is modelling structural systems as linear elements in one, two or three dimensions to reflect actual behaviour. This is where the importance of 'Matrix Structural Analysis' is to be seen and a need to present the basic features along with intricacies in application to actual problems backed up by numerical examples, forms the main focus and objective of writing this book. With the authors' experience in teaching this course to graduate students in NIT Tiruchirappali for the last several years; the book reflects many of the problems and queries the students have in understanding the basics of matrix representation and analysis.

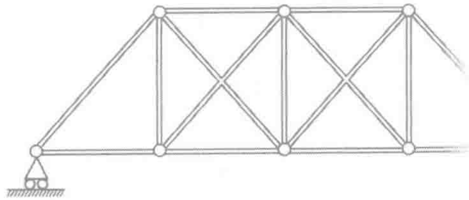
The book is conveniently divided into nine chapters with first two chapters covering basics of matrices, followed by three chapters specific to structural systems. Having provided the foundation for matrix structural representation, the next three chapters look at dimensional and behavioural aspects to classify into 'pin-jointed systems', then onto beams and finally three-dimensional rigid jointed systems. Ninth chapter devotes to special topics and techniques in using matrices for structural analysis. Each chapter contains number of numerical examples so that student can get confidence in working numerically. Besides, MATLAB codes are given at the end to illustrate interfacing with standard computing tool. After gaining proper understanding of analysis techniques based on first principles, the students are encouraged to make use of this time saving MATLAB codes.

The first author would like to place on record his gratitude to NIT Trichy for supporting this venture. He also thanks his mother Thanulekshmi, wife Rema, daughters Thanulekshmi Kameshraja, Madhavi, Meenu and grandson Dhiren Kamesh for their co-operation extended

while making this book possible. The second author expresses her sincere gratitude and profound thanks to her co-author, Prof. C. Natarajan, for igniting the idea of developing this book. She also would like to mention the moral supports of her husband Krishnakumar, daughter Jayshitha, ever great friend Robert and all her family members who contributed in making this book possible. This book is been dedicated to her late father Purushothaman.

The authors welcome suggestions from readers for improving this book in any manner. The next edition is sure to take into account all such suggestions.

C. Natarajan
P. Revathi



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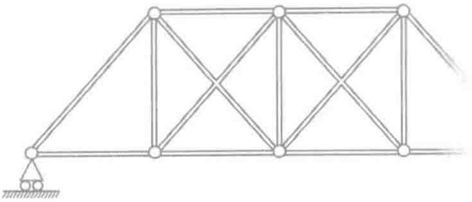


1

Basic Concepts

1.1 INTRODUCTION

The classification of structure depends upon the type of structure itself, the type of analysis used and often habit. Actually, no classification is explicit, fully descriptive or satisfactory. Certain aspects of the structure are always emphasized and certain aspect is ignored for simplicity when any classification is used. Hence, different classifications of the structure are discussed in this Chapter.

1.2 CLASSIFICATION OF STRUCTURES

A stable structure will support any admissible system of applied loads, resisting these loads elastically, and the strength of all members and the capacity of all supports being considered is infinite. In other words, stability of a structure depends on the number and arrangement of reaction components rather than strengths of the supports and structural elements. Even though a structure is stable for a given system of loads, sometimes the same may not be stable for another system of loads and hence they are also termed as unstable structures. Since a structure to be classified as stable, must be stable under any admissible system of loads, it is recommended to omit all the loads while considering stability and determinateness. Therefore, no loads will be shown on the illustrations in the subsequent sections.

1.2.1 Axial and Bending Structures

This is a classification based on the structural action of the given structure to the externally applied loads. When the loads are applied it is resisted by the structure through several predominant structural actions, viz., axial deformation, bending, twisting, etc. The structures which resist the applied external loads through predominant axial deformation are termed as trusses. This is possible only when these structural forming elements are connected by hinges, thus eliminating moment in the members.

Bending structures are the structures which resist the externally applied loads through predominant bending action. These structures are connected by rigid joints which are capable of resisting and transferring the moments induced. These structures are subjected to transverse loads only. Beams and frames are the structures which come under this category. More specifically there are also other types of structures in which tension is predominant such as cables and structures in which compression is predominant such as arches. Grids which are transversely loaded are under predominant torsion.

1.2.2 Plane and Space Structures

Plane structures are those for which the axes of all the members of the structure or its mid-surface lie in the same plane and are subjected to loads in plane only. *Space structures* are structures other than plane structures. Plane beams, plane trusses, plane frames and flat plates which are loaded in their plane are examples of plane structures. Beams loaded in more than one plane, grids loaded out of their plane, plates loaded in bending, three-dimensional frames and trusses are examples of space structures.

1.2.3 Two-dimensional and Three-dimensional Structures

This classification is based on the fact that all the member axes (or the mid surface) lie in one plane in two-dimensional structures, which is not the case for three-dimensional structures. In addition, for a two-dimensional structure, if the loading is acting in the same plane, then it is classified as plane structure; and if the loading is not acting in the same plane, then it is classified as space structure according to the definition given in Section 1.2.2.

1.2.4 Discrete Elements and Continuum Systems

Almost all the structures are three-dimensional continuous systems. Obviously, examples of such systems are arch dams, soil media and thick plates. Less obvious examples are rigid-building frames and plane trusses. In the case of a plane truss, although the centroidal axes of its members can be represented by straight lines connected by hinges, in reality such members are three-dimensional and the structure as a whole is three-dimensional. In the case of hinged joints, no moments can be transmitted at the joints; but in the cases of welded joints, this is not true. A representation of a plane truss as a series of one-dimensional bars connected by hinges is, therefore, an idealization of the real structure.

1.2.5 Stable and Unstable Structures

A structure is considered to be stable if the number of external supports and internal members and joints is sufficient to determine all external reactions and internal actions uniquely. Generally, it can be stated that if the number of supports, members and joints is atleast equal to the number needed to determine their value using the laws of static equilibrium, then the structure is stable. If the number of any of these elements (supports, members or joints) is insufficient, then the structure is unstable externally (supports) and/or internally (members and/or joints).

1.2.6 Determinate and Indeterminate Structures

Most structures fall into one of the following three classifications: beams, frames or trusses. A beam is said to be completely analyzed when the shear and bending moment diagrams are found. A frame is completely analyzed when the variations in direct stress, shear and bending moment along the lengths of all the members are found. Similarly, a truss is said to be completely analyzed when the direct stresses in all the members are determined.

Shear and bending moment diagram of beams can be drawn when the external reactions are known. In the study of the equilibrium of a coplanar parallel-force system, it has been proved that not more than two unknown forces can be found by the principle of statics. In the case of beams these two unknown forces are usually reactions. Thus, two reactions in simple, overhanging and cantilever beams can be determined by the equations of statics. Hence, these types of beams are known as *statically determinate*.

If, however, a beam rests on more than two supports or in addition one or both end supports are fixed, there are more than two external reactions to be determined. These reactions cannot be determined by the equations of statics alone, and beams with such reactions are called *statically indeterminate* beams. The degree of indeterminacy is given by the number of extra or redundant reactions. More about static and kinematic indeterminacy is discussed in the subsequent sections.

1.2.7 Supports and Restraints

In order to clearly understand the concept of indeterminacies, let us first of all discuss on supports and restraints. Most structures are either partly or completely restrained so that they cannot move freely in space. Such restrictions on the free motion of a body are called restraints and are supplied by supports that connect the structure to some external stationary body. For example, consider a planar structure such as the bar AB shown in Figure 1.1(a). This bar would move freely in space with some combined translatory and rotational motion, if this bar were a free body and were acted upon by a force P. If a restraint were introduced in the form of a hinge that connected the bar to some stationary body at point A, then the motion of the body will be only of rotational movement about the hinge (Figure 1.1(b)). However, point B would move along an arc with point A as the centre. Therefore, another restraint is required at B to prevent completely the free motion of the bar.

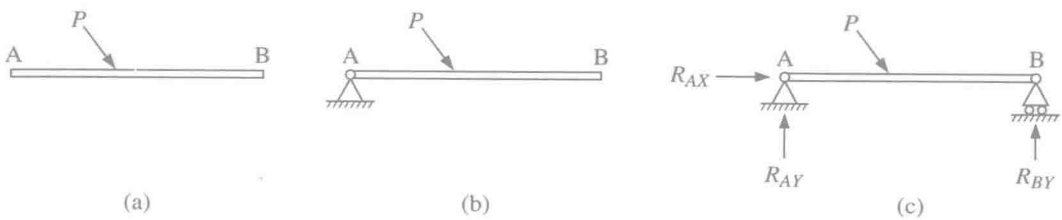


Figure 1.1 Support reactions.

The supports at A and B, in restricting the free motion of the bar, are called upon to resist the action that the force P imposes upon them through the bar. The resistances they develop to counteract the action of the bar upon them are called supports. The effect of these supports may, therefore, be replaced by the reactions that they supply to the structure (Figure 1.1(c)). Any support would offer restraint and some degree of freedom; restraints may be replaced by reactions (force/moment) and degree of freedom may be represented by displacements (deflections/rotation).

Table 1.1 Types of Supports and their Reactions

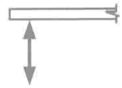
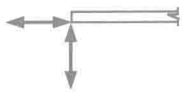
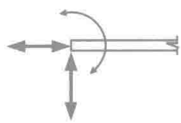
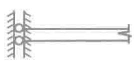
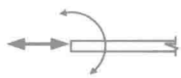
Type of support	Symbol	Reactions or static degree of freedom	Displacements or kinematic degree of freedom
Roller support	 or 		One translational and one rotational
Pinned support			One rotational
Fixed support			All arrested
Guided fixed support			One translational

Table 1.1 gives the reactions and the degrees of freedom in some of the ideal supports encountered in common engineering structures. It is interesting to note that reactions are developed only at the constrained degrees of freedom. For any support the sum of reactions and degrees of freedom is always three in these 2D problems. Hence, these are known as structures with three degrees of freedom per node. Sometimes, some of these degrees of freedom are arrested/constrained, which in turn induces reactions.

1.3 LEVELS OF STRUCTURAL ANALYSIS

There are various levels of structural analysis. It depends on the various assumptions related to the structure (determinate or indeterminate), loading history (static or dynamic), behaviour of the material and geometry (linear or nonlinear) and their uncertainties (deterministic or stochastic). In reality all structural problems are highly indeterminate. We reduce the degree of indeterminacy by making suitable assumptions. For instance, we assume that, the axial and shear deformations are negligible compared to flexural deformations in framed structures. The less is the number of unknowns, easier the method of analysis. Also, if the static indeterminacy is less than the kinematic indeterminacy, force method is preferred in which the forces are the unknowns. If the kinematic indeterminacy is less than the static indeterminacy displacement method is preferred as the unknowns in which displacements are the unknowns. Through proper modelling and structural idealization, it is possible to reduce the indeterminacy and thereby reduce the computational effort in analysis. It is also possible to reduce the effort in analysis by taking advantage of symmetry or anti-symmetry, if any, present in the structure.

1.3.1 Static and Dynamic Analysis

A structure is said to be in static condition, if it is in a state of rest (or uniform motions); such that there are no acceleration in any part of the structure. Such a static response is possible, only when the applied load and the structure characteristics are static (do not vary with time). Loads, especially dead loads are fixed in position and do not vary with respect to time, in magnitude or direction and thereby result in a static response. However, in reality, we encounter many other loads such as live loads, wind loads, earthquake loads, impact loads and wave loads, which vary in magnitude and position with time. These loads result in a time-dependent response. Dynamic analysis is required in such circumstances to predict such responses. It is also necessary to account for the inertial forces (additional forces associated with acceleration), when the structure is not in a state of static equilibrium. If these inertial forces are small in magnitude, the dynamic problem can be approximated to a static problem, and it suffices to do a static analysis, which is much simpler than dynamic analysis. Due to the difficulties associated with dynamic analysis, structural designers prefer to do quasi-static analysis or an equivalent static analysis. This is usually achieved by applying some suitable enhancement factor. These factors are also called impact factors in case of bridges, or gust factor in case of wind resistant design. However, dynamic analysis is unavoidable in some cases, such as asymmetric or irregular structures subject to earthquake loading even in regions of moderate seismicity.

1.3.2 Linear and Nonlinear Analysis

In most of the structural analysis, it is assumed that the behaviour is linear elastic, which enables us to apply the Principle of Superposition. It states that the responses due to several loads on the structure can be obtained by superposing, algebraically, the individual responses due to the various loads acting one at a time. This principle can be used for forces and displacements. This assumption can result in underestimation of the response, when there are relatively large deformations involved or there are axial forces inducing so-called *P*-delta effects, requiring considerations of equilibrium in the deformed configuration. This underestimation is also possible, when the stress strain behaviour of the structural material is nonlinear or exceeds the elastic limit. The former induces geometric nonlinearity, while latter induces material nonlinearity. Problems involving material nonlinearity require a step by step iterative solution procedure, which is difficult and sometimes has problems related to a lack of convergence in the response. When the deformations are large and the material yields, nonlinear analysis must be carried out for an accurate solution.