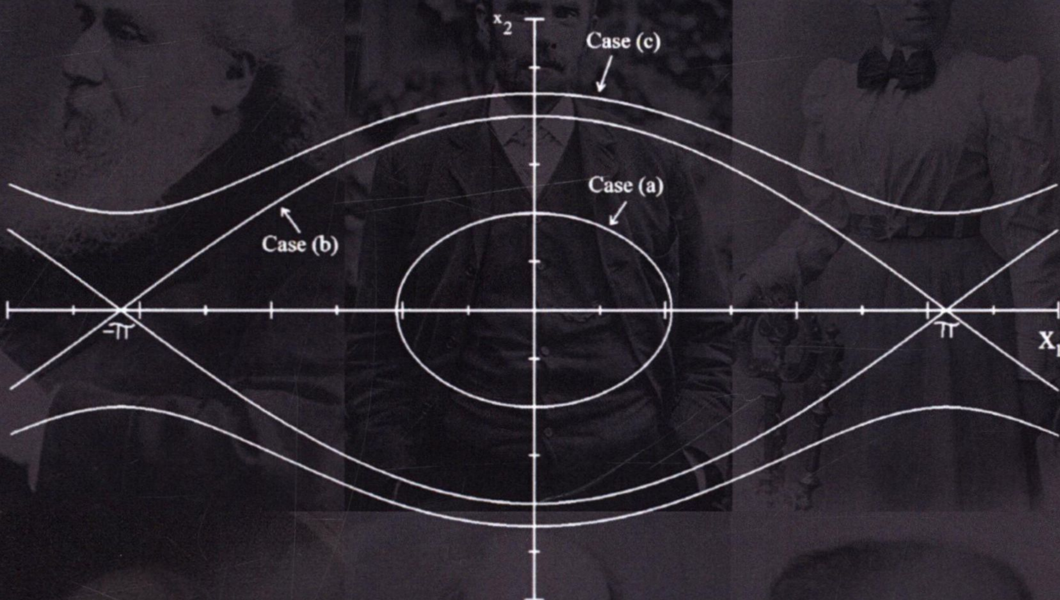
TEXTBOOKS IN MATHEMATICS

Invitation to Linear Algebra



David C. Mello

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Invitation to Linear Algebra

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Invitation to Linear Algebra

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To the Instructor

This book is intended as an introductory course in linear algebra for sophomore or junior students majoring in mathematics, computer science, economics, and the physical sciences. Lofty assumptions have not been made about the level of mathematical preparation of the typical student; in fact, it is only assumed that the typical student has completed a standard calculus course, and is familiar with basic integration techniques.

As a fellow instructor, I know that this course is probably the typical student's first encounter with the requirement of formulating mathematical proofs, and dealing with mathematical formalism. For this reason, each definition has been carefully stated, and detailed proofs of the key theorems have been provided. More importantly, in each proof, the motivation for each step has been given, along with the "intermediate steps" that are normally omitted in most texts.

Unlike most books of this type, the book has been organized into "lessons" rather than chapters. This has been done to limit the size of the mathematical morsels that must be digested by your students during each class, and to make it easier for you, the instructor, to budget class time. Most lessons can be covered in a standard class period.

Considerably more material has been provided than is normally covered in a first course. For example, several advanced topics such as Jordan canonical form, and matrix power series have been included. This is to make the book more flexible, and allow you to choose enrichment material which may reflect your interest and that of your students.

In addition, numerous applications of the course material to mathematics and to science appear in the exercises. The special applications, consisting of the application of linear algebra to both linear and nonlinear dynamical systems appear in Lessons 36-38 at the end of the text.

I would like to thank my colleague Dr. Adam Hartman, for carefully reviewing the manuscript, and for his many helpful comments. If you should have any ideas or suggestions for improving the book, please feel free to email me directly at **dmello@jwu.edu**.

David C. Mello
Providence, Rhode Island
October, 2016

To the Student

This book has been written with you, the student, in mind. It has been designed to help you learn the key elements of linear algebra in an enjoyable fashion. Hopefully, it will give you a glimpse of the intrinsic beauty of this subject, and how it can be used in your chosen field of study.

For most students such as yourself, linear algebra is probably your first encounter with formal mathematics and in constructing proofs of mathematical propositions. In using the book, you should pay close attention to the definitions of new concepts, and to the various theorems; these items have been clearly designated throughout the entire text.

Whenever possible, you should try to work through the proof of each theorem. Pay attention to each step, and make sure it makes sense to you, before proceeding to the next step. If you do this, you will be rewarded with a deeper understanding of the course material, and it will make the exercises involving proofs much easier.

I often tell my students that learning mathematics is like learning to play a musical instrument. Listening to someone else play a beautiful melody might be enjoyable, but it doesn't help you master the instrument yourself. So, you have to take the time to actually "do mathematics" in order to play its music.

But exactly how do you "do mathematics?" Here are some helpful hints for learning and doing mathematics:

1. Always read the text with a pencil in hand, and take the time to work through each example provided in the text. Numerous examples have been provided to help you master the course material.

2. Before attempting to solve a problem that involves constructing your own proof, look at the relevant *definitions* of the concepts involved so you can clearly understand exactly what has to be demonstrated.

3. Be sure to do the homework problems that are assigned by your instructor. If you are unable to do some of the assigned work, or if you are confused about some point, then don't get discouraged, but be sure to ask your instructor for help.

4. Prepare for each class by reading the assigned material in the text *before* class. If you take the time to do this, you'll find that you will spend less time taking notes, and more time really understanding and enjoying each class.

I sincerely hope that you will find the textbook to be extremely helpful and that you will enjoy "doing mathematics." Don't be afraid of new concepts if you don't fully understand them at your first reading; a great physicist once said that "if you give mathematics a chance, and if you learn to love mathematics, then it will love you back!"

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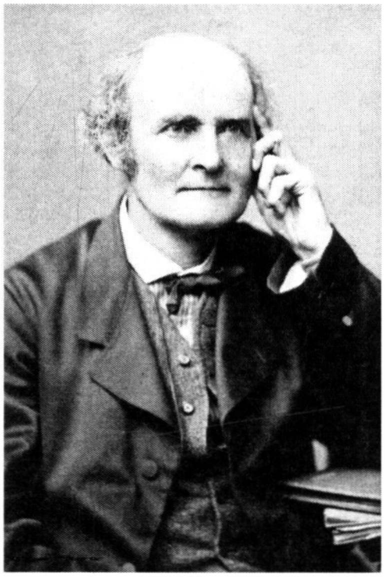

Arthur Cayley (1821-1895)
(Portrait in London by Barraud and Jerrard)

It is interesting to note that the general rules of matrix algebra were first elucidated by the English mathematician Arthur Cayley, Sadlerian professor of mathematics at Cambridge University. In an important paper, published in 1855, Cayley formally defined the concept of a matrix, introduced the rules of matrix algebra, and examined many of the important properties of matrices.

Although the basic properties of matrices were known prior to Cayley's work, he is generally acknowledged as the creator of the theory of matrices because he was the first mathematician to treat matrices as distinct mathematical objects in their own right, and to elucidate the formal rules of matrix algebra. Cayley received many honors for his mathematical work and made major contributions to the theory of determinants, linear transformations, the analytic geometry of n dimensional spaces, and the theory of invariants.