



FREE LOGIC

SELECTED
ESSAYS

KAREL LAMBERT

Free Logic

Paperback Re-issue

Free logic is an important field of philosophical logic that first appeared in the 1950s. Karel Lambert was one of its founders and coined the term itself.

The essays in this collection (written over a period of forty years) explore the philosophical foundations of free logic and its application to areas as diverse as the philosophy of religion and computer science. Among the applications are those to the analysis of existence statements, to definite descriptions, and to partial functions. The volume contains a proof that free logics of any kind are nonextensional and then uses that proof to show that Quine's theory of predication and referential transparency must fail.

The purpose of this collection is to bring an important body of work to the attention of a new generation of professional philosophers, computer scientists, and mathematicians.

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 0-521-03922-3



9 780521 039222 >

COVER DESIGN BY JAMES F. BRISSON

LAMBERT

REFLECTIONS

CAMBRIDGE

Free Logic

Selected Essays

KAREL LAMBERT

University of California, Irvine

University of Salzburg



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521818162

© Karel Lambert 2003

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2003
This digitally printed version 2007

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Lambert Karel, 1928–
Free logic: selected essays / Karel Lambert.
p. cm.
Includes bibliographical references and index.
ISBN 0-521-81816-8
I. Free logic. I. Title.

BC129.L34 2002

160–dc21

2002020177

ISBN 978-0-521-81816-2 hardback
ISBN 978-0-521-03922-2 paperback

Free Logic

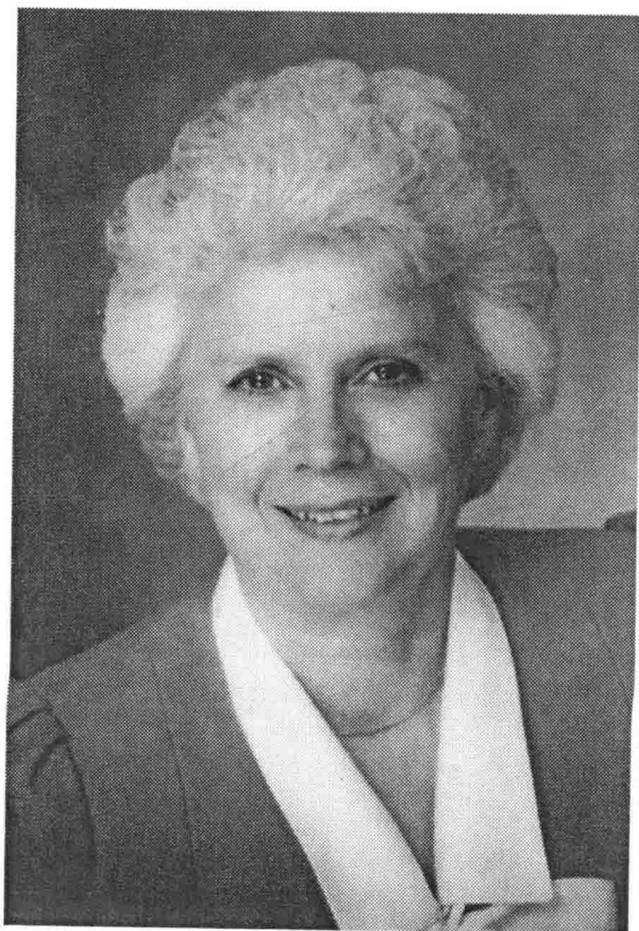
Selected Essays

Free logic is an important field of philosophical logic that first appeared in the 1950s. Karel Lambert was one of its founders and coined the term itself.

The essays in this collection (written over a period of forty years) explore the philosophical foundations of free logic and its application to areas as diverse as the philosophy of religion and computer science. Among the applications are those to the analysis of existence statements, to definite descriptions, and to partial functions. The volume contains a proof that free logics of any kind are nonextensional and then uses that proof to show that Quine's theory of predication and referential transparency must fail.

The purpose of this collection is to bring an important body of work to the attention of a new generation of professional philosophers, computer scientists, and mathematicians.

Karel Lambert is Research Professor of Logic and Philosophy of Science at the University of California, Irvine.



*To my dear Carol, whose beauty, sweetness of nature, capacity to laugh,
and unfailing dignity have sustained me for more than half a century.*

Introduction

In one way or another the nine chapters of this book all have to do with free logic. Most are updated revisions in and adaptations of previously published papers. The exceptions are Chapters 4 and 5, though Chapter 4 contains a revised segment from a recently published paper.

Chapter 1 began as an invited address to the Western Division of the American Philosophical Association Meetings in 1991, and at the request of the organizers of those meetings was subsequently published in slightly revised form in *Philosophical Studies*, 65 (1992), pp. 153–167. It is a critical analysis of Russell's famous theory of definite descriptions of which there are two quite distinct versions. The defining feature of either version is that definite descriptions are not singular terms. That the essence of Russell's theory has to do with logical grammar was stressed in my 'Explaining away singular existence statements', *Dialogue*, 1 (1963), pp. 381–389, and later, independently, by David Kaplan in 'What is Russell's theory of descriptions?' *Physics, History and Logic* (eds. W. Yourgrau and A. Breck), Plenum Press, New York (1970), pp. 277–288.

Chapter 2 is an adaptation of several papers. The main essays are 'Existential import revisited', *Notre Dame Journal of Formal Logic*, 4 (1963), pp. 288–292, 'Notes on E! III: A theory of descriptions', *Philosophical Studies*, 13 (1963), pp. 5–59, 'Notes on E! IV: A reduction in free quantification theory with identity and definite descriptions', *Philosophical Studies*, 15 (1964), pp. 85–88, and 'Free logic and the concept of existence', *Notre Dame Journal of Formal Logic*, 8 (1967), pp. 133–144.

This chapter lays out a motivation for, and the first axiomatic formulation of, a (positive) free logic. It presents the original semantical foundations, and applies that logic to the analysis of singular existence and definite descriptions. Indeed, it contains the first consistent and complete free theory of definite descriptions.

Chapter 3 is essentially an adaptation of two essays. The first is ‘On the reduction of two paradoxes and the significance thereof’, and appeared in a volume in honor of Gerhard Scheibe entitled *Physik, Philosophie, und die Einheit der Wissenschaften* (Hrsg. Lorenz Krüger and Brigitte Falkenburg), Spectrum: Heidelberg (1995), pp. 21–33. The second is ‘A theory about logical theories of “expressions of the form ‘the so and so’ where ‘the’ is in the singular”’, and appeared in a memorial issue of *Erkenntnis* (35 [1991], pp. 337–346) in honor of Rudolf Carnap and Hans Reichenbach. This chapter shows how two paradoxes discovered by Russell, one in Meinong’s theory of objects and the other in a Frege-inspired formulation of set theory, stem from the same source, the naïve theory of definite descriptions. It uses this information to provide an explanation of the origins of the various traditions in the treatment of definite descriptions exactly parallel to the explanation of various approaches to set theory in the wake of Russell’s paradox. Finally, it details some benefits of the free definite description theory approach especially as they concern the logics of definite descriptions in Russell and Frege.

Chapter 4 is new in this book. It is a formal and philosophical examination of the original (informal) theory of definite descriptions of David Hilbert and Paul Bernays, and also of certain neo-Hilbert-Bernays approaches, especially one due to Sören Stenlund. I am very much indebted to Paul Schweizer for his help in the formalization of the *original* Hilbert and Bernays theory. The critique of Stenlund’s approach is my own, and Schweizer is blameless. The upshot is that Stenlund’s theory, despite his claim to the contrary, is not a free definite description theory, but an interesting development hovering somewhere between the original Hilbert-Bernays treatment (as formalized by Schweizer and me) and free definite description theory.

Chapter 5 presents the foundations of much unpublished work in the 1980s and 1990s done by Peter Woodruff and me on what Bas van Fraassen originally called “the spectrum” of (positive) free definite description theories. It lays out the motivation for one approach to the

subject and provides a uniform procedure for proving the completeness of various theories in what I prefer to call – less misleadingly – the hierarchy of positive free definite description theories. It is of a piece with Kripke's semantical analysis of the Lewis hierarchy of modal logics.

Chapter 6 is the updated adaptation of two essays. The first is 'Predication and extensionality', *The Journal of Philosophical Logic*, 3 (1974), pp. 255–264, and the second is 'Fixing Quine's theory of predication', *Dialectica*, 52 (1998), pp. 153–161. This chapter contains the proof that Quine's theory of predication (and hence his theory of referential transparency) is non-extensional, a proof to which Quine himself devoted some attention in his post humous essay, 'Confessions of a confirmed extensionalist'. It also seeks to restore the extensional features of the theory without recourse to the elimination of singular terms.

Chapter 7 is a considerably revised version of the essay entitled 'Nonextensionality'. It was published in a volume in honor of Franz von Kutschera entitled *Das weite Spektrum der analytischen Philosophie* (Hrg. Wolfgang Lenzen), de Gruyter, Berlin (1997). It shows how the logical dependence of two notions of extensionality – the truth-value dependence conception and the *salva veritate* substitution conception – can be restored despite a forceful argument that they are not in a language that contains singular terms without existential import.

Chapter 8 is a much revised version of my Collège de France lectures on the philosophical foundations of free logic, lectures I was invited to give in the spring of 1980. They were published in the journal *Inquiry*, 24 (1981), pp. 147–203 under essentially that title. These lectures, awarded the Medal of the Collège de France, comprehensively examined motivations, discussed confusions, considered applications and laid down now widely adopted conventions.

Chapter 9 is a considerable revision of 'Logical truth and microphysics'. It was published initially in a memorial volume in honor of Henry Leonard entitled *The Logical Way of Doing Things* (ed. Karel Lambert), Yale University Press, New Haven (1969). It was the first application of van Fraassen's technique of supervaluations to a topic in the philosophy of science. His method initially was invented to provide a completeness proof for (positive) free logic that did not depend on model structures utilizing inner and outer domains. Chapter 9

shows that van Fraassen's method can also be used to reconstruct the Reichenbachian interpretation of elementary microphysical statements without appeal to a third truth-value along with a defense of that reconstruction against complaints by Michael Scriven and Wesley Salmon, among others.

There are many to thank in the development and preparation of this book. In particular, I am indebted to Paul Schweizer in the formal development of the original Hilbert-Bernays theory of definite descriptions, and to Peter Woodruff in the essay on the hierarchy of positive free definite descriptions theories. I have enjoyed Woodruff's companionship for nearly forty years. Nothing can detract from his enormously powerful intellect and good nature. Whether the huge amount of work we have done together on definite descriptions ever gets into public print is in the lap of the gods. More generally, I am especially indebted to Bas van Fraassen, Robert K. Meyer, and Brian Skyrms for years of friendship and support, and with each of whom I have been privileged to work on various topics. I am also indebted to longtime personal associations with Edgar and Inge Morscher at the University of Salzburg, Wolfgang and Ulli Spohn at the University of Konstanz, and the late Jules Vuillemin of the Collège de France. I should also like to thank John and Julie Trafford, the former for having discovered the Trafford Eclipse, and the latter for restoring the Sun. Thanks are also due to my good friend Lee Sandler for his many kindnesses. To my children, Kal, Kathryn, and Christopher I am grateful for taking it upon themselves to keep my professional efforts in perspective via loving irreverence. For the preparation of the current book, I thank Terence Moore and his colleagues at Cambridge University Press. They are a firm, decisive, and very helpful group of people.

Contents

<i>Introduction</i>	<i>page ix</i>
1 Russell's Version of the Theory of Definite Descriptions	1
2 Existential Import, 'E!' and 'The'	16
3 The Reduction of Two Paradoxes and the Significance Thereof	33
4 The Hilbert-Bernays Theory of Definite Descriptions	44
5 Foundations of the Hierarchy of Positive Free Definite Description Theories	69
6 Predication and Extensionality	92
7 Nonextensionality	107
8 The Philosophical Foundations of Free Logic	122
9 Logical Truth and Microphysics	176

Russell's Version of the Theory of Definite Descriptions

1. INTRODUCTION

It is mildly ironic that the title of this chapter is an unfulfilled (or improper) definite description because Russell really had two versions of the theory of definite descriptions. The two versions differ in primary goals, character and philosophical strength.

The first version of Russell's theory of definite descriptions was developed in his famous essay of 1905, 'On Denoting'.¹ Its primary goal was to ascertain the logical form of natural language statements containing denoting phrases. The class of such statements included statements with definite descriptions, a species of denoting phrase,² such as 'The Prime Minister of England in 1904 favored retaliation' and 'The gold mountain is gold'. So the theory of definite descriptions contained in what Russell himself regarded as his finest philosophical essay is a theory about how to *paraphrase* natural language statements containing definite descriptions into an incompletely specified

¹ Bertrand Russell, 'On Denoting', *Mind*, New Series: XIV (1905), pp. 479–493; Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, (Second Printing), Cambridge, At the University Press (1910), Volume I. 'On Denoting' is reprinted in *Bertrand Russell: Logic and Knowledge* (editor, Robert C. Marsh), George Allen and Unwin, Ltd., London (1956), pp. 41–56. All references here to 'On Denoting' are to the reprinted version in the Marsh collection.

² Dismissed by G. F. Stout as rubbish, 'On Denoting' was praised by F. P. Ramsey as a paradigm of philosophical analysis. Russell's own opinion of the quality of his famous essay is reported on page 39 in the Marsh collection cited in the previous note.

formal language about propositional functions. Russell used this version of his theory to disarm arguments such as Meinong's arguments for beingless objects. Such reasoning, he said, is the product of a mistaken view about the logical form of statements containing definite descriptions.

The second and later version is presented in that epic work of 1910, *Principia Mathematica* (hereafter usually *Principia*). Its primary goal, in contrast to the first version, was to provide a foundation for mathematics, indeed, to reduce all of mathematics to logic. In chapter *14 Russell introduces a special symbol, the inverted iota, and uses it to make singular term-like expressions out of quasi-statements. They serve as the formal counterpart of definite descriptions, and the expression 'definite description' is extended to cover the formal counterparts themselves, not an uncommon procedure in logic. Then contextual definitions are offered which are said to "define" definite descriptions in all the possible statements in which they can occur. Definite descriptions are regarded not as a referring kind of expression but as a certain variety of "incomplete symbol". So, in *Principia*, Russell's theory of definite descriptions is a theory about how to treat the logical counterpart of natural language expressions of the form 'the so and so' where 'the' is used in the singular. As such it is a *definitional extension* of a formal language, the first order fragment of which is similar to the predicate logic found in most contemporary textbooks of symbolic logic, minus names. Russell uses definite descriptions in *Principia* for all sorts of purposes; for example, he uses them to define descriptive functions.

The chronological order of the two versions will be reversed and the second version will be discussed first. It is the most complicated of the two versions, is more prone to technical complaint, and mainly because of these same complaints, it is weaker in philosophical strength than the first version of the theory. In fact, the first version is a very natural antidote to many of the problems besetting the second version.

2. RUSSELL'S THEORY IN *PRINCIPIA MATHEMATICA*

In what follows Russell's inverted iota is replaced by a smaller case 'i', the dot notation is replaced by parentheses, '&' replaces '.', his sign for conjunction, and the higher case English letters 'P' and 'Q' replace his Greek symbols 'Φ' and 'Ψ'.