6th Edition

Nalluri & Featherstone's

# Civil Engineering Hydraulics

Essential Theory with Worked Examples

MARTIN MARRIOTT



WILEY Blackwell

#### Nalluri & Featherstone's

#### **Civil Engineering Hydraulics**

**Essential Theory with Worked Examples** 

6th Edition

Martin Marriott
University of East London

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#### **Preface to Sixth Edition**

This book has regularly been on reading lists for hydraulics and water engineering modules for university civil engineering degree students. The concise summary of theory and the worked examples have been useful to me both as a practising engineer and as an academic.

The fifth edition aimed to retain all the good qualities of Nalluri and Featherstone's previous editions, with updating as necessary and with an additional chapter on environ-

mental hydraulics and hydrology.

The latest sixth edition now adds a new chapter on coastal engineering prepared by my colleague Dr Ravindra Jayaratne based on original material and advice from Dr Dominic Hames of HR Wallingford. As before, each chapter contains theory sections, after which there are worked examples followed by a list of references and recommended reading. Then there are further problems as a useful resource for students to tackle. The numerical answers to these are at the back of the book, and solutions are available to download from the publisher's website: http://www.wiley.com/go/Marriott.

I am grateful to all those who have helped me in many ways, either through their advice in person or through their published work, and of course to the many students with whom

I have enjoyed studying this material.

Martin Marriott University of East London 2016

#### **About the Author**

This well-established text draws on Nalluri and Featherstone's extensive teaching experience at Newcastle University, including material provided by Professor J. Saldarriaga of the University of Los Andes, Colombia. The text has been updated and extended by Dr Martin Marriott with input from Dr Ravindra Jayaratne of the University of East London and Dr Dominic Hames of HR Wallingford.

Martin Marriott is a chartered civil engineer, with degrees from the Universities of Cambridge, Imperial College London and Hertfordshire. He has wide professional experience in the UK and overseas with major firms of consulting engineers, followed by many years of experience as a lecturer in higher education, currently at the University of East London.

## Symbols

The following is a list of the main symbols used in this book (with their SI units, where appropriate). Various subscripts have also been used, for example to denote particular locations. Note that some symbols are inevitably used with different meanings in different contexts, and so a number of alternatives are listed below. Readers should be aware of this, and check the context for clarification.

- a area (m²); distance (m); acceleration (m/s²)
- b width (m); probability weighted moment of flows (m<sup>3</sup>/s)
- c wave celerity (m/s)
- d diameter (m); water depth (m)
- f force (N); function; silt factor; frequency
- g gravitational acceleration ( $\approx 9.81 \text{ m/s}^2$ )
- h height (m); pressure head difference (m); head loss (m)
- i rank in descending orderj rank in ascending order
- k radius of gyration (m); roughness height (m); constant; coefficient
- m metacentric height (m); mass (kg)
- n Manning's coefficient; exponent; number; wave steepness; group velocity parameter
- p pressure (N/m<sup>2</sup>)
- q discharge per unit width (m<sup>2</sup>/s)
- r radius (m); discount rate
- s relative density; distance (m); sinuosity; standard deviation of sample
- t time (s); L-moment ratios
- u velocity (m/s); parameter
- υ velocity (m/s)
- w velocity (m/s)
- x distance (m); variable
- y distance (m); reduced variate; depth (m)
- z elevation (m); vertical distance (m)
- A area (m<sup>2</sup>)
- B width (m); centre of buoyancy; benefit
- C constant; centre of pressure; coefficient; cost

- D diameter (m)
- E specifie energy (J/N = m); elastic modulus (N/m<sup>2</sup>); wave energy (J/m<sup>2</sup>)
- F force (N); head loss coefficient (s<sup>2</sup>/m); annual probability of non-exceedance
- Fr Froude number
- G centroid
- H height (m); head (m); wave height (m)
- I second moment of area (m<sup>4</sup>); inflow (m<sup>3</sup>/s)
- J junction or node
- K bulk modulus of elasticity (N/m<sup>2</sup>); coefficient; conveyance (m<sup>3</sup>/s); circulation (m<sup>2</sup>/s)
- L length (m); L-moment of flows (m<sup>3</sup>/s); wavelength (m)
- M metacentre; mass (kg)
- N number; rotational speed (rev/min)
- P height of weir (m); wetted perimeter (m); power (W); annual exceedance probability; wave power (W/m)
- Q discharge (m<sup>3</sup>/s)
- R resultant force (N); hydraulic radius (m); radius (m)
- Re Reynolds number
- S slope; energy gradient; storage volume (m<sup>3</sup>); wave spectrum (m<sup>2</sup>s)
- T thrust (N); time period (s); return period (years); surface width (m); thickness (m); wave period (s)
- U velocity (m/s)
- V volume (m<sup>3</sup>); velocity (m/s)
- W weight (N); fall velocity (m/s)
- We Weber number
- Z elevation (m); section factor ( $m^{5/2}$ )
- α angular acceleration (rad/s²); angle (rad); Coriolis coefficient; parameter
- β momentum correction factor (Boussinesq coefficient); parameter; slope
- $\gamma$  specific weight (N/m<sup>3</sup>)
- $\delta$  boundary layer thickness (m)
- ζ factor
- $\eta$  efficiency; wave profile (m)
- $\theta$  angle (radian or degree); slope; wave direction
- κ constant
- λ Darcy-Weisbach friction factor; scale
- $\mu$  dynamic or absolute viscosity (Ns/m<sup>2</sup>); ripple factor; mean
- $\nu$  kinematic viscosity (m<sup>2</sup>/s)
- ξ spillway loss coefficient; displacement (m)
- $\pi$  circle circumference-to-diameter ratio ( $\approx 3.142$ ); Buckingham dimensionless group
- $\rho$  mass density (kg/m<sup>3</sup>)
- $\sigma$  surface tension (N/m); safety factor
- $\tau$  shear stress (N/m<sup>2</sup>)
- $\phi$  function; potential (m<sup>2</sup>/s); transport parameter; angle of repose (degree)
- $\psi$  stream function (m<sup>2</sup>/s); flow parameter
- ω angular velocity (rad/s)
- $\Delta$  increment; submerged relative density

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### Chapter 1 Properties of Fluids

#### 1.1 Introduction

A fluid is a substance which deforms continuously, or flows, when subjected to shear stresses. The term fluid embraces both gases and liquids; a given mass of liquid will occupy a definite volume whereas a gas will fill its container. Gases are readily compressible; the low compressibility, or elastic volumetric deformation, of liquids is generally neglected in computations except those relating to large depths in the oceans and in pressure transients in pipelines.

This text, however, deals exclusively with liquids and more particularly with Newtonian liquids (i.e. those having a linear relationship between shear stress and rate of deformation).

Typical values of different properties are quoted in the text as needed for the various worked examples. For more comprehensive details of physical properties, refer to tables such as Kaye and Laby (1995) or internet versions of such information.

#### 1.2 Engineering units

The metre-kilogram-second (mks) system is the agreed version of the international system (SI) of units that is used in this text. The physical quantities in this text can be described by a set of three primary dimensions (units): mass (kg), length (m) and time (s). Further discussion is contained in Chapter 9 regarding dimensional analysis. The present chapter refers to the relevant units that will be used.

The unit of force is called newton (N) and 1 N is the force which accelerates a mass of 1 kg at a rate of 1 m/s<sup>2</sup> (1 N = 1 kg m/s<sup>2</sup>).

The unit of work is called joule (J) and it is the energy needed to move a force of 1 N over a distance of 1 m. Power is the energy or work done per unit time and its unit is watt (W) (1 W = 1 J/s = 1 N m/s).

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#### 1.3 Mass density and specific weight

Mass density ( $\rho$ ) or density of a substance is defined as the mass of the substance per unit volume (kg/m<sup>3</sup>) and is different from specific weight ( $\gamma$ ), which is the force exerted by the earth's gravity (g) upon a unit volume of the substance ( $\gamma = \rho g$ : N/m<sup>3</sup>). In a satellite where there is no gravity, an object has no specific weight but possesses the same density that it has on the earth.

#### 1.4 Relative density

**Relative density** (s) of a substance is the ratio of its mass density to that of water at a standard temperature (4°C) and pressure (atmospheric) and is dimensionless.

For water,  $\rho = 10^3 \text{ kg/m}^3$ ,  $\gamma = 10^3 \times 9.81 \approx 10^4 \text{ N/m}^3$  and s = 1.

#### 1.5 Viscosity of fluids

Viscosity is that property of a fluid which by virtue of cohesion and interaction between fluid molecules offers resistance to shear deformation. Different fluids deform at different rates under the action of the same shear stress. Fluids with high viscosity such as syrup deform relatively more slowly than fluids with low viscosity such as water.

All fluids are viscous and 'Newtonian fluids' obey the linear relationship

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y}$$
 (Newton's law of viscosity) [1.1]

where  $\tau$  is the shear stress (N/m<sup>2</sup>), du/dy the velocity gradient or the rate of deformation (rad/s) and  $\mu$  the coefficient of dynamic (or absolute) viscosity (N s/m<sup>2</sup> or kg/(m s)).

Kinematic viscosity (v) is the ratio of dynamic viscosity to mass density expressed in metres squared per second.

Water is a Newtonian fluid having a dynamic viscosity of approximately  $1.0 \times 10^{-3}$  N s/m<sup>2</sup> and kinematic viscosity of  $1.0 \times 10^{-6}$  m<sup>2</sup>/s at 20°C.

#### 1.6 Compressibility and elasticity of fluids

All fluids are compressible under the application of an external force and when the force is removed they expand back to their original volume, exhibiting the property that stress is proportional to volumetric strain.

The bulk modulus of elasticity, 
$$K = \frac{\text{pressure change}}{\text{volumetric strain}}$$

$$= -\frac{\text{d}p}{(\text{d}V/V)}$$
[1.2]

The negative sign indicates that an increase in pressure causes a decrease in volume.

Water with a bulk modulus of  $2.1 \times 10^9$  N/m<sup>2</sup> at 20°C is 100 times more compressible than steel, but it is ordinarily considered incompressible.

#### 1.7 Vapour pressure of liquids

A liquid in a closed container is subjected to partial vapour pressure due to the escaping molecules from the surface; it reaches a stage of equilibrium when this pressure reaches saturated vapour pressure. Since this depends upon molecular activity, which is a function of temperature, the vapour pressure of a fluid also depends upon its temperature and increases with it. If the pressure above a liquid reaches the vapour pressure of the liquid, boiling occurs; for example, if the pressure is reduced sufficiently, boiling may occur at room temperature.

The saturated vapour pressure for water at  $20^{\circ}$ C is  $2.45 \times 10^{3}$  N/m<sup>2</sup>.

#### 1.8 Surface tension and capillarity

Liquids possess the properties of cohesion and adhesion due to molecular attraction. Due to the property of cohesion, liquids can resist small tensile forces at the interface between the liquid and air, known as surface tension ( $\sigma$ : N/m). If the liquid molecules have greater adhesion than cohesion, then the liquid sticks to the surface of the container with which it is in contact, resulting in a capillary rise of the liquid surface; a predominating cohesion, in contrast, causes capillary depression. The surface tension of water is  $73 \times 10^{-3}$  N/m at  $20^{\circ}$ C.

The capillary rise or depression h of a liquid in a tube of diameter d can be written as

$$h = \frac{4\sigma\cos\theta}{\rho gd} \tag{1.3}$$

where  $\theta$  is the angle of contact between liquid and solid.

Surface tension increases the pressure within a droplet of liquid. The internal pressure p balancing the surface tensional force of a small spherical droplet of radius r is given by

$$p = \frac{2\sigma}{r} \tag{1.4}$$

#### **Worked examples**

#### Example 1.1

The density of an oil at 20°C is 850 kg/m<sup>3</sup>. Find its relative density and kinematic viscosity if the dynamic viscosity is  $5 \times 10^{-3}$  kg/(m s).

Solution:

Relative density, 
$$s = \frac{\rho \text{ of oil}}{\rho \text{ of water}}$$
$$= \frac{850}{10^3}$$
$$= 0.85$$

Kinematic viscosity, 
$$v = \frac{\mu}{\rho}$$

$$= \frac{5 \times 10^{-3}}{850}$$

$$= 5.88 \times 10^{-6} \text{ m}^2/\text{s}$$

#### Example 1.2

If the velocity distribution of a viscous liquid ( $\mu = 0.9 \text{ N s/m}^2$ ) over a fixed boundary is given by  $u = 0.68y - y^2$ , in which u is the velocity (in metres per second) at a distance y (in metres) above the boundary surface, determine the shear stress at the surface and at y = 0.34 m.

Solution:

$$u = 0.68y - y^2$$
$$\Rightarrow \frac{du}{dy} = 0.68 - 2y$$

Hence,  $(du/dy)_{y=0} = 0.68 \text{ s}^{-1}$  and  $(du/dy)_{y=0.34\text{m}} = 0$ .

Dynamic viscosity of the fluid,  $\mu = 0.9 \text{ N s/m}^2$ 

From Equation 1.1,

shear stress 
$$(\tau)_{y=0} = 0.9 \times 0.68$$
  
= 0.612 N/m<sup>2</sup>

and at  $y = 0.34 \,\text{m}$ ,  $\tau = 0$ .

#### Example 1.3

At a depth of 8.5 km in the ocean the pressure is  $90\,\mathrm{MN/m^2}$ . The specific weight of the sea water at the surface is  $10.2\,\mathrm{kN/m^3}$  and its average bulk modulus is  $2.4\times10^6\,\mathrm{kN/m^2}$ . Determine (a) the change in specific volume, (b) the specific volume and (c) the specific weight of sea water at 8.5 km depth.

Solution:

Change in pressure at a depth of 8.5 km, 
$$dp = 90 \text{ MN/m}^2$$
  
=  $9 \times 10^4 \text{ kN/m}^2$ 

Bulk modulus, 
$$K = 2.4 \times 10^6 \text{ kN/m}^2$$
  
From  $K = -\frac{dp}{(dV/V)}$   
 $\frac{dV}{V} = \frac{-9 \times 10^4}{2.4 \times 10^6} = -3.75 \times 10^{-2}$ 

Defining specific volume as  $1/\gamma$  (m<sup>3</sup>/kN), the specific volume of sea water at the surface =  $1/10.2 = 9.8 \times 10^{-2}$  m<sup>3</sup>/kN.

Change in specific volume between that at the surface and at 8.5 km depth, dV=  $-3.75 \times 10^{-2} \times 9.8 \times 10^{-2}$ =  $-36.75 \times 10^{-4}$  m<sup>3</sup>/kN