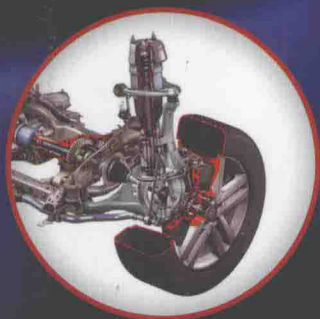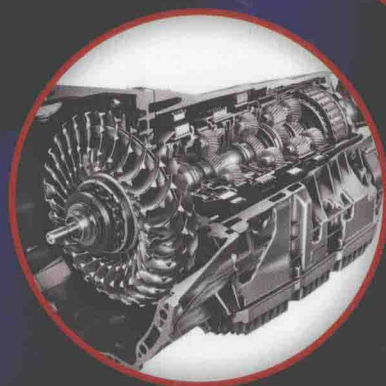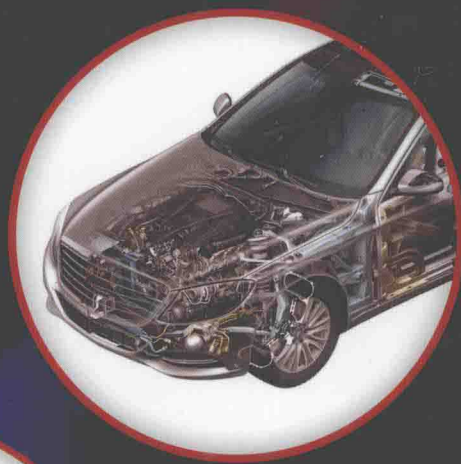# VEHICLE DYNAMICS

MARTIN MEYWERK

VEHICLE DYNAMICS

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WILEY

Foreword

This book is an extract of lectures on vehicle dynamics and mechatronic systems in vehicles held at the Helmut-Schmidt-University, University of the Federal Armed Forces, Hamburg, Germany. The lectures have been held since 2002 (Vehicle Dynamics) and 2009 (Vehicle Mechatronics). The book is an introduction to the field of vehicle dynamics and most parts of the book should be comprehensible to undergraduate students with a knowledge of basic mathematics and engineering mechanics at the end of their Bachelor studies in mechanical engineering. However, some parts require advanced methods which are taught in graduate studies (Master programme in mechanical engineering).

I wish to thank Mrs Martina Gerds for converting the pictures to Corel Draw with LaTeX labels and for typing Chapter 9. My thanks go to Mr Darrel Fernandes, B.Sc., for the pre-translation of my German scripts. I especially wish to thank Mr Colin Hawkins for checking and correcting the final version of the book with respect to the English language. My scientific assistants, especially Dr Winfried Tomaske and Dipl.-Ing. Tobias Hellberg, I thank for proofreading, especially with regard to the technical aspects. Special thanks for assistance in preparing a number of Solid Works constructions for pictures of suspensions and transmissions as well for help in preparing some MATLAB diagrams go to Mr Hellberg. Last but not the least, my thanks go to my family, my wife, Dr Annette Nicolay, and my children, Sophia, Aljoscha, Indira and Felicia, for their patience and for giving me a lot of time to prepare this book.

Series Preface

The automobile is a critical element of any society, and the dynamic performance of the vehicle is a key aspect regarding its value proposition. Furthermore, vehicle dynamics have been studied for many years, and provide a plethora of opportunities for the instructor to teach her students a wide variety of concepts. Not only are these dynamics fundamental to the transportation sector, they are quite elegant in nature linking various aspects of kinematics, dynamics and physics, and form the basis of some of the most impressive machines that have ever been engineered.

Vehicle Dynamics is a comprehensive text of the dynamics, modeling and control of not only the entire vehicle system, but also key elements of the vehicle such as transmissions, and hybrid systems integration. The text provides a comprehensive overview of key classical elements of the vehicle, as well as modern twenty-first century concepts that have only recently been implemented on the most modern commercial vehicles. The topics covered in this text range from basic vehicle rigid body kinematics and wheel dynamic analysis, to advanced concepts in cruise control, hybrid powertrain design and analysis and multi-body systems. This text is part of the *Automotive Series* whose primary goal is to publish practical and topical books for researchers and practitioners in industry, and post-graduate/advanced undergraduates in automotive engineering. The series addresses new and emerging technologies in automotive engineering supporting the development of next generation transportation systems. The series covers a wide range of topics, including design, modelling and manufacturing, and it provides a source of relevant information that will be of interest and benefit to people working in the field of automotive engineering.

Vehicle Dynamics presents a number of different designs, analysis and implementation considerations related to automobiles including power requirements, converters, performance, fuel consumption and vehicle dynamic models. The text is written from a very pragmatic perspective, based on the author's extensive experience. The book is written such that it is useful for both undergraduate and post-graduate courses, and

is also an excellent reference text for those practicing automotive systems design and engineering, in the field. The text spans a wide spectrum of concepts that are critical to the understanding of vehicle performance, making this book welcome addition to the *Automotive Series*.

Thomas Kurfess
October 2014

Preface

This books covers the main parts of vehicle dynamics, which is divided into three topics: longitudinal, vertical and lateral dynamics. It also explains some applications, especially those with a mechatronic background, and outlines some components.

Figure 1 provides an overview of the chapters of the book. The main parts (longitudinal, vertical and lateral) as well as applications and component chapters are grouped together. Many principal aspects of dynamics are explained by using simple mechanical models (e.g. quarter-vehicle model and single-track model). As the virtual development process with very complex multi-body systems (MBS) is used in the design of modern cars, this simulation technique is described very briefly in the last chapter. Although these MBS models are able to predict many details, the user of such models should understand the principles of how vehicles behave, and the main theory behind dynamic behaviour. It is therefore important to learn the basic dynamic behaviour using the simple models described in this book.

Chapter 1 contains some general data for vehicles. These remarks are followed by an introduction to some of the basics of frames and axis systems. This introduction should be read by everyone. Following this are the three groups of longitudinal, vertical and lateral dynamics, which are largely independent. The longitudinal and the vertical parts are completely independent of the other parts and may be read and understood without any knowledge of the other parts. The third group, containing the lateral dynamics part, includes a number of aspects that may be difficult to understand without first reading the longitudinal or the vertical part.

The application chapters can only partly be understood without reading the corresponding theory chapter. Readers are therefore recommended to start with the basic parts: the basics for Chapters 7 (Hybrid Powertrains), 8 (Adaptive Cruise Control) and 17 (Torque and Speed Converters) can be found in the longitudinal dynamics chapters, while lateral dynamics is important for Chapter 16 (Suspension Systems), in the case of Chapter 18 (Shock Absorbers, Springs and Brakes) a knowledge is required of vertical dynamics as well as some aspects of longitudinal and lateral dynamics. Chapter 19 (Active Longitudinal and Lateral Systems), as the name reveals, involves longitudinal and lateral aspects. Chapter 20 is nearly independent of the theoretical considerations.

Figure 1 includes the letters B and M, which stand for Bachelor and Master, behind every chapter. Chapters of level B should be comprehensible for undergraduate
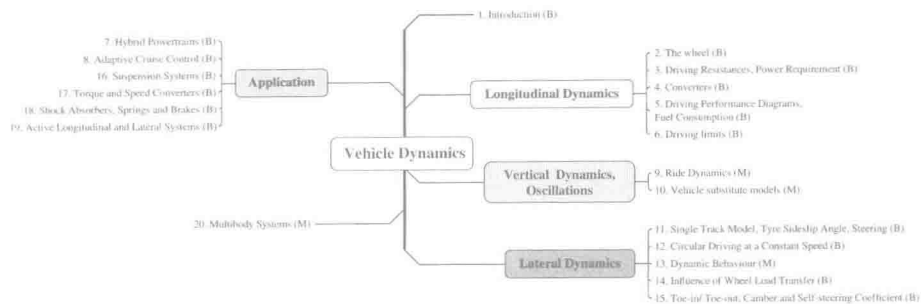
Figure 1   Chapters of the book
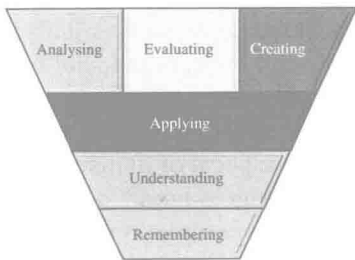
Figure 2   Bloom's taxonomy of learning

students with a knowledge of engineering mechanics and mathematics at the end of their Bachelor studies in mechanical engineering. Topics covered are: algebra; trigonometric functions; differential calculus; linear algebra; vectors; coordinate systems; force, torque, equilibrium; mass, centre of mass, moment of inertia; method of sections, friction, Newton's laws, Lagrange's equation. In chapters followed by level M, an advanced knowledge is useful, as is usually taught to graduate students: ordinary differential equations (ODE), stability of ODEs, Laplace transformation, Fourier transformation, stochastic description of uneven roads and spectral densities.

At the end of nearly each chapter, you will find some questions and exercises. These are for monitoring learning progress or for applying the material learned to some small problems. For this reason, the questions and tasks are arranged in classes according to Bloom's taxonomy of learning (cf. Figure 2).

The simplest class is *Remembering*, which means that you only have to remember the correct content (e.g. a definition or a formula). You should be able to answer the questions of the second class *Understanding* if you have understood the content. The tasks of the third class *Application* involve applying the content to some unknown problem. The remaining three classes *Analysing*, *Evaluating* and *Creating* are more suited to extended student works, such as Bachelor or Master theses, and are therefore rarely included in this book.

List of Abbreviations and Symbols

The tables on the following pages summarize the mathematical symbols and abbreviations used in this book. In most cases (but not in all), the indices used with the symbols indicate the following:

- v vehicle
- b body
- w wheel
- t tyre
- x, y, z: $\vec{e}_{x*}, \vec{e}_{y*}, \vec{e}_{z*}$

Sometimes a symbol, which is needed only in a very local part of the book, could be used in another meaning than it is described in the following tabular, as well as sometimes the units can differ from those given in the tabular. Symbols that occur only in a small part of the book are not listed in the tabular.

Table 1   List of symbols

Symbol	Description	Units	Page
a	Acceleration of the vehicle $a = \ddot{x}_v$	m/s^2	84
A	Aerodynamic area	m^2	27
α	Tyre slip angle	rad	177
	(or locally used angle)		
ABS	Anti-lock braking system	–	283
A_c	Clothoid parameter	m	171
ACC	Adaptive cruise control	–	107
α_g	Angle of inclination of the road	rad	11, 29
α_{gz}	Progression ratio of a transmission	1	48
	$\alpha_{gz} = i_{z-1}/i_z, z = 2, \dots, N_{z\text{max}}$		
α_j	Tyre slip angle ($j = 1$ front; $j = 2$ rear)	rad	186
$\overline{\alpha}_j$	Average tyre slip angle ($j = 1$ front; $j = 2$ rear)	rad	219
α_{ji}	Inner tyre slip angle ($j = 1$ front; $j = 2$ rear)	rad	217
α_{jo}	Outer tyre slip angle ($j = 1$ front; $j = 2$ rear)	rad	217
ASF	Active front steering	–	297
ASR	Anti-slip regulation	–	293
β	Vehicle sideslip angle	rad	170
γ	Camber angle	rad	229
c_α	Cornering stiffness	N/rad	180
$\overline{c}_\alpha$	Mean cornering stiffness	N/rad	219
c_d	Aerodynamic drag coefficient	1	27
c_{lj}	Aerodynamic lift coefficient ($j = 1$ front; $j = 2$ rear)	1	76
CPVA	Centrifugal pendulum vibration absorber	–	144
c_y	Aerodynamic crosswind coefficient	1	214
DAE	Differential algebraic equations	–	315
δ_{10}	Toe-in $\delta_{10} > 0$, toe-out $\delta_{10} < 0$	rad	229
ΔF_{zji}	Wheel load change at the inner wheel j; ($j = 1$ front; $j = 2$ rear)	N	217
ΔF_{zjo}	Wheel load change at the outer wheel j; ($j = 1$ front; $j = 2$ rear)	N	217
$(O, \vec{e}_{ix}, \vec{e}_{iy}, \vec{e}_{iz})$	Inertial reference frame	–	6
η_d	Efficiency of the differential	1	49
η_e	Efficiency of the engine	1	67
$\overline{\eta}_e$	Mean efficiency of the engine	1	63
η_t	Efficiency of the drivetrain (transmission and differential)	1	58
$\overline{\eta}_t$	Mean efficiency of the drivetrain (speed and torque converter, differential)	1	63
η_z	Efficiency of the zth gear of the transmission	1	49
ESP	Electronic stability programme	–	294

Table 2 List of symbols

Symbol	Description	Units	Page
$(S_{cp}, \vec{e}_{tx}, \vec{e}_{ty}, \vec{e}_{tz})$	Tyre reference frame	–	7
$(S_{cm}, \vec{e}_{vx}, \vec{e}_{vy}, \vec{e}_{vz})$	Vehicle reference frame	–	6
e_w	Eccentricity at the wheel	m	12
e_{wj}	Eccentricity at the wheels of axle j ($j = 1$ front; $j = 2$ rear)	m	74
$(S_{cmw}, \vec{e}_{wx}, \vec{e}_{wy}, \vec{e}_{wz})$	Wheel reference frame, $\vec{e}_{wz}$ perpendicular to the road plane	–	7
F_a	Aerodynamic drag in the longitudinal direction for simplified models in longitudinal dynamics	N	27
F_{ax}	Aerodynamic drag force in the $\vec{e}_{vx}$-direction for single track model and wheel load transfer	N	172
F_{ay}	Aerodynamic drag force in the $\vec{e}_{vy}$-direction	N	172
F_{az}	Aerodynamic lift force in the $\vec{e}_{vz}$-direction	N	76
F_{basic}	Basic demand of tractive force: $F_{\text{basic}} = F_r + F_a$	N	35
f_e	Frequency of the received signal	Hz	112
F_g	Gradient resistance	N	29
$F_{g/i}$	Combined gradient and inertial resistance for $y = p + \lambda \ddot{x}_v/g$	N	35
F_{ideal}	Ideal (demand) characteristic map of tractive force	N	37
F_i	Acceleration or inertial resistance	N	33
FMCW	Frequency modulated continuous wave	–	112
f_r	Rolling resistance coefficient	1	15
f_s	Frequency of the transmitted signal	Hz	112
$\tilde{f}_{ri}$	Coefficients for f_r approximation ($i = 0, 1, 4$)	1	16
F_r	Rolling resistance	N	15
F_{tot}	Total tractive force demand: $F_{\text{tot}} = F_a + F_i + F_g + F_r$	N	34
F_{wsz}	Force supplied at the wheel from the powertrain for gear z	N	50
F_x	Section force for tyre-road	N	13
F_{xj}	Section force for tyre-road ($j = 1$ front; $j = 2$ rear)	N	32
F_{zj}	Section force (wheel load) ($j = 1$ front; $j = 2$ rear)	N	74
$F_{zj\,\text{aero}}$	Wheel load aerodynamic portion ($j = 1$ front; $j = 2$ rear)	N	76
$F_{zj\,\text{stat}}$	Wheel load static portion ($j = 1$ front; $j = 2$ rear)	N	75

Table 3 List of symbols

Symbol	Description	Units	Page
$F_{zj\,\mathrm{dyn}}$	Wheel load dynamic portion ($j = 1$ front; $j = 2$ rear)	N	78
F_z	Wheel or axle load in the $\vec{e}_{wz}$-direction	N	12
g	Gravitational acceleration	m/s^2	13
G_{aj}	Weight of the axle j ($j = 1$ front; $j = 2$ rear)	N	75
G_b	Weight of body (sprung mass)	N	75
h	Distance: centre of mass S_{cm} – road	m	86
h_b	Distance: centre of mass of the body – road	m	75
h_{cm}	Distance: centre of mass S_{cm} – road	m	77
HP	Pump	–	297
h_{pp}	Distance: Centre of pressure S_{pp} – road for air flow in the $\vec{e}_{vx}$-direction	m	76
HSV	High-pressure selector valve	–	297
i_d	Transmission ratio of the differential (final drive)	1	44
i_g	Transmission ratio of the gearbox; for a stepped transmission: $i_g = i_z$	1	30
i_t	Total transmission ratio $i_t = i_z i_d$	1	58
i_z	Transmission ratio of gear z of the transmission, $z = 1, \ldots, N_{z\,\max}$	1	48
J_{aj}	Moment of inertia of the axle j	kg m^2	30
J_c	Moment of inertia of gear, differential, Cardan shaft	kg m^2	30
J_e	Moment of inertia of engine, clutch	kg m^2	30
J_z	Moment of inertia of the vehicle with respect to the $\vec{e}_z$-axis	kg m^2	172
κ	Angle of rotation of the body of the vehicle	rad	222
κ_{cc}	Instantaneous curvature ($\kappa_{cc} = 1/\rho_{cc}$) of the vehicle path	1/m	171
κ_w	Wavenumber of an uneven road	rad/m	164
ℓ	Wheelbase; distance between front and rear axle	m	75
ℓ_1	Distance in the $\vec{e}_{vx}$ direction between front axle centre of mass and centre of mass S_{cm} of the vehicle	m	75
ℓ_2	Distance in the $\vec{e}_{vx}$ direction between rear axle centre of mass and centre of mass S_{cm} of the vehicle	m	75
λ	Rotational mass factor	1	33
λ_e	Eigenvalue with respect to time	1/s	207
ℓ_{cm}	Distance: centre of gravity S_{cm} – centre of pressure S_{pp} in the $\vec{e}_{vx}$ direction	m	172

Table 4 List of symbols

Symbol	Description	Units	Page
$\mu = \mu(S)$	Tyre longitudinal force coefficient	1	21
M_a	Aerodynamic moment	Nm	74
μ_a	Coefficient of adhesion	1	21
m_{aj}	Mass of the axle ($j = 1$ front; $j = 2$ rear)	kg	32
M_{aj}	Section moment at the axle j ($j = 1$ front; $j = 2$ rear)	Nm	32
m_b	Mass of the body or sprung mass of the vehicle	kg	31
MBS	Multi-body systems	–	4
M_{cc}	Centre of curvature	–	170
M_{cr}	Instantaneous centre of rotation	–	173
M_e	Torque supplied from the engine	N	58
M_{100}	Full load moment of the engine	Nm	37
M_i	Input moment (e.g. at input of transmission or clutch)	Nm	45
$M(P_{\max})$	Moment where the power of the engines reaches a maximum	Nm	37
M_l	Torque loss from the engine	N	59
$M_{\max}$	Maximum torque of the engine	Nm	55
$M(n_{\max})$	Full load moment of the engine at $n_{\max}$	Nm	37
$M(n_{\min})$	Full load moment of the engine at $n_{\min}$	Nm	37
M_o	Output moment (e.g. at input of transmission or clutch)	Nm	45
μ_s	Coefficient of pure sliding	1	22
m_{tot}	Total mass (sprung and unsprung mass)	kg	29
M_{ws}	Torque supplied at the wheel from the powertrain	N	58
n_c	Total caster trail $n_c = n_{kc} + n_{tc}$	m	188
n_e	Engine speed (revolutions)	rad/s	44
n_i	Input speed or revolutions (e.g. at input of transmission or clutch)	rev/s	45
n_{iz}	Input speed (revolutions) of transmission at gear z	rad/s	48
n_{kc}	Kinematic caster trail	m	182
$n_{\max}$	Maximum speed of the engine	rpm	37
$n_{\min}$	Minimum speed of the engine	rpm	37
n_o	Output speed or revolutions (e.g. at input of transmission or clutch)	rev/s	45
$n(P_{\max})$	Engine speed where the power of the engines reaches a maximum	rpm	37
$n_{w\,\max}$	Maximum revolutions per minute of the wheel	rpm	44
n_{oz}	Output speed (revolutions) of transmission at gear z	rad/s	48

Table 5   List of symbols

Symbol	Description	Units	Page
n_{tc}	Tyre caster trail	m	177
n_w	Wheel speed (revolutions)	rad/s	44
$N_{z\,\text{max}}$	Number of gears in a transmission	1	48
ODE	Ordinary differential equation	–	5
OEM	Original equipment manufacturer	–	121
ω_i	Input angular velocities (e.g. at input of transmission or clutch)	rad/s	45
ω_o	Output angular velocities (e.g. at input of transmission or clutch)	rad/s	45
p	Gradient (inclination) of a road $p = \tan \alpha_g$	1	29
ψ	Yaw angle	rad	7
φ	Roll angle	rad	7
P_{100}	Full load power of the engine	$W = Nm/s$	37
P_a	Power of aerodynamic drag force ($S = 0$: $P_a = F_a v_v$)	$W = Nm/s$	35
φ_{aj}	Angle of rotation of the axle j ($j = 1$ front; $j = 2$ rear)	rad	32
φ_b	Pitch angle of the body	rad	159
P_{basic}	Basic demand of power: $P_{\text{basic}} = P_r + P_a$	N	35
$\dot{\varphi}_c$	Angular velocity gear, differential, Cardan shaft	rad/s	30
P_e	Power supplied from the engine	$W = Nm/s$	46
$\dot{\varphi}_e$	Angular velocity engine, clutch	rad/s	30
$P_{g/i}$	Power of combined gradient and inertial resistance	$W = Nm/s$	35
P_g	Power of gradient resistance	$W = Nm/s$	35
Φ_h	Spectral density of the stochastic road surface irregularity	m^3	131
$\Phi_h(\Omega_0)$	Coefficient of roughness	m^3	131
P_{basic}	Total power demand: $P_{\text{basic}} = P_r + P_a$	N	34
P_{ideal}	Ideal (demand) characteristic map of power at the wheel	N	37
P_i	Input power (e.g. at input of transmission) in Chapter 4	$W = Nm/s$	45
P_i	Power of inertia forces in Chapter 3	$W = Nm/s$	35
P_{max}	Maximum power of the engine reaches a maximum	$W = Nm/s$	37
P_o	Output power (e.g. at input of transmission)	$W = Nm/s$	45
P_r	Power of rolling resistance ($S = 0$: $P_r = F_r v_v$)	$W = Nm/s$	34
P_{tot}	Total power demand: $P_{\text{tot}} = P_r + P_a + P_g + P_i$	N	34
φ_w	Rotational angle of the wheel w.r.t the $\vec{e}_{wy}$-axis	rad	14

Table 6 List of symbols

Symbol	Description	Units	Page
P_{wsi}	Power supplied at the wheel from the powertrain for gear i	W = Nm/s	50
P_w	Power at the wheel	W = Nm/s	63
ρ_a	Mass density of air	kg/m^3	27
ρ_{cc}	Instantaneous radius of curvature of the vehicle path	m	171
r_k	Scrub radius: distance between the intersection of the steering axis with road and the centre of the contact patch	m	183
r_σ	Kingpin offset between the wheel centre and the steering axis	m	184
R_{w0}	Dynamic rolling radius	m	20
R_{w0j}	Dynamic rolling radius ($j = 1$ front; $j = 2$ rear)	m	32
r_{wst}	Static radius of a wheel	m	14
r_{wstj}	Static radius of the wheels of the axle j ($j = 1$ front; $j = 2$ rear)	m	32
σ	Inclination angle of the steering axis; angle from the $\vec{e}_{Vz}$ direction to the projection of the steering axis on to the $\vec{e}_{Vz}$-$\vec{e}_{Vy}$-plane	rad	181
S_j	Slip at wheels of the axle j ($j = 1$ front; $j = 2$ rear)	1	32
s_j	Track of the axle j ($j = 1$ front; $j = 2$ rear)	m	219
S_{cm}	Centre of mass of the vehicle (sprung and unsprung mass)	–	6
S_{cmw}	Centre of mass of a wheel	–	7
SOV	Switch over valve	–	297
S_{pp}	Centre of pressure	–	76
S_{cp}	Centre point of the contact patch	–	7
σ_n	Normal stress distribution in the contact patch	N/m^2	12
SSF	Static stability factor	–	220
τ	Caster angle; angle from the $\vec{e}_{vz}$ direction to the projection of the steering axis on to the $\vec{e}_{vz}$-$\vec{e}_{vx}$-plane	rad	181
t_b	Pressure build-up time	s	80
t_{fb}	Foot force build-up time	s	80
t_f	Duration of full braking process	s	82
t_r	Reaction time	s	79
t_t	Transmission time	s	80
ϑ	Pitch angle	rad	7
v	Absolute value of $\vec{v}_v$: $v = |\vec{v}_v| = v_v$	m/s	27
v_a	Velocity of wind	m/s	27
$\vec{v}_a$	Velocity vector of wind	m/s	214

Table 7 List of symbols

Symbol	Description	Units	Page
v_c	Circumferential velocity of wheel	m/s	20
v_{ch}^2	Characteristic velocity squared	m^2/s^2	197
v_{crit}	Critical velocity	m/s	209
$\vec{v}_r$	Resultant velocity vector (wind and vehicle)	m/s	214
$\vec{v}_v$	Velocity of the vehicle at S_{cm}; $$v_v = |\vec{v}_v|; \vec{v} = \vec{v}_v$$ $$\vec{v} = (v_{vx}, v_{vy}, v_{vz}) \cdot (\vec{e}_{ix}, \vec{e}_{iy}, \vec{e}_{iz})^T$$	m/s	214
v_w	Velocity of wheel	m/s	179
w	Waviness of an uneven road	1	131
W_w	Work at the wheel	J = Nm	63
X	Section force between wheel and body	N	13
X_j	Section force axle-vehicle at the axle j $(j = 1$ front; $j = 2$ rear)	N	33
$\ddot{x}_v$	Acceleration of the vehicle	m/s^2	30
x_{aj}	Coordinate of the axle j $(j = 1$ front; $j = 2$ rear)	m	32
x_v, y_v, z_v	S_{cm} vehicle coordinates w.r.t. $(O, \vec{e}_{ix}, \vec{e}_{iy}, \vec{e}_{iz})$	m	6
x_w, y_w, z_w	S_{cmw} wheel coordinates w.r.t. $(S_{\mathrm{cm}}, \vec{e}_{vx}, \vec{e}_{vy}, \vec{e}_{vz})$	m	14
Y	Section force between wheel and body	N	178
Z	Section force between wheel and body	N	13
$\mathcal{Z}$	Braking ration $\mathcal{Z} = -a/g$	1	84
z_1	Displacement of the wheel	m	264
z_2	Displacement of the body	m	264
z_3	Displacement of the seat	m	136
z_b	Displacement of the body	m	156
z_w	Displacement of the wheel	m	156

Contents