Valentin L. Popov · Markus Heß

Method of Dimensionality Reduction in Contact Mechanics and Friction

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Valentin L. Popov
Institute of Mechanics
Technische Universität Berlin
Berlin
Germany

Markus Heß
Abteilung IC Studienkolleg
Technische Universität Berlin
Berlin
Germany

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... it is more difficult to break a prejudice than an atom.

A. Einstein

Foreword

Contact and friction are phenomena that are of extreme importance in uncountable technical applications. Simultaneously, they are phenomena which cause difficulties in their theoretical consideration and numerical simulation. This book presents a method that trivializes two classes of contact problems to such a degree that they become accessible even for first semester engineering students who possess an elementary understanding of mathematics and physics. Furthermore, this method presents a very simple way to numerically simulate contact and frictional forces.

The "trivialization" occurs with the help of the method of dimensionality reduction, which is the primary focus of this book. This method is based on the analogy between certain classes of three-dimensional contacts and contacts with one-dimensional elastic or viscoelastic foundations. Within the framework of the method of dimensionality reduction, three-dimensional contacts are replaced by a series of one-dimensional elastic or viscoelastic elements. In doing this, we would like to strongly accentuate the fact that this *is not an approximation*: Certain macroscopic contact properties correspond *exactly* with those of the original three-dimensional contact.

The method of dimensionality reduction offers a *two-fold* reduction: First, a three-dimensional system is replaced by a one-dimensional system, and second, the resulting degrees of freedom for the one-dimensional system are independent of one another. Both of these properties lead to an enormous simplification in the treatment of contact problems and a qualitative acceleration of numerical simulations.

The method of dimensionality reduction distinguishes itself by four essential properties: It is *powerful*, it is *simple*, it is *proven*, and it is *counterintuitive*. It is difficult to be convinced of its validity. Every expert in the field of contact mechanics who has not yet engaged himself in the detailed proofs of the reduction method would immediately form the opinion that it cannot possibly work. It appears to completely contradict a healthy intuition that a system with another spatial dimension, and furthermore, independent degrees of freedom can correctly agree with a three-dimensional system with spatial interactions. And nevertheless, it works! This book is dedicated to the reasons for and under which limitations this is the case.

In writing this book, we have followed two main goals. The first is the simplest possible presentation of the rules of application of the method. The second is to prove the assertions of the reduction method with strict mathematical evidence, so that even the most rigorous practitioner in contact mechanics may be convinced of the correctness of this method. We have attempted to keep these two goals separated. We attempted to keep the mathematical proofs to a minimum in the chapters in which the fundamentals of practical application are explained. This is primarily in Chap. 3, but also in the immediately following Chaps. (4–7) as well as Chap. 10, which is dedicated to the contact mechanics of rough surfaces. It is clear to us that this has not been possible at every point.

Above all, the method of dimensionality reduction offers the engineer a practical tool. In order to stress the practicality of the method even more, we have included many problems at the end of most chapters, which serve for a better understanding of the use of the reduction method and its areas of application. Therefore, this book can also be used as a textbook in a tribologically oriented course of studies.

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Chapter 10: Normal Contact of Rough Surfaces—together with R. Pohrt
Chapter 11: Frictional Force—together with S. Kürschner
Chapter 12: Frictional Damping—together with E. Teidelt
Chapter 13: The Coupling of Macroscopic Dynamics—together with E. Teidelt
Chapter 14: Acoustic Emission in Rolling Contacts—together with M. Popov und J. Benad
Chapter 15: Coupling on the Microscale—together with R. Pohrt
Chapter 19: Appendix 3: Replacing the Material Properties with Radok's Method of Functional Equations—together with S. Kürschner

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Berlin, May 2014

Valentin L. Popov
Markus Heß

Author Biographies

Prof. Dr. rer. nat. Valentin L. Popov studied physics (1976–1982) and obtained his doctorate in 1985 from the Moscow State Lomonosov University. He worked at the Institute of Strength Physics of the Russian Academy of Sciences. After a guest-professorship in the field of theoretical physics at the University of Paderborn (Germany) from 1999 to 2002, he has headed since 2002 the Department of System Dynamics and the Physics of Friction in the Institute of Mechanics at the Berlin University of Technology.

Address: Technische Universität Berlin, Institute of Mechanics, Sekr. C8-4, Str. des 17. Juni 135, 10623 Berlin, Germany.
E-mail: v.popov@tu-berlin.de

Dr. Markus Heß studied physical engineering sciences at TU Berlin. He obtained his doctorate in 2011, receiving in the same year the Research Prize from the German Tribological Society for his dissertation. Since 2011, he has worked for the Studienkolleg at TU Berlin, where he teaches engineering mathematics.

Address: Technische Universität Berlin, IC Studienkolleg, Sekr. EB15, Str. des 17. Juni 145, 10623 Berlin, Germany.
E-mail: markus.hess@tu-berlin.de

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Chapter 1
Introduction

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1.1 Goal of This Book

The goal of this book is to describe the *method of dimensionality reduction* in contact mechanics and friction. Contacts between three-dimensional bodies arise in a wide variety of applications. Therefore, their simulation, both analytically and numerically, are of major importance. From a mathematical point of view, contacts are described using integral equations having mixed boundary conditions. Furthermore, the stress distribution, the displacements of the surface, and even the shape of the contact domain are generally not known in such problems [1]. It is, therefore, astounding that a large number of classical contact problems can be mapped to one-dimensional models of contacts with a properly defined linearly elastic foundation (Winkler foundation). This means that the results of the one-dimensional model correspond *exactly* to those of a three-dimensional model. According to this mapping concept, solving contact-mechanical problems is trivialized is such a way as to require no special knowledge other than the fundamentals of algebra and calculus.

The healthy intuition of a specialist in contact mechanics completely discards the possibility of such an *exact* mapping of a three-dimensional problem with extensive interactions to the banal one-dimensional foundation of independent elements (spring or spring–dashpot combinations). Yet even the finest intuition must come to terms with mathematical truths: It has been rigorously proven mathematically for a large variety of contact problems that the one-dimensional mapping of three-dimensional contact problems results in an exact solution [2]. This book offers the required evidence for interested readers.

Just like every model, the method of dimensionality reduction has its domain of application. There are problems which cannot be exactly solved with this method as well as domains for which the method is not exact, but provides a very good approximation. Of course, there are also boundaries beyond which this

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method is no longer applicable. The method of dimensionality reduction provides exact solutions for normal and tangential contacts with *arbitrary axially-symmetric* bodies. Already here, the following argument may be voiced: "That may well be, but contact problems with axially-symmetric bodies can also be solved in three-dimensions. The method of dimensionality reduction does not present anything new!" This statement is fundamentally correct. However, the abundance of exact solutions for three-dimensional problems is strewn throughout the one-hundred year development of contact mechanics in hundreds of original publications. The authors of this book deal with contact mechanics on a daily basis and still we must admit that it took us months and years to gather and assemble the necessary solutions. The method of dimensionality reduction places this abundance of knowledge at the disposal of every engineer in a simple form, here and now, effectively and without reservation. It is, therefore, correct to say that "the method of dimensionality reduction is *nothing new* for axially-symmetric bodies." However, it reproduces the results of a three-dimensional contact problem exactly, thereby, lending itself to being a kind of pocket edition of contact mechanics.

We would like to add that many contact problems with axially symmetric bodies are solved "in principle," however, their application is extremely difficult when, for example, it comes to dynamic contacts. Also here, the method of reduction of dimensionality can be helpful, because due to its trivial formulation, it can be applied very easily either analytically or numerically and provides a convenient "thinking tool."

A second large field of application for this method is the contact between rough surfaces. Not all problems involving rough surfaces can be solved with the reduction method, but only those that deal with forces and relative displacements, such as problems dealing with contact stiffness, electrical or heat conduction, and frictional force. The area of application is limited but very large and includes many problems which have meaningful implications in engineering. There are no exact solutions when it comes to rough surfaces. Therefore, we are dependent on comparisons with three-dimensional numerical solutions for the purpose of verification. Due to the fact that this method is meant to be an engineering tool, it was very important for us to ensure its applicability for rough surfaces. For this purpose, extensive three-dimensional simulations of rough surfaces with elastic [3] and viscous [4] media were conducted in the Department of System Dynamics and the Physics of Friction at the Technische Universität Berlin. In doing this, we have investigated the entire spectrum of rough surfaces, from "white noise" to smooth single contacts (see Chap. 10). Over this span, the reduction method results in either an (asymptotically) exact solution or a very good approximation. Here as well, the book presents evidence to these findings.

The mapping of real contact area remains *beyond* the realm of application. The method of dimensionality reduction is able to map contact areas for the very short initial stage of indentation, but not in a general case [5].

With this book, we wish to introduce practitioners to methods which are in our opinion extremely simple, elegant, and effective. We are certain that they will find direct application in numerical simulation methods in the future.

The prospective primary application of this method lies not in the field where it yields exact solutions, but rather in the field of the contact mechanics and friction of rough surfaces. The most important advantage is the speed at which the calculations may be carried out, due to the one-dimensionality and the independent degrees of freedom. Therefore, it allows for a direct simulation of multi-scaled systems for which both the macroscopic system dynamics as well as the microscopic contact mechanics can be combined into one model.

1.2 Method of Dimensionality Reduction as the Link Between the Micro- and Macro-Scales

Since the classical works of Bowden and Tabor [6], it is generally known that the surface roughness has a deciding influence on tribological contacts. Without roughness, these contacts would have completely different properties. If this were the case, Coulomb's law would not even be approximately valid. Furthermore, adhesive forces would be orders of magnitude larger than those typically observed in macroscopic tribological systems. The world of atomically smooth surfaces exhibits an entirely different nature than that of the real world with rough surfaces! As early as the 1950s, it was determined that the roughness of real surfaces features a complicated multi-scaled structure, which can be characterized as being "fractal." Many physical surfaces (e.g., fractured or worn surfaces or surfaces produced using current technologies) have fractal characteristics, meaning they exhibit roughnesses on all scales from the atomic to the macroscopic. Above all, it became clear through the works of Archard [7] that this fractality has a significant influence on the properties of real contacts and is the actual cause for the approximate validity of Coulomb's law. Contact mechanics is, therefore, a multi-scaled phenomenon. This multi-scaled nature begs the question: Which methods can be used to take *all* relevant scales of a dynamic system into account? One of the possibilities consists of dividing the considered scales into three domains: Micro, Meso, and Macro. On the macroscopic scales, the system is simulated explicitly with a typical "monoscale method," for instance, finite element methods. On the smallest microscales, the approach remains the same as in the past and a "microscopic law of friction" is defined. This can either be determined through molecular-dynamic simulations or through empirical observation. The scales between micro and "macro" must somehow be bridged with a method which reproduces these scales with sufficient accuracy but is no more extensive than necessary so that the respective simulations are able to be carried out on realistic time scales. For this, the method of dimensionality reduction described in this book is an excellent candidate.