

Mathematical Engineering

Dieter Gerling

Electrical Machines

Mathematical Fundamentals of Machine
Topologies



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Electrical Machines

Mathematical Engineering

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Preface

Calculation and design of electrical machines and drives remain challenging tasks. However, this becomes even more and more important as there are increasing numbers of applications being equipped with electrical machines. Some recent examples well-known to the public are wind energy generators and electrical traction drives in the automotive industry. To realize optimal solutions for electrical drive systems it is necessary not only to know some basic equations for machine calculation, but also to deeply understand the principles and limitations of electrical machines and drives.

To foster the know-how in this technical field, this book *Electrical Machines* starts with some basic considerations to introduce the reader to electromagnetic circuit calculation. This is followed by the description of the steady-state operation of the most important machine topologies and afterwards by the dynamic operation and control methods. Continuously giving detailed mathematical deductions to all topics guarantees an optimal understanding of the underlying principles. Therefore, this book contributes to a comprehensive expert knowledge in electrical machines and drives. Consequently, it will be very useful for academia as well as for industry by supporting senior students and engineers in conceiving and designing electrical machines and drives.

After introducing Maxwell's equations and some principles of electromagnetic circuit calculation, the first part of the book is dedicated to the steady-state operation of electrical machines. The detailed description of the brushed DC-machine is followed by the rotating field theory, which in particular explains in detail the winding factors and harmonics of the magneto-motive force of distributed windings. On this basis, induction machines and synchronous machines are described. This first part of the book is completed by regarding permanent magnet machines, switched reluctance machines, and small machines for single-phase use.

Dynamic operation and control of electrical machines are the topics of the second part of this book, starting with some fundamental considerations. Next, the dynamic operation of brushed DC-machines and their control is described (in particular cascaded control using PI-controllers and their adjustment rules). A very important concept for calculating the dynamic operation of rotating field machines is the space vector theory; this is deduced and explained in detail in the following chapter. Then, the dynamic behavior of induction machines and synchronous machines follows, including the description of important control methods like field-

oriented control (FOC) and direct torque control (DTC). The permanent magnet machine with surface mounted magnets (SPM) or interior magnets (IPM) is explained concerning the differences of both in torque control and concerning the maximum torque per ampere (MTPA) control method. The last chapter gives an overview of latest research results concerning concentrated windings.

In spite of being a new contribution to a comprehensive understanding of electrical machines and the respective actual developments the reader may find parts of the contents even in different literature, as this book explains the fundamentals of electrical machines (steady-state and dynamic operation as well as control). Concerning these fundamentals it is nearly impossible to list all relevant literature during the text layout. Therefore, the most important references are given at the end of each chapter. In addition, parts of the lectures of Prof. H. Bausch (Universitaet der Bundeswehr Munich, Germany) and Prof. G. Henneberger (RWTH Aachen, Germany) were used as a basis.

The author deeply wishes to express his grateful acknowledgment to all team members of his Chair of Electrical Drives and Actuators at the Universitaet der Bundeswehr Munich and of the spin-off company FEAAM GmbH for their most valuable discussions and support. In particular this holds for (in alphabetical order) Dr.-Ing. Gurakuq Dajaku, Mrs. Lara Kauke, and most notably Dr.-Ing. Hans-Joachim Koebler. Without their beneficial contributions this book would not have been possible in such a high quality.

Last, but not least the author exceedingly thanks his wife and his daughters for their respectfulness and understanding not only concerning the effort being accompanied by writing this book, but even concerning the expenditure of time the author dedicates to professional activities.

Munich, April 2014

Dieter Gerling

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1 Fundamentals

1.1 Maxwell's Equations

1.1.1 The Maxwell's Equations in Differential Form

The basis for all following considerations are the Maxwell's equations. In differential form these are (the time-dependent variation of the displacement current $\vec{D}$ can always be neglected against the current density $\vec{J}$ for all technical systems regarded here):

1. Maxwell's equation

$$\text{rot}\vec{H} = \vec{J} + \frac{d\vec{D}}{dt} \approx \vec{J} \quad (1.1)$$

2. Maxwell's equation

$$\text{rot}\vec{E} = -\frac{d\vec{B}}{dt} \quad (1.2)$$

3. Maxwell's equation

$$\text{div}\vec{B} = 0 \quad (1.3)$$

4. Maxwell's equation

$$\text{div}\vec{D} = \rho \quad (1.4)$$

The material equations are:

$$\vec{B} = \mu\vec{H} \quad (1.5)$$

$$\vec{D} = \epsilon\vec{E} \quad (1.6)$$

$$\vec{J} = \gamma \vec{E} \quad (1.7)$$

The used variables have the following meaning:

- $\vec{H}$ the vector field of the magnetic field strength;
- $\vec{J}$ the vector field of the electrical current density;
- $\vec{D}$ the vector field of the displacement current;
- $\vec{E}$ the vector field of the electric field strength;
- $\vec{B}$ the vector field of the magnetic flux density;
- ρ the scalar field of the charge density;
- μ the scalar field of the permeability (in vacuum or air there is: $\mu = \mu_0$);
- ε the scalar field of the dielectric constant (in vacuum or air there is: $\varepsilon = \varepsilon_0$);
- γ the scalar field of the electric conductivity.

The expression “vector field” means that the vector quantity depends on all (usually three) geometric coordinates; the expression “scalar field” means that scalar quantity depends on all geometric coordinates.

In the case of homogeneous, isotropic materials the scalar fields μ , ε and γ are reduced to space-independent material constants.

1.1.2 The Maxwell's Equations in Integral Form

1.1.2.1 Ampere's Law (First Maxwell's Equation in Integral Form)

The first Maxwell's equation in integral form is

$$\oint \vec{H} d\vec{\ell} = \int_A \vec{J} d\vec{A} \quad (1.8)$$

The line integral of the magnetic field strength $\vec{H}$ on a closed geometric integration loop $\vec{\ell}$ (“magnetic circulation voltage”) is equal to the total electric current flowing through the area A limited by this loop (“magneto-motive force”, “ampere-turns”), if the displacement current is neglected.

For graphical explanation see Fig. 1.1.

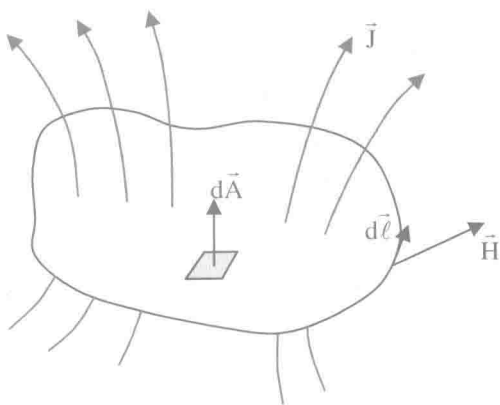


Fig. 1.1. Explanation of Ampere's Law.

1.1.2.2 Faraday's Law, Law of Induction (Second Maxwell's Equation in Integral Form)

The second Maxwell's equation in integral form is

$$\oint \vec{E} d\vec{\ell} = -\frac{d}{dt} \int_A \vec{B} d\vec{A} \quad (1.9)$$

with the magnetic flux being

$$\int_A \vec{B} d\vec{A} = \Phi \quad (1.10)$$

The line integral of the electric field strength $\vec{E}$ on a closed geometric integration loop $\vec{\ell}$ ("electric circulation voltage") is equal to the negative time-dependent variation of the total magnetic flux, that penetrates the area A limited by this loop.

For graphical explanation see Fig. 1.2.