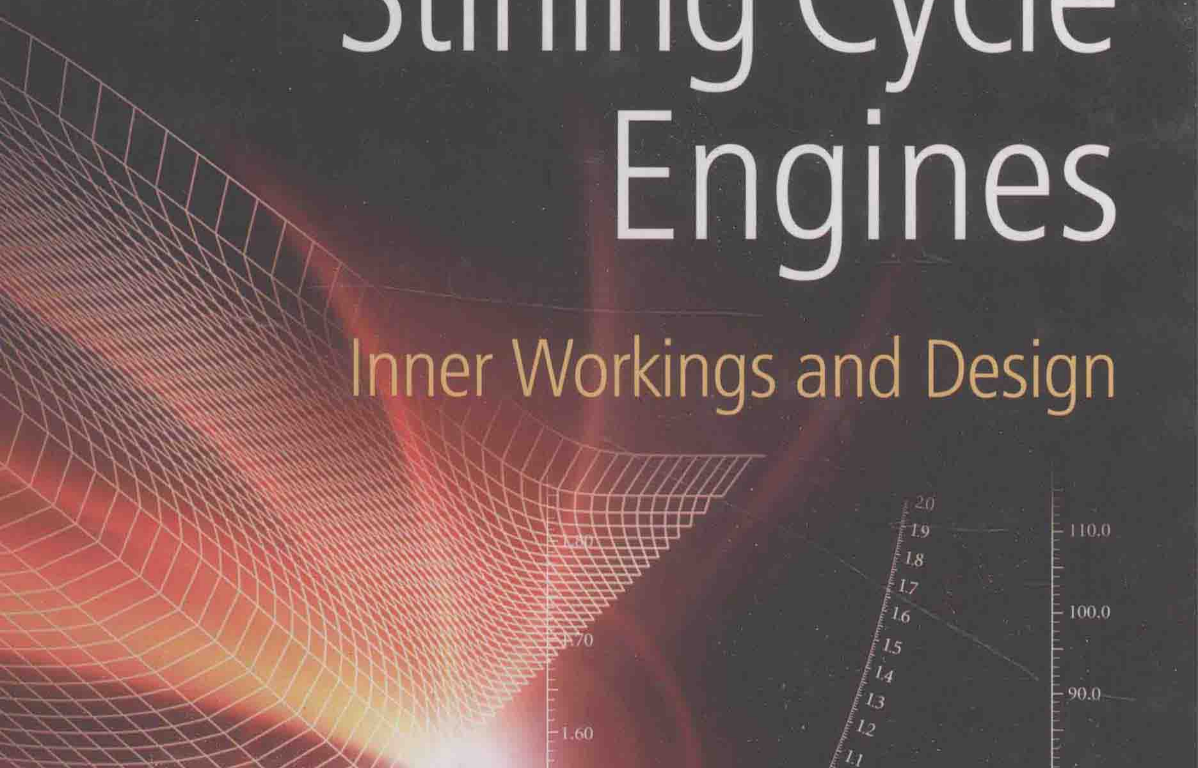
Stirling Cycle Engines

Inner Workings and Design



Allan J Organ

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STIRLING CYCLE ENGINES

About the Author

Apart from a six-year spell developing tooling for high-speed metal forming, the author's academic research career – which concluded with 20 years in Cambridge University Engineering Department where he learned more than he taught – has focused on Stirling cycle machines.

It would be gratifying to be able to claim that this commitment had resulted in his becoming an authority – perhaps *the* authority. However, it is a feature of the Stirling engine – perhaps its irresistible attraction – that it does not yield up its subtleties quite so readily. Thus the author writes, not as a self-styled expert, but as a chronic enthusiast anxious that the results of his most recent enquiries should not expire with him! It must now fall to a more youthful intellect to pursue matters.

With the 200th anniversary of the 1816 invention fast approaching, the author remains optimistic of a commercial future for a design targeting appropriate applications and undertaken with realistic expectations. Readers interested in this vision may wish to look out for a work of fiction entitled '*The Bridge*'.

Foreword

What is the source of the Stirling cycle's siren song? Why has it attracted so many for so long (including yours truly over some 30-plus years)? In part, it is simply that Stirling engines are not in common use, so that each idealist who stumbles upon some brief description of such, perceives that he has stumbled upon hidden gold, a gift to the world that needs only to be unwrapped and presented to a grateful society that will receive its promise of clean, quiet, reliable power. No valves, no timing, no spark, no explosions – what could be simpler? Arthur C. Clarke said "Any sufficiently developed technology is indistinguishable from magic". Surely, then, with just a bit of development, or better materials, or the keen insight of one who sees what others have missed, the Stirling engine will blossom into the fruition of its magical promise.

And yet. …

The first such machines were built two centuries ago, though only retrospectively identified with their eponymous creator, Rev. Robert Stirling. That naming came mostly by virtue of his invention of the "economizer" (what we now recognize as the regenerator in a reciprocating machine of this type), and was promoted most effectively by Rolf Meijer, who led the change from air-charged machines to light-gas, pressurized devices at Philips in the 1940s. Dr. Meijer might be said to be the father of the modern Stirling engine, as the earlier work did not have the benefits of thermodynamics as a science or modern materials like stainless steel.

And yet. …

It is remarkable that some 70 years after the Philips Stirling generator sets were produced, then abandoned as unprofitable, they remain one of the benchmarks against which novel attempts at Stirling engines of practical utility are measured. Few have succeeded in bettering their technical performance, and fewer still have achieved any greater commercial returns. Countless hobbyists, dozens of corporate ventures, and even a few large-scale government projects have come and gone.

And yet. …

Not one living person in a million today has seen, used, or been empowered by a Stirling engine. Why then, do we persist in our apparently sisyphean pursuit of this esoteric system? And commensurately, why is the present book important?

It has been noted that most diversity exists at the interfaces among ecosystems: that the junctions of field and forest, sea and shore, or sky and soil support more life in more forms than the depths of any one such domain. It is sure that these are the sites of evolution. As an inventor–instructor–entrepreneur, it has been apparent to me that a similar effect applies to intellectual pursuits: the greatest opportunities for development are to be found at the intersections of disciplines, rather than at their cores. Recent advances in combination fields such as mechatronics, evolutionary biology, and astrophysics might be evidence of some truth in this observation. Let us consider the Stirling engine in this light, the better to appreciate the value of this book.

Thermodynamics, heat transfer, fluid science, metallurgy, structural mechanics, dynamics of motion: all these and more are essential elements in Stirling embodiments and their mastery serves as arrows in the quiver of the developer aiming for Stirling success. Perhaps this is one reason why we are attracted to the Stirling – each finds his own expertise essential to it. And perhaps this is one reason why success is so hard to achieve, for who has all these skills at hand? And if success demands such polymath capacities, where is one to begin?

There are many published works on the broad topic of Stirling cycles, engines, and coolers, especially if the reader seeks descriptive or historical information. A selection of analytical texts can be found for those seeking guidance on the first-principles design of aspects of a new engine, including some worthy treatises on the numerical simulation of complete engines (or coolers). Yet none provides a technically sound, computationally compact path to buildable, valid engines.

This new work builds on the author's earlier focus on the essential regenerator and the application of similarity principles, validated by well-documented machines that serve here as a basis for scaling rules and the design of new engines for applications and operating conditions that superficially differ greatly from prior examples. It must be at those new conditions, perhaps at the intersections of conventional mechanics with micro-, or bio-, or other technologies, that new and evolved implementations of Stirling technology will arise and become, perhaps, successful. In this offering, Dr. Organ does the world of Stirling developers (and would-be Stirling magnates) a great service. For many times, new energy has been brought to this field and applied without reference to the experiences, successes, and failures of the past, here applied to great effect.

That tendency to dive in without a thorough grounding in the prior art is due only in part to the aforementioned siren song of Stirling and its addling effect on the newly captured. Such repetitive waste is also driven by the relative inaccessibility of much of the greater body of Stirling technical literature (e.g., I watched the published output of one famous free-piston company go through several 2-year cycles of re-inventions, as successive tiers of graduate students rotated through!).

The challenge is that, even when accessible as correct content, much of what is published in Stirling literature is either uselessly facile or excruciatingly partial in scope, so as to preclude its ready application to new designs. Tools to fit that job have not heretofore been available to those not willing or able to amass and absorb a gargantuan (if dross-filled) library of publications and apply that through associated years of experimental training. This book, through the author's elegant nomographic presentation – fully sustained by clear text and mathematical underpinnings – provides just that holistic entry point, presented with wit and minimal pain in calculations.

Hints of whole-physics participation in Stirling analysis abound here, not least in the dismissal of Schmidt models for their gross errors and oversimplifications that have led to conceptual misunderstandings and hampered many development efforts. I am particularly pleased at the refutation of long-standing shibboleths such as the "evil" of dead space; shattered here with clear and concrete constructions of the actual effects of dead space, and its value in the right places and amounts. In my own work of recent years, which has merged a long

Stirling experience with more recently developed thermoacoustic science, the key has been full consideration of the inertial properties of the cycle fluids, which is ignored by most Stirling models and simulations. Here, those effects are illustrated and their contributions to actual Stirling device behaviors discussed in a unique bridge between closed-form, analytical methods, and the full physics of numerical simulation, including a proper dressing-down and reformulation of the steady-flow correlations so often misapplied to this oscillatory system. The resulting graphics are both useful and beautiful.

Dr. Organ's offered tools, *FastTrack, FlexiScale, and ReScale* fulfill his promise of guiding the designer of a new Stirling engine to a safe island in the sea of possibilities. This traceable relation from technically successful engines of the past (although I am, of course, crushed that none of mine are among those cited), without the need to extract and refine the data from disparate sources elsewhere, opens the possibility of building a useful Stirling engine to a much larger population of aspirants. Perhaps by this means, some clever member thereof will at last find the sweet spot for commercial success; but at the very least, innumerable hours that would otherwise have been wasted in blind stabs can now be channelled into production refinements on a sturdy base. This is indeed a grand achievement, and being provided in so readable a volume is all the more so: a gift to the Stirling Community sure to be acclaimed throughout.

I am honored to call this author my friend and fellow explorer, and to introduce you, the reader, to this work with the certainty that if you have heard already that siren song of Stirling, this book by Allan Organ can lead you to safe harbor in plotting your course in response. Gentlefolk, Start Your Engines!

Dr John Corey

Preface

If the academic study of the Stirling engine began with Gustav Schmidt in 1861, then it has been more than a century and a half in the making. This might be deemed more than sufficient time to achieve its purpose – which must surely be to put itself out of a job.

A symbolic date looms: Tuesday 27 September 2016 – the bi-centenary of Stirling's application for his first patent – for his 'economizer' (regenerator). The Stirling engine will by then have been under development – admittedly intermittent development – for two centuries. Yet a would-be designer continues to be faced with the unhappy choice between (a) proceeding by trial and error and (b) design from thermodynamic first principles. Either course can be of indeterminate cost and duration.

In principle, nothing stands in the way of an approach to thermodynamic design which is both general and at the same time 'frozen' – general in the sense of coping with arbitrary operating conditions (*rpm*, charge pressure, working gas) and 'frozen' to the extent that thermodynamic design (numbers, lengths and cross-sectional dimensions of flow channels, etc.) are read from graphs or charts, or acquired by keying operating conditions and required performance into a lap-top computer or mobile phone 'app'. The possibility arises because, from engine to engine, the gas process interactions by which heat is converted to work have a high degree of *intrinsic similarity*. Physical processes which are formally similar are *scalable*: once adequately understood and rendered in terms of the appropriate parameters, no compelling reason remains for ever visiting them again.

Market prospects must surely be improved by relegating the most inscrutable – and arguably most daunting – aspect of the design of a new engine to a few minutes' work.

The present account is motivated by a vision along the foregoing lines. Utopia remains on the horizon, but there is progress to report:

> Wherever possible, working equations are reduced to three- and four-scale nomograms. The format affords better resolution and higher precision than the traditional $x - y$ plot, and allows a range of design options to be scanned visually in less time than it takes to launch equivalent software on a computer.
>
> New, independently formulated algorithms for thermodynamic scaling endorse the original method (*Scalit*) and increase confidence in this empirical approach to gas path design. The scaling sequence now reduces to the use of nomograms.
>
> There is a novel – and unprecedentedly simple – way of inferring loss per cycle incurred in converting net heat input to indicated work.
>
> Steady-flow heat transfer and friction correlations appear increasingly irrelevant to conditions in the Stirling engine. Attention shifts to the possibility of correlating *specific thermal load* per

> tube against a *Reynolds parameter* for the multi-tube exchanger assembly tested *in situ*. Results are promising. Given that they derive from Stirling engines under test, relevance to the unsteady flow conditions is beyond question.
>
> Progress has resulted from noting that the context does not call for a comprehensive picture of regenerator transient response: the interests of the mathematician do not coincide with the realities and requirements of satisfactory engine operation. Focusing on the relevant margin of the potential operating envelope allows respective temperature excursions of gas and matrix to be explored independently.
>
> The kinetic theory of gases is mobilized in an attempt to dispense once for all with the suspect steady-flow flow correlations. To convey the resulting insights calls for animated display. This is not yet a feature of the conventional hard-copy volume. Selected still frames give an impression.

(Full exploitation of the kinetic theory formulation awaits the next generation of computers, so another book looms. Another book – another preface!)

> Certain entrenched tenets of thermodynamic design have been found to be faulty. These are remedied.
>
> Versatility and utility of the design charts (nomograms) have been enhanced: The range of equations susceptible to traditional methods of construction is limited. The technique of 'anamorphic transformation' has been corrected relative to published accounts, and now allows display of functions of the form $w = f(u,v)$ in nomogram form. Function w can be the result of lengthy numerical computation. (The display remains confined to the range over which the variation in w is monotonic.)
>
> The 'hot-air' engine receives a measure of long-overdue attention.

The writer has benefited from long hours of dialogue with Peter Feulner, with Geoff Vaizey, with Camille van Rijn, with Peter Maeckel and with R G 'Jimmy' James. The influence of Ted Finkelstein endures.

Constructive criticism is always welcome at *allan.j.o<at>btinternet.com*.

This material appears in print thanks largely to a unique combination of persistence, patience and diplomacy applied over a period of 18 months by Eric Willner, executive commisioning editor at John Wiley & Sons. The project has since become reality in the hands of Anne Hunt, associate commissioning editor and Tom Carter, project editor.

Notation

Variables Having Dimensions

A_{ff}	free-flow area	m^2
A_w	wetted area	m^2
b	width of slot	m
c_v, c_p	specific heat at constant volume, pressure	J/kgK
d_x	internal diameter of heat exchanger duct	m
D	inside diameter of cylinder, or outside diameter of displacer, as per context	m
e	désaxé offset	m
f	cycle frequency ($= \omega/2\pi$)	s^{-1}
g	radial width of annular gap	m
h	coefficient of convective heat transfer	W/m^2 K
	specific enthalpy	J/kgK
H	enthalpy	J
H_C	clearance height	m
k	thermal conductivity	W/mK
L_d	axial length displacer shell	m
L_{ref}	reference length $V_{sw}^{1/3}$	m
L_x	length of heat exchanger duct	m
m	mass (of gas)	kg
M	total mass of gas taking part in cycle	kg
M_w	mass of matrix material	kg
p_{ref}	reference pressure – max/min/mean cycle value	Pa
p_w	wetted perimeter	m
Q	heat	J
r	linear distance coordinate in radial direction	m
	radius – e.g., of crank-pin offset	m
r_h	hydraulic radius	m
q'	heat rate	W
q''	heat rate per unit length of exchanger	W/m
R	specific gas constant	J/kgK
S_p, S_d	stroke of work piston and displacer respectively	m
T_E, T_C	temperatures of heat source and sink	K

T_w	temperature of solid surface	K
T	temperature of gas	K
t	time	s
	thickness in radial coordinate direction	m
u	velocity in x coordinate direction	m/s
	specific internal energy	J/kgK
U	internal energy	J
V_{sw}	swept volume	m^3
W	work	J
W'	work rate	W
X, Y	linear distances in x and y coordinate directions	m
z	linear offset in kinematics of crank-slider mechanism	m
α	thermal diffusivity $k/\rho c_p$	m^2/s
$\underline{\varepsilon}_T$	mean temperature perturbation or 'error' $\underline{T - T_w}$	K
μ	coefficient of dynamic viscosity	Pas
ρ	density	kg/m^3
ω	angular speed	s^{-1}

Dimensionless Variables

a, b, c, d	coefficients and indices as required	-
a	coefficients of linear algebraic equations	-
C	numerical constant (as required)	-
CI	'cycle invariant' defined at point of use	-
C_f	friction factor $\Delta p/\tfrac{1}{2}\rho u^2$	-
DG	dimensionless group defined at point of use	-
Ma	Mach number $u/\sqrt{\gamma RT}$	-
n	polytropic index: n_e expansion phase; n_c expansion phase	-
n_{Tx}	number of exchanger tubes	-
N_B	specific cycle work: power/$fV_{sw}p_{ref}$	-
N_F	Fourier modulus $\alpha/\omega {r_0}^2$	-
N_{FL}	Flush ratio: ratio of mass of gas per uni-directional blow to instantaneous mass of gas in regenerator void volume.	-
N_{MA}	characteristic Mach number $\omega L_{ref}/\sqrt{RT_C}$	-
N_{Nu}	Nusselt number hr_h/k	-
N_{RE}	characteristic Reynolds number $N_{SG}{N_{MA}}^2$	-
N_{SG}	Stirling parameter $p_{ref}/\mu\omega$	-
N_T	characteristic temperature ratio T_E/T_C	-
N_{TCR}	thermal capacity ratio $\rho_w c_w T_C/p_{ref}$	-
NTU	<u>N</u>umber of <u>T</u>ransfer <u>U</u>nits $StL_x/r_h = hT_C/\omega p_{ref}S$ in Carnot cycle study	-
P	parameter in eq'n. which relates L_x/d_x to L_x/L_{ref}	-
$P(\varphi)$, $Q(\varphi)$	consolidated coefficients of first-order differential equation	-
Pr	Prandtl number $\mu c_p/k$	-
QI	'quasi-invariant' defined at point of use	-
r_v	volumetric compression ratio (e.g., V_1/V_3 of Carnot cycle)	-
r_p	pressure ratio p_{max}/p_{min}	-

Re	Reynolds number $4\rho\lvert u\rvert\mu/r_h$	-
RE_ω	Reynolds number characteristic of exchanger operation over a cycle	-
S	linear scale factor $L_d/L_p = L_{derivative}/L_{prototype}$	-
Sg	Stirling number $pr_h/\mu\lvert u\rvert$ (see Stirling *parameter* above)	-
St	Stanton number $h/\rho\lvert u\rvert c_p$	-
TCR	net thermal capacity ratio (Chapter 17)	-
U( )	step function used in ideal adiabatic cycle	-
x	numerical scale factor	-
x, y, z	cartesian coordinates	-
XQ_x	specific thermal load on exchanger assembly	-
XQ_{XT}	specific thermal load on individual tube of exchanger assembly	-
Z	work quantity (e.g., loss per cycle) normalized by MRT_C or by $p_{ref}V_{sw}$	-
α	phase advance of events in expansion space over those in compression space	-
β	phase advance of displacer motion over that of work piston	-
γ	isentropic index – specific heat ratio	-
Δ	any finite difference or change	-
ΔT	temperature difference $T - T_w$	-
δ	dimensionless dead space V_d/V_{sw}	-
φ	crank angle = ωt	-
θ	angular coordinate in circumferential direction	-
	angle through which coordinate frame is rotated in process of dealing with molecular collision	-
κ	thermodynamic volume ratio V_C/V_E	-
λ	ratio of volume swept by work piston to that swept displacer $\approx S_p/S_d$ in parallel-cylinder, coaxial 'beta' machine	-
Λ	Hausen's 'reduced length' – equivalent to NTU	-
ν	Finkelstein's dimensionless dead space $2\Sigma\upsilon_i T_i/T_C$	-
ψ	specific pressure p/p_{ref}	-
Π	Hausen's 'reduced period' $hA_w L_r/fM_w c_w$	-
ρ	composite dimensionless function arising in Finkelstein's formulation of Schmidt analysis: $\sqrt{\{N_T^{-2} + \kappa^2 + 2\kappa\cos(\alpha)/N_T\}}/(N_T^{-1} + \kappa + \nu)$	-
σ	specific mass $mRT_C/p_{ref}V_{sw}$	-
σ'	specific mass rate $d\sigma/d\varphi = mRT_C/\omega p_{ref}V_{sw}$	-
ς	dimensionless length x/L_{ref}	-
τ	Finkelstein's temperature ratio $T_C/T_E = N_T{}^{-1}$	-
υ	Finkelstein's proportional dead space V_{di}/V_E	-
Σ	total inventory of specific mass $MRT_C/p_{ref}V_{sw} = \sigma_e + \sigma_c + \nu\psi$ – assumed invariant	-
$¶_v$	volume porosity	-

Subscripts

$_{comb}$	combustion
$_C$	relating to compression or to compression space
$_d$	relating to displacer

env	'envelope' of overlapping displacements of piston and displacer in coaxial, 'beta' machine
exp	experimental
E	relating to expansion or to expansion space
f	relating to flow friction
ff	free-flow (area)
i	in or inlet
j,k	array subscripts
L	lost – as in lost available work
o	out or outlet
p	relating to piston
q	relating to heat transfer
r	regenerator
ref	reference value of variable
rej	(heat) rejected
sw	swept (volume)
w	wetted (area), wall or wire, as per context
x	exchanger: $_{xe}$ – expansion exchanger; $_{xc}$ – compression exchanger
∞	free-stream

Superscripts

deriv	derivative design of scaling process
prot	relating to specification of prototype of scaling process
T	relating to the new (transposed) reference frame
+	extra or additional (dead space)

Contents