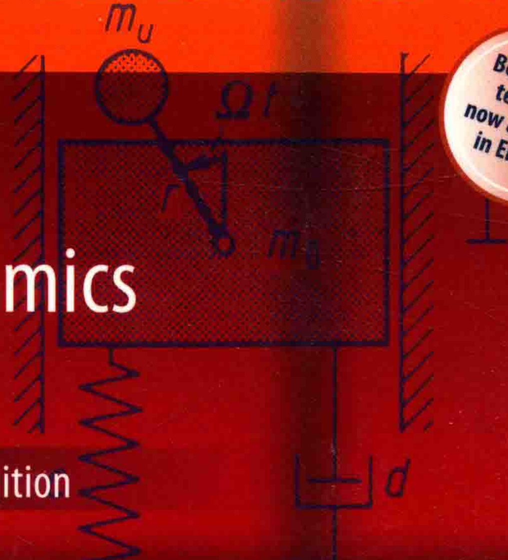
Dietmar Gross · Werner Hauger
Jörg Schröder · Wolfgang A. Wall
Sanjay Govindjee

Engineering Mechanics 3

Dynamics



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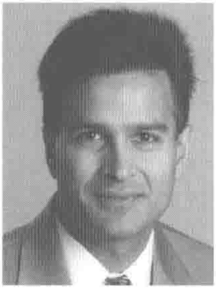

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Preface

Dynamics is the third volume of a three-volume textbook on Engineering Mechanics. Volume 1 deals with *Statics*; Volume 2 contains *Mechanics of Materials*. The original German version of this series is the bestselling textbook on Engineering Mechanics in German speaking countries; its 12th edition has just been published.

It is our intention to present to engineering students the basic concepts and principles of mechanics in the clearest and simplest form possible. A major objective of this book is to help the students to develop problem solving skills in a systematic manner.

The book developed out of many years of teaching experience gained by the authors while giving courses on engineering mechanics to students of mechanical, civil and electrical engineering. The contents of the book correspond to the topics normally covered in courses on basic engineering mechanics at universities and colleges. The theory is presented in as simple a form as the subject allows without being imprecise. This approach makes the text accessible to students from different disciplines and allows for their different educational backgrounds. Another aim of the book is to provide students as well as practising engineers with a solid foundation to help them bridge the gaps between undergraduate studies, advanced courses on mechanics and practical engineering problems.

A thorough understanding of the theory cannot be acquired by merely studying textbooks. The application of the seemingly simple theory to actual engineering problems can be mastered only if the student takes an active part in solving the numerous examples in this book. It is recommended that the reader tries to solve the problems independently without resorting to the given solutions. In order to focus on the fundamental aspects of how the theory is applied, we deliberately placed no emphasis on numerical solutions and numerical results.

We gratefully acknowledge the support of Dr.-Ing. Vera Ebbing and the cooperation of the staff of Springer who were responsive to our wishes and helped to create the present layout of the books.

Darmstadt, Essen, München, and Berkeley, Spring 2014

D. Gross
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Introduction

The primary task of mechanics is the description and prediction of the motion of bodies along with the associated forces. The subject of mechanics can be broken into the disciplines of *statics* and *dynamics*. The subject of statics is the study of bodies in equilibrium. Dynamics, on the other hand, deals with bodies in motion. It is further sub-divided into the subjects of *kinematics* and *kinetics*. Kinematics is the study of the geometry and time evolution of motion independent of the forces causing the motion, while kinetics concerns itself with the interplay between forces and motion.

Statics as a subject has its origin in antiquity. Dynamics, in comparison, is a much younger discipline. The first systematic studies in dynamics were undertaken by Galileo Galilei (1564-1642). With the help of a series of brilliantly designed experiments, he was able to determine the laws of motion governing bodies in free fall and those of projectiles, as well as the law of inertia in 1638. To fully appreciate Galilei's achievements, one should note that differential and integral calculus were unknown in his time and instruments to precisely measure time were non-existent.

The scientific foundations for dynamics were laid down by Isaac Newton (1643-1727), who in 1687 formulated what we now know as *Newton's Laws of Motion*. Newton's Laws were able to accurately explain all experimental evidence at that time and the conclusions drawn from them have been confirmed to accurately predict the motion of all macroscopic bodies. We will treat these laws as axiomatic in character – they are not subject to mathematical proof.

Before we can study the interplay of forces and motion, it will be useful to first consider the purely geometric aspects of motion (kinematics). In this regard, we will carefully introduce the notions trajectory, velocity, and acceleration. Depending upon the type of motion (e.g. rectilinear, planar, or three-dimensional) we will describe these concepts using a variety of variables and coordinate systems. Our point of departure for the study of dynamics will be Newton's Laws of Motion. We will restrict our attention to the study of point masses and rigid bodies. With the help of these idealizations, we will see that we can effectively treat a wide variety of complex technical problems and arrive at useful solutions.

Newton's Laws of Motion are valid only in *inertial frames of reference*. However, it is often more convenient to formulate problems relative to *moving* frames of reference. In this regard, we will also briefly treat the topic of relative motion.

Newton's Laws of Motion are equivalent to the so-called *principles of mechanics* – the virtual power or work principles. In the solution of some problems it is useful to employ these alternate forms of the fundamental laws. We will restrict ourselves to the presentation of d'Alembert's principle and Lagrange equations of the 2nd type.

In the study of dynamics we will reuse many concepts we have already introduced in the study of statics, e.g. space, mass, force, moment, and idealizations such as point masses, rigid bodies, and point forces. Fundamental concepts from statics such as section cuts, the action-reaction law, and the force parallelogram law will also be employed. In the solution of concrete problems, we will also see that free-body diagrams will play a central role, just as they did in the study of statics. For the study of motion, we will further see that we will have to introduce a new fundamental variable, time, which was unnecessary in statics. With the introduction of time, we will find the need to define new dynamical concepts (e.g. velocity, acceleration, impulse, kinetic energy) and dynamical laws (e.g. impulse law and the work-energy theorem); it is with these concepts and related ideas that we will occupy ourselves in the chapters to follow.

Chapter 1

Motion of a Point Mass

1

1 Motion of a Point Mass

Objectives: We will first learn how one describes the motion of a point mass by its position, velocity, and acceleration in different coordinate systems and how such quantities can be determined. Subsequently, we will concern ourselves with the equations of motion, which prescribe the relation between forces and motion. An important role will again be played by the free-body diagram with whose help we will be able to properly derive the equations of motion. Further, we will discuss important physical concepts such as momentum, angular momentum, and work-laws and their applications.

1.1 Kinematics

1.1.1 Velocity and Acceleration

The subject of kinematics is the description of motion in space. Kinematics can be thought of as the geometry of motion independent of the cause of the motion.

The position of a point mass M in space is given by a point P and is uniquely described by its *position vector* $\boldsymbol{r}$ (Fig. 1.1a). This vector shows the momentary or instantaneous location of M relative to a fixed reference point in space, 0. If M changes location with time t, then $\boldsymbol{r}(t)$ describes the *trajectory* or *path* of M.

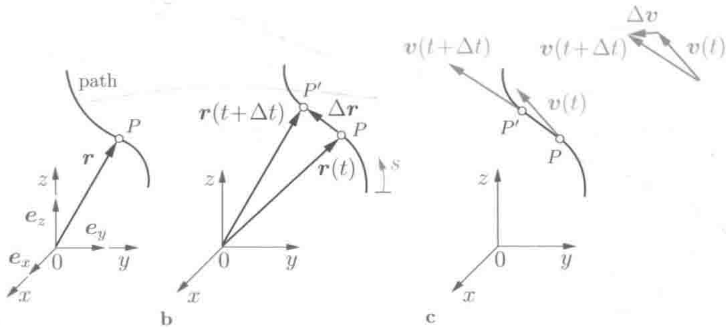

Fig. 1.1

Let us now consider two neighboring locations for M, P and P', at two different times t and $t+\Delta t$ (Fig. 1.1b). The change in the position vector over the time interval Δt is given by $\Delta\boldsymbol{r} = \boldsymbol{r}(t+\Delta t) - \boldsymbol{r}(t)$. The *velocity* of M is defined as the limit of the change in position with respect to time:

$$\boldsymbol{v} = \lim_{\Delta t\to 0}\frac{\boldsymbol{r}(t+\Delta t)-\boldsymbol{r}(t)}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta\boldsymbol{r}}{\Delta t} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \dot{\boldsymbol{r}}\,. \quad (1.1)$$

Thus, the velocity $\boldsymbol{v}$ is the time derivative of the position vector $\boldsymbol{r}$. We will usually denote time derivatives with a superposed dot.

Velocity is a vector. Since the change of the position vector, $\Delta\boldsymbol{r}$, in the limit as $\Delta t \to 0$ points in the direction of the tangent to the trajectory of M, the velocity is always *tangent* to this curve. The velocity points in the direction that the mass traverses the path in space. In order to determine the magnitude of the velocity vector, we introduce the *arc-length* s as a measure of distance covered by M along its trajectory. Assume that the mass has moved a distance s up to the location P and a distance $s + \Delta s$ up to the location P'. With $|\Delta \boldsymbol{r}| = \Delta s$, one obtains from (1.1) the *speed*

$$|\boldsymbol{v}| = v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{\mathrm{d}s}{\mathrm{d}t} = \dot{s}\,. \tag{1.2}$$

Velocity and speed have dimensions of distance/time and are often measured in units of m/s. The units of km/h, which are used in transportation applications, are related as $1\,\mathrm{km/h} = \frac{1000}{3600}$ m/s $= \frac{1}{3.6}$ m/s or 1 m/s = 3.6 km/h.

In general, velocity depends on time. In neighboring positions P and P' (Fig. 1.1c) the considered point mass has velocities $\boldsymbol{v}(t)$ and $\boldsymbol{v}(t + \Delta t)$. Thus, the change in the velocity is given by $\Delta \boldsymbol{v} = \boldsymbol{v}(t + \Delta t) - \boldsymbol{v}(t)$. The *acceleration* is defined as the limit of this change with respect to time:

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\boldsymbol{v}(t + \Delta t) - \boldsymbol{v}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \dot{\boldsymbol{v}} = \ddot{\boldsymbol{r}}\,. \tag{1.3}$$

Thus the acceleration $\boldsymbol{a}$ is the first derivative of $\boldsymbol{v}$ and the second derivative of $\boldsymbol{r}$. Acceleration is a vector. But since $\Delta \boldsymbol{v}$ (see Fig. 1.1c) does not have an obvious relation to the trajectory, we can not easily make statements about its direction and magnitude. Acceleration has dimensions of distance/time^2 and is often measured in units of m/s^2.

Velocity and acceleration have been introduced independent of a coordinate system. However, to solve specific problems, it is useful to introduce particular coordinates. In what follows, we will consider three important coordinate systems.

1.1.2 Velocity and Acceleration in Cartesian Coordinates

If we want to describe motion in Cartesian coordinates, we can choose 0 as the origin of a fixed (in space) system x, y, z. With unit vectors (basis vectors) $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ in the three coordinate directions (Fig. 1.1a), the position vector is given as

$$\boldsymbol{r}(t) = x(t)\,\boldsymbol{e}_x + y(t)\,\boldsymbol{e}_y + z(t)\,\boldsymbol{e}_z\,. \tag{1.4}$$

This is a parametric description of the trajectory with t as the parameter. If one can eliminate time from the three component relations in (1.4), then one has a time independent geometric description of the trajectory (cf. e.g. Section 1.2.2).

Using (1.1), one finds the velocity via differentiation (the basis vectors do not depend on time):

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = \dot{x}\,\boldsymbol{e}_x + \dot{y}\,\boldsymbol{e}_y + \dot{z}\,\boldsymbol{e}_z\,. \tag{1.5}$$

Further differentiation gives the acceleration as

$$\boldsymbol{a} = \dot{\boldsymbol{v}} = \ddot{\boldsymbol{r}} = \ddot{x}\,\boldsymbol{e}_x + \ddot{y}\,\boldsymbol{e}_y + \ddot{z}\,\boldsymbol{e}_z\,. \tag{1.6}$$

Thus the components of the velocity and acceleration in Cartesian coordinates are given as

$$\begin{aligned} v_x &= \dot{x}, & v_y &= \dot{y}, & v_z &= \dot{z}, \\ a_x &= \dot{v}_x = \ddot{x}, & a_y &= \dot{v}_y = \ddot{y}, & a_z &= \dot{v}_z = \ddot{z}. \end{aligned} \tag{1.7}$$

The magnitudes follow as

$$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \quad \text{and} \quad a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}. \tag{1.8}$$

1.1.3 Rectilinear Motion

Rectilinear motion is the simplest form of motion. Even so, it has many practical uses. For example, the free fall of a body in the