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T.R. Hurd

Contagion! Systemic Risk in Financial Networks

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Preface

This slim volume logs the development of a cascade of contagious ideas that has occupied my space, time and mind in recent years. There was a clear triggering event that occurred in April 2009. Late in that month, Michael Lynch and his colleagues at MITACS Canada brought together a host of scientists, mathematicians and finance industry participants for three days to brainstorm about underlying causes of the ongoing financial crisis and how mathematical thinking could be brought to bear on it. My role there was as gadfly to provoke discussion on a special topic no one at the meeting was very aware of, namely financial systemic risk.

Since that event introduced me to the subject, I have had many opportunities to present to a diversity of audiences an evolving view of how the architecture of the financial system can be described in terms of network science, and how such a network formulation can be made amenable to a certain type of mathematical analysis. This book is not intended to be a definitive work on the subject of financial systemic risk, and does not try to represent a broad consensus. Instead, it is a personal attempt to crystallize the early results of research that focusses on the basic modelling structure that ensures some kind of mathematical tractability, while allowing a great deal of both reality and complexity in the actual finance network specification. I owe a debt of thanks to a great number of people, especially graduate students and research colleagues, who have listened, commented and added new nodes to this complex network of ideas.

My McMaster colleague, Matheus Grasselli, was instrumental in many ways, not least in providing the original impetus to write this SpringerBrief. Nizar Touzi encouraged and supported me in my first attempt at delivering a minicourse on systemic risk. The scope of this minicourse grew over time: Jorge Zubelli hosted me for an extended period at IMPA, where I delivered another version; Peter Spreij arranged a session for me to speak at the Winter School on Financial Mathematics in Lunteren; James Gleeson provided me with multiple invitations to Limerick. The Fields Institute for Research in Mathematical Sciences gave me encouragement and organized multiple events relevant to my work. The Global Risk Institute for Financial Services, in particular Michel Maila and Catherine Lubochinsky, has

provided substantial financial and moral support for this research. I give my hearty thanks to Mario Wüthrich and Paul Embrechts who hosted my extended stay at ETH Zürich in 2014 where I was extremely fortunate to be able to deliver a Nachdiplom lecture series based on the material contained in this book. Finally, to my wife, Rita Bertoldi, I offer my affectionate acknowledgment of her patient support throughout my lengthy exposure to this dangerous contagion.

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Chapter 1

Systemic Risk Basics

Annual income twenty pounds, annual expenditure nineteen nineteen six, result happiness. Annual income twenty pounds, annual expenditure twenty pounds ought and six, result misery. The blossom is blighted, the leaf is withered, the god of day goes down upon the dreary scene, and—and, in short, you are for ever floored (Charles Dickens, *David Copperfield*, Chap. 12, p. 185 (1950). First published 1849–1850.).

Abstract Attempts to define systemic risk are summarized and found to be deficient in various respects. This introductory chapter, after considering some of the salient features of financial crises in the past, focusses on the key characteristics of banks, their balance sheets and how they are regulated.

Keywords Systemic risk definition • Financial stability • Balance sheet • Contagion channel • Macroprudential regulation

Bankruptcy! Mr. Micawber, David Copperfield's debt-ridden sometime mentor, knew first hand the difference between surplus and deficit, between happiness and the debtors' prison. In Dickens' fictional universe, and perhaps even in the real world of Victorian England, a small businessman's unpaid debts were never overlooked but always lead him and his loved ones to the unmitigated misery of the poorhouse. On the other hand, it seems that an aristocrat would usually escape from his debtors to the comforts of his dining club.

For people, firms, and in particular banks, bankruptcy nowadays is more complicated yet still retains some of the flavour of the olden days. When a bank fails, it often seems that the rich financiers responsible for its collapse and the collateral damage it inflicts walk away from the wreckage with intact compensation packages and bonuses. When a particularly egregious case arises and a scapegoat is needed, a middle rank banker is identified who takes the bullet for the disaster. A cynic might say that despite the dictates of Basel I, II, III, . . . ∞ , bank executives remain free to take excessive risks with their company, receiving a rich fraction of any upside while insulating themselves from any possible disaster they might cause.

As we learn afresh during every large scale financial crisis, society at large pays the ultimate costs when banks fail. Spiking unemployment leads to the poverty of the less well-to-do, while salary freezes and imploded pension plans lead to belt-tightening and delayed retirement for the better-off. Those at the top of the pile, even those responsible, often do just fine. Banks that are too big to fail are propped up, while failed banks are bailed out by governments, their debts taken over and paid by the taxpayers.

If anything is different since the crisis of 2007–2008, perhaps it is the widespread recognition that society needs to find ways and means to ensure that the responsible parties pay the downside costs of bank failure. New ideas on bank resolution, including contingent capital and bail-in regulation, aim to force the financial stakeholders, not the central bank, to pay much higher fractions of the costs of failure. Banks' creditors, bondholders and equity investors should in the future be forced to take their fair share of losses. When banking incentives and regulation are better aligned with the needs of society, we might hope bank failures will be better anticipated, prepared for and managed to reduce their most catastrophic social consequences.

1.1 The Nature of This Book

The title “Contagion! Systemic Risk in Financial Networks” is intended to suggest that financial contagion is analogous to the spread of disease, and that damaging financial crises may be better understood by bringing to bear ideas gained from studying the breakdown of other complex systems in our world. It also suggests that the aim of systemic risk management is similar to the primary aim of epidemiology, namely to identify situations when contagion danger is high, and then to make targeted interventions to damp out the risk.¹

The primary goal of this book is to present a unified mathematical framework for the transmission channels for damaging shocks that can lead to instability in financial systems. Models in science and engineering can usually be described as either explanatory or predictive. In the early stages of research in a field, explanatory models may make dramatic oversimplifications or counterfactual assumptions that are only justifiable to the extent they highlight and explain the most critical mechanisms underlying the phenomenon of interest. Later, when guided by such improvements in understanding, predictive models become feasible. Certainly, predictive models will be more complex, and must be carefully calibrated to the details of the observed system in question. Since financial systemic risk is a rather new field, this book focusses on certain explanatory models developed by economists that aim to explore how disruptions can arise in large financial systems. We will therefore make certain

¹Interestingly, I found on Wikipedia that epidemiology has a code of nine principles, called the “Bradford Hill criteria”, that should be considered to help assess evidence of a causal relationship between an incidence and a consequence. Perhaps, researchers can codify an analogous set of principles for assessing systemic risk.

dramatic oversimplifications in the hope of gaining mathematical clarity and analytic tractability that can improve understanding of the different ways financial instability can arise.

This introductory chapter will develop the concepts and setting for systemic risk in financial networks. It provides a brief survey of how people have viewed and defined financial crises and systemic risk. It looks at how banks' balance sheets reflect the type of business they deal with, and the ways adverse shocks between banks can be transmitted and amplified. Finally, we review the key aspects of the new international regulatory regime for banks that is designed to safeguard global financial stability.

From Chap. 2 onwards, we delve more deeply into the mechanics of the interactions between banking counterparties. Chapter 2 puts a sharp focus on the type of bank behaviour that can negatively impact the functioning of the entire system, by surveying, dissecting and classifying a number of economic models for financial contagion that have been proposed in recent years. We will make the important discovery that a common mathematical structure unifies a variety of financial cascade mechanisms, namely such crises proceed through cascade mappings that approach a cascade equilibrium. To address the intrinsic opacity of financial institutions and their interconnections, we identify a particular point of view developed by Gai and Kapadia [44], Amini et al. [7] and others that argues for the usefulness of *random financial networks*, a statistical representation of networks of banks, their interconnections and their balance sheets. The design of this concept reflects the type of models that network science, reviewed in the book [73], has already developed in other domains.

The remainder of the book is devoted to studying cascade models on large random financial networks. Chapter 3 provides the mathematical underpinning we need by developing and adapting the theory of random graphs which describes the *skeleton* structure at the heart of the random financial network. Two distinct classes of random graphs, the Assortative Configuration Graph model and the Inhomogeneous Random Graph model, are characterized in detail by their stochastic construction algorithms. The first class, which will form the framework underlying the cascade channels studied in the remaining chapters, is an extensive generalization of the well-known configuration graph model that incorporates *assortative wiring* between nodes that represent banks, which means wiring probabilities depend on banks' degree. It has not been well studied before so we spend time to develop its key mathematical properties, the most important of which we call the *locally tree-like property*. The second class of random graph will in principle enable the meaning of nodes to represent *types* of financial institutions other than banks, such as asset classes or hedge funds. Chapter 4 is devoted to understanding the relation between the Watts 2002 model of information cascades [85] and the concept of bootstrap percolation in random networks, studied recently in [10]. The Watts model will be fully analyzed from first principles, providing us with a template for results on more specific cascade mechanisms on financial networks. We shall learn that its properties can be determined using the mathematics of *percolation*, the theory of the size distribution of connected network components. Chapter 5 returns to focus on the zero recovery default cascade mechanism introduced by Gai and Kapadia [44]. It develops a purely

analytical method for computing the large network asymptotics of cascade equilibria, based on the locally treelike property of assortative configuration graphs. The main theorem on the asymptotic form of the default cascade extends the work of Amini, Cont and Minca in certain respects, and requires new proof techniques not previously developed. This theory provides us with a computational methodology that is independent of and complementary to the usual Monte Carlo simulation techniques used everywhere in network science. Finally in Chap. 6 we indicate some of the ways this theory can be extended to encompass more complex contagion channels.

Do there exist classes of mathematical systemic risk models that provide a degree of realism, but at the same time are sufficiently tractable that all critical parameters can be varied at will and resulting network characteristics computed? Can these model systems be tested for their resilience and stability in all important dimensions? Are the mathematical conclusions robust and relevant to the real world of financial crisis regulation? We hope this book will be viewed as providing an emphatic “YES” in answer to these questions.

1.2 What Is Systemic Risk?

First it is helpful to identify what systemic risk is not. Duffie and Singleton [34] identify five categories of risk faced by financial institutions: (i) market risk: the risk of unexpected changes in market prices; (ii) credit risk: the risk of changes in value due to unexpected changes in credit quality, in particular if a counterparty defaults on one of their contractual obligations; (iii) liquidity risk: the risk that costs of adjusting financial positions may increase substantially; (iv) operational risk: the risk that fraud, errors or other operational failures lead to loss in value; (v) systemic risk: the risk of market wide illiquidity or chain reaction defaults. To the extent that the first four risk categories are focussed on individual institutions, they are not deemed to be systemic risk. However, each of the four also has market wide implications: such market wide implications are wrapped up into the fifth category, systemic risk.

Kaufman and Scott [61], John B. Taylor [81] and others all seem to agree that the concept of systemic risk must comprise at least three ingredients. First, a triggering event. Second, the propagation of shocks through the financial system. And third, significant impact of the crisis on the macroeconomy. Possible triggers might come from outside the financial system, for example a terrorist attack that physically harms the system. Or triggers might come internally, such as the surprise spontaneous failure of a major institution within the system. Propagation of shocks may be through direct linkages between banks or indirectly, such as through the impact on the asset holdings of many banks caused by the forced sales of a few banks or through a crisis of confidence. The impact of systemic crises on the macroeconomy may take many forms: on the money supply, on the supply of credit, on major market indices, on interest rates, and ultimately on the production economy and the level of employment.

As Admati and Hellwig [3] have argued, ambiguity in the definition of systemic risk implies that mitigation of systemic risk might mean different things to different people. One approach might seek to reduce impact on the financial system, whereas a different approach might instead try to mitigate the damage to the economy at large. These aims do not necessarily coincide: the demise of Lehman Bros. illustrates that key components of the financial system might be sacrificed to save the larger economy during a severe crisis. It is therefore important to have an unambiguous definition of systemic risk supported by a widespread consensus.

1.2.1 Defining SR

The economics literature has used the term *systemic risk* in the context of financial systems for many years. Nonetheless, Kaufman and Scott, Taylor and many others argue that there is as yet no generally accepted definition of the concept, and furthermore, that without an agreed definition, it may be pointless and indeed dangerous to implement public policy that explicitly aims to reduce systemic risk. To see that there is no consensus definition over the years, consider the following examples of definitions proposed in the past.

1. Mishkin 1995 [68]: “the likelihood of a sudden, usually unexpected, event that disrupts information in financial markets, making them unable to effectively channel funds to those parties with the most productive investment opportunities.”
2. Kaufman 1995 [60] “The probability that cumulative losses will accrue from an event that sets in motion a series of successive losses along a chain of institutions or markets comprising a system... That is, systemic risk is the risk of a chain reaction of falling interconnected dominos.”
3. Bank for International Settlements 1994 [41] “the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties.”
4. Board of Governors of the Federal Reserve System 2001 [75] “In the payments system, systemic risk may occur if an institution participating on a private large-dollar payments network were unable or unwilling to settle its net debt position. If such a settlement failure occurred, the institution’s creditors on the network might also be unable to settle their commitments. Serious repercussions could, as a result, spread to other participants in the private network, to other depository institutions not participating in the network, and to the nonfinancial economy generally.”

In the light of the 2007–2008 financial crisis, the above style of definitions, deficient as they are in several respects, can be seen to miss or be vague about one key attribute of any systemic crisis, namely that it also causes damage outside the network, through its failure to efficiently perform its key function of providing liquidity, credit and services. S.L. Schwarcz’ definition [77] of systemic risk explicitly includes this important aspect:

Systemic risk: a definition The risk that (i) an economic shock such as market or institutional failure triggers (through a panic or otherwise) either (X) the failure of a chain of markets or institutions or (Y) a chain of significant losses to financial institutions, (ii) resulting in increases in the cost of capital or decreases in its availability, often evidenced by substantial financial-market price volatility.

While the Schwarcz definition is hardly elegant in its phrasing, it has received support from a rather broad range of practitioners. We will therefore accept it as the closest thing we have to a concise definition of the spirit of systemic risk.

If this definition captures much of the spirit of systemic risk, it fails to address how to measure or quantify the level of systemic risk, and how it might be distributed over the network. Much of current research on systemic risk is dedicated to defining measures of systemic risk and identifying where it is concentrated. Some of the important concepts are *counterparty value at risk (CoVaR)* introduced by Adrian and Brunnermeier [4]; *systemic expected shortfall* introduced by Acharya et al. [2]; and *marginal expected shortfall* introduced by Acharya et al. [83]. For a recent and comprehensive review of these and many other systemic risk measures, please see [12].

1.2.2 Haldane's 2009 Speech

In 2009, in the aftermath of the crisis, Andrew G. Haldane, Executive Director of Financial Stability at the Bank of England, gave a provocative and visionary talk, entitled "Rethinking the Financial Network" [49]. In this brilliant summary of the nature of networks, he compares the 2002 SARS epidemic to the 2008 collapse of Lehman Bros, with the aim to inspire efforts to better understand the nature of systemic risk. For a free thinking overview, we cannot do better than summarize the high points of his speech.

In these two examples of contagion events he identifies the following pattern:

- an external event strikes;
- panic ensues and the complex system seizes up;
- *collateral damage* is wide and deep;
- in hindsight, the trigger event was modest;
- during the event itself, dynamics was chaotic.

He claims this type of pattern is a manifestation of any complex adaptive system, and should be the target where we need to direct our attention.

So, in more detail, what went wrong with the financial network in 2008? Haldane identifies two contributing trends: increasing complexity and decreasing diversity. In real world networks these two trends are observed to lead to fragility, and ring alarm bells for ecologists, engineers, geologists. Figure 1.1 illustrates how the global financial network has grown in complexity. Highly connected, heterogeneous networks may be *robust yet fragile*, meaning they may be resistant to average or typical shocks, yet highly susceptible to an attack that targets a highly connected or dominant node.

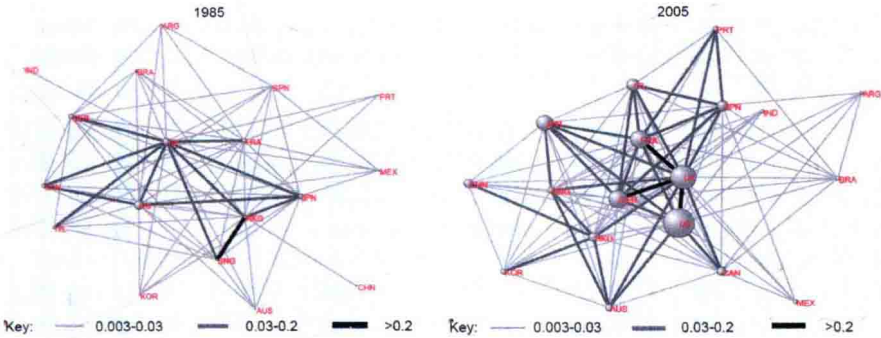


Fig. 1.1 The global financial network in 1985 (left) and 2005 (right). Here line thickness denotes link strength as fraction of total GDP (figure taken from Haldane [49])

In such networks, connections that we think of as shock absorbers may turn out to act as shock amplifiers during a crisis. There may be a sharp *tipping point* that separates normal behaviour from a crisis regime. Thus, a network with a fat-tailed *degree distribution* (i.e. where there is a significant number of highly connected nodes) may be robust to random shocks while vulnerable to shocks that preferentially target these highly connected nodes.

In both of Haldane’s examples of contagion events, agents exhibit a variety of behavioural responses that create feedback and influence the stability of the network. In epidemics, two classic responses, “hide” or “flee”, may prevail and the virulence of the event is highly dependent on which behaviour dominates. In a financial crisis, two likely responses of banks are to hoard liquidity or to sell assets. Both responses are rational, but both make the systemic problem worse. Massive government intervention to provide liquidity and restore capital to banks in a timely manner may be needed in order to curtail systemic events.

Financial networks generate chains of claims and at times of stress, these chains can amplify uncertainties about true counterparty exposures. In good times, counterparty risk is known to be small, and thus “Knightian” uncertainty² is small, and in such times we might expect that stability will improve with connectivity. In bad times, counterparty risk can be large and highly uncertain, due to the complicated web and the nature of the links: we then expect stability to decline with connectivity. Financial innovation, particularly *securitization*, created additional instability. As CDOs, MBSs, RMBSs and similar high dimensional products proliferated internationally, they dramatically expanded the size and scope of the precrisis bubble (see [78]). The structure of these contracts was opaque not transparent. They dramatically increased the connectedness and complexity of the network, and moreover adverse selection made them hard to evaluate. As Haldane wrote:

²In *Knightian* terms, *uncertainty* describes modelling situations where probabilities cannot plausibly be assigned to outcomes. On the other hand, *risk* describes situations where uncertainty can be adequately captured in a probability distribution.