Interpreting Gödel

Critical Essays

EDITED BY
Juliette Kennedy

Interpreting Gödel

The logician Kurt Gödel (1906–1978) published a paper in 1931 formulating what have come to be known as his "incompleteness theorems," which prove, among other things, that within any formal system with resources sufficient to code arithmetic, questions exist which are neither provable nor disprovable on the basis of the axioms which define the system. These are among the most celebrated results in logic today. In this volume, leading philosophers and mathematicians assess important aspects of Gödel's work on the foundations and philosophy of mathematics. Their essays explore almost every aspect of Godel's intellectual legacy including his concepts of intuition and analyticity, the Completeness Theorem, the set-theoretic multiverse, and the state of mathematical logic today. This ground-breaking volume will be invaluable to students, historians, logicians, and philosophers of mathematics who wish to understand the current thinking on these issues.

Contributors

John P. Burgess, Michael Detlefsen, Janet Folina, Curtis Franks, Juliette Kennedy, Charles Parsons, Bjorn Poonen, Saharon Shelah, John R. Steel, Jouko Väänänen,

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

Cover illustration: Kurt Gödel (1906–1978).
Photo by Alfred Eisenstaedt / Time Life Pictures / Getty Images.

ISBN 978-1-316-63977-1

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Cover designed by Hart McLeod Ltd

Kennedy *Interpreting Gödel*

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781316639771

First published 2014
First paperback edition 2016

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Interpreting Gödel : critical essays / edited by Juliette Kennedy, University of Helsinki.
pages cm
ISBN 978-1-107-00266-1 (Hardback)
1. Logic, Symbolic and mathematical. 2. Gödel, Kurt. 3. Mathematics–Philosophy.
I. Kennedy, Juliette, 1955– editor of compilation.
QA9.2.158 2014
511.3–dc23 2014007629

ISBN 978-1-107-00266-1 Hardback
ISBN 978-1-316-63977-1 Paperback

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This book is dedicated to the memory of my mother, Poppy Kennedy

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Acknowledgements

I wish to thank Hilary Gaskin of Cambridge University Press for her support of this volume. The volume was completed while I was a member of the School of Historical Studies of the Institute for Advanced Study. I wish to express my gratitude to the Institute and to the School of Historical Studies through its Otto Neugebauer Fund, as well as to the Academy of Finland and the Väisälä Foundation for this support. Finally I wish to thank Marcia Tucker, Librarian of the Historical Studies and Social Science Library of the IAS, and her staff, for all of their cheerful assistance over the years.

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CHAPTER I

Introduction. Gödel and analytic philosophy: how did we get here?

Juliette Kennedy

1 Introduction

It is often said about Kurt Gödel that he was the greatest logician of the twentieth century. His work in mathematical logic, when it does not constitute the very ground out of which its various subfields grew and developed, made the continuation of the subject possible at a time when fundamental concepts had not even been identified, and proofs of key theorems – in those cases when they had been stated – had not materialized in anything like their final form. This is not to say that Gödel was intellectually infallible; one could also point to the richness of Gödel's logical milieu. But there is no doubt that a gigantic intelligence had turned to the field of mathematical logic – and how much better off the subject was for it!

Gödel's philosophical work on the other hand, work to which he devoted himself almost exclusively from the mid-1940s until his death in 1978, has not been as well received. Put another way, any praise of Gödel's contributions to the foundations of mathematics has largely been limited to his theorems.[1] Gödel the *philosopher* – and indeed even today it is a matter of debate, whether Gödel can be regarded as a philosopher at all – has traditionally been seen as advocating a crude form of Platonism in his philosophical writings, one entangled with the views of Kant and Leibniz in a way which was seen as philosophically naive and primarily historical; and one which, anachronistically, seemed to give no quarter to what turned out to be the single most important development in twentieth

[1] See for example Boolos's introduction (Gödel 1995, pp. 290–304) to Gödel's posthumously published 1951 Gibbs Lecture ("Some basic theorems of the foundations of mathematics and their philosophical implications," reprinted in Gödel (1995), pp. 304–323):

> What may be found problematic in Gödel's judgement that his conclusion is of philosophical interest is that it is certainly not obvious what it means to say that the human mind... is a Turing machine.

century (analytic) philosophy, namely the so-called linguistic turn inaugurated by Frege, Russell and Moore. To the contrary, Gödel's Platonism, that is to say his various formulations of the view that mathematics is contentual, or in other versions that mathematical truth is bivalent, or in still other versions that mathematical objects enjoy some positive sense of existence, were seen by philosophers – when they did not simply bypass his work – as the antiquarian views of an old-fashioned, albeit great mathematician, untrained in philosophy and nostalgic for the days when the concept of mathematical truth was considered to be beyond criticism – an ironic development in the light of Gödel's actual discoveries.

With this volume we wish to effect a change in the philosophical body politic; to call attention to threads in Gödel's thinking which have turned out to be, in light of the directions in which philosophy has developed since Gödel's time, either newly or persistently important. We wish to reassess Gödel's practice of *philosophy as mathematics*; in a word, to reassess his philosophical work in the light of possibly favorable developments. Recent excursions into mathematical naturalism, for example, to be found in works by Penelope Maddy and others, have brought into the philosophy of mathematics a newly invigorated focus on mathematical practice – a nonnegotiable, core commitment for Gödel. Of course, much of the writing on Gödel's philosophical work has focused on his avowed Platonism. And while there is every reason to expect that Gödel will continue to be a canonical representative of that view in the minds of many philosophers, others have gained philosophical traction in areas of Gödel's writings which are less overtly metaphysical and more oriented toward actual mathematics, set theory in particular, but also other material which is "closer to the ground" mathematically and logically.

Of Gödel's philosophically informed logical work, his Completeness Theorem is a fundamental technical result. But the resurgence of interest in logical consequence places it at the center of contemporary philosophical focus. As Curtis Franks puts it in Chapter 5,

> While the theorem contained in Gödel's thesis is a cornerstone of modern logic, its far more sweeping and significant impact is the fact that, through its position in a network of technical results and applications, the way of thinking underlying the result has come to seem definitive and necessary, to the extent that we have managed to forget that it has not always been with us.

Franks's observation that as far as the concept of logical consequence goes, our world is Gödelian through and through, could equally well apply to